P. Diamond - Lecture Notes

From Microscopic { Fluctuations / Interaction / ⋮ } to

⇒ Macroscopic Transport

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⑨

Zwanzig - Mori

Formalism :

<u>Problem</u> <u>Reduction</u> via

<u>Scale</u> <u>Elimination</u>

→ A General structure ....

→ obtains relaxation/memory function

cf: R. Zwanzig; "Non-Equlbrm Statistical Mechanics"

Rationale:

- 3 'types' of transport problems addressed here:

  → collisional, radiative ⇒ {Boltzmann H-Thm}

  → ordered macroscopic flows ⇒ {Flow structure}
  non-chaotic

  (*) → turbulence, interacting waves, spatio-temporal chaos ....

(*) → { multi-scale, strongly interacting, statistical } chaotic ... → interesting relevant but **hard**

⇒ No straightforward counter-part of Boltzmann - Chapman - Enskog Theory ?!?

→ Usual Approach (Caveat Emptor)

- convert multi-scale problem to kinetics problem (via <u>scale elimination</u>)

↔

- develop analogy with:

→ Brownian motion / Fokker-Planck Theory

$$\partial_t V + \gamma V = \tilde{f}_0$$

→ Boltzmann Equation, etc.

$$\partial_t + v \cdot \nabla f = C(f)$$

here, need extract: $\left\{ \begin{array}{l} \gamma, \tilde{f}_0 \\ C(f) \\ \text{etc.} \end{array} \right.$

→ Scale Elimination / Coarse Graining

Modes

$$\underline{k}_1, \underline{k}_2, \ldots \underline{k}_M \gg \underline{k}_{M+1}, \ldots \underline{k}_N$$

$1/T_i$ ← relaxation rates → $1/T_j$

$$- T_i^{-1} < T_j^{-1} \Rightarrow \text{"slow"} \quad 1 \to M$$
$$\text{"fast"} \quad M+1 \to N$$

idea:

- let fast modes 'equilibrate' / relax (we hope...)

- calculate slow evolution in presence of fast

i.e. → 'slave' fast to slow (adiabatics ...).

→ 2 Paradigmatic Examples

ⓐ Zwanzig - Mori Theory

e.x. concentration transport

slow ← fast

prototype → 1, 2, -- m ∫∫ m+1, --- N

(general, instructive, ... ponderous)

slaving → { relaxation rate, noise

ⓑ 1D Plasma Quasi-linear Theory

e.x. 1D plasma wave turbulence $\begin{cases} \frac{dx}{dt} = v \\ \frac{dv}{dt} = \frac{e}{m} E \end{cases}$

{ Mean Field Theory } → 1 ∫∫ 2 --- N

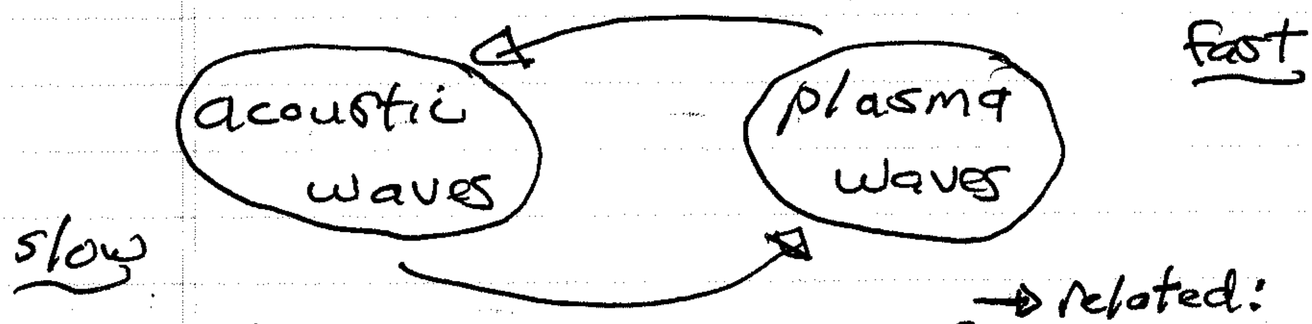
average out all but largest scale slowest

↓

work horse of transport theory → { fluctuation - driven diffusion

ⓒ Plasma Langmuir Turbulence

e.x. Plasma waves + acoustic waves



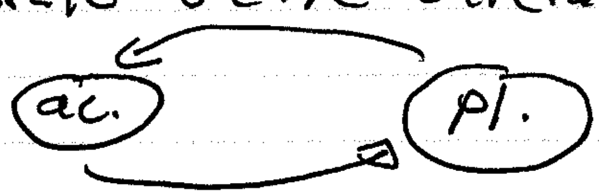
→ related:
{wave ↔ mean flow interaction}

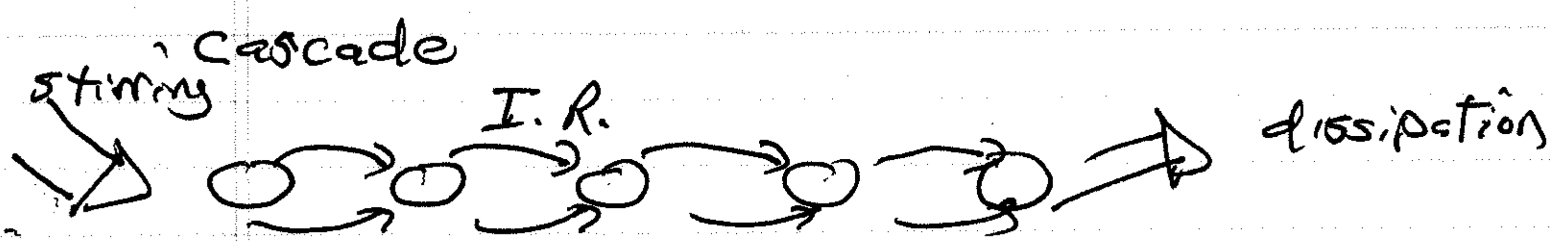
slaving with adiabatic modulation feedback

N.B.: ① ⓒ extends ⓐ, ⓑ

→ allows adiabatic evolution of coarse-grained scales.

② Contrast:
disparate scale interaction





→ Some Issues to Keep in Mind!?

- Irreversibility
  - where from?
    usually no counterpart H-thm.
  - characteristic time scale?

- Scale separation
  - what is required?
  - is dynamics self-consistent with assumed ranges?

- Equilibration/statistics → intermittency?
  - do fast modes in fact "thermalize"?
  - do moments of statistical distribution exist, actually.