Transport by large-scale flows or Large-scale flows in stars

Michel Rieutord

Laboratoire d'Astrophysique de Toulouse-Tarbes,
Observatoire Midi-Pyrénées, France

7 juillet 2010

Outline

Outline

Introduction

Why studying transport in stars ?

- The abundance of elements at the surface of stars cannot be explained by the standard, hydrostatic models
- Rotational mixing seems necessary, although...

Introduction

Why studying transport in stars ?

- The abundance of elements at the surface of stars cannot be explained by the standard, hydrostatic models
- Rotational mixing seems necessary, although...

Introduction
Data from LMC/SMC

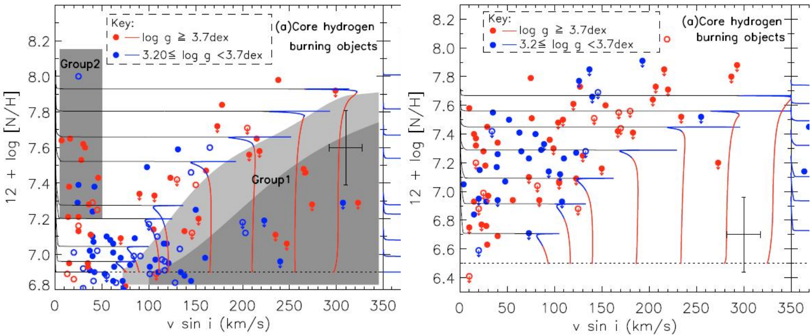


FIG.: Abundance of Nitrogen as a function of the rotational velocity for LMC (left) and SMC(right) (from Hunter et al. 2009).

Introduction

Why studying transport in stars ?

- But transport in stars is either very fast (thermal convection) or very slow (radiative regions).
- The difficulties have been increased by the misconceptions introduced by the Eddington-Sweet mechanism.

Introduction

Why studying transport in stars ?

- But transport in stars is either very fast (thermal convection) or very slow (radiative regions).
- The difficulties have been increased by the misconceptions introduced by the Eddington-Sweet mechanism.

Outline

The equation of the flow

$$\rho\left(\frac{D\vec{v}}{Dt}+\underbrace{2\vec{\Omega}\wedge\vec{v}}_{\mathrm{I}}+\underbrace{\Omega^2\vec{s}}_{\mathrm{II}}\right)=-\vec{\nabla}P+\underbrace{\rho\vec{g}}_{\mathrm{III}}$$

Terms I,II,III characterize the problem

- The Coriolis force $2\vec{\Omega}\wedge\rho\vec{v}$ insures the conservation of angular momentum.
- The centrifugal force $-\Omega^2\vec{s}$ induces the baroclinicity of the set-up.
- The buoyancy force converts density fluctuations into fluid motion.

Outline

The waves

Two sorts of waves arise in for a stably and rotating system

- Inertial waves due to Coriolis
- Gravity waves due to buoyancy

Together they make gravito-inertial (or inertia-gravity) waves where two subgroups are distinguished :

- Rossby waves
- Baroclinic waves

The waves

Two sorts of waves arise in for a stably and rotating system

- Inertial waves due to Coriolis
- Gravity waves due to buoyancy

Together they make gravito-inertial (or inertia-gravity) waves where two subgroups are distinguished :

- Rossby waves
- Baroclinic waves

Inertial waves

Let the density be constant

$$\begin{cases} \rho\dfrac{\partial\vec{u}}{\partial t} + \vec{e}_z \wedge \rho\vec{u} = -\vec{\nabla}P \\ \\ \vec{\nabla}\cdot\vec{u} = 0 \end{cases} \tag{1}$$

If $\vec{u} = \vec{u}(\vec{r})e^{i\omega t}$ then

$$\frac{\omega}{2\Omega} \leq 1$$

Inertial waves are low frequency waves.

Inertial waves
The Poincaré equation

If the velocity is eliminated, pressure perturbations verify :

$$\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} - \left(\frac{4\Omega^2 - \omega^2}{\omega^2}\right)\frac{\partial^2 P}{\partial z^2} = 0$$

which is **Poincaré equation**.

Inertial modes : their singularity

The Poincaré equation is spatially hyperbolic making the eigenmmodes solution of an ill-posed problem. Singularities arise !
Examples :

Inertial modes

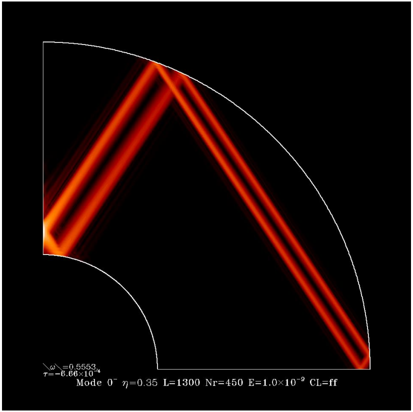


FIG.: A singular inertial mode.

Inertial modes

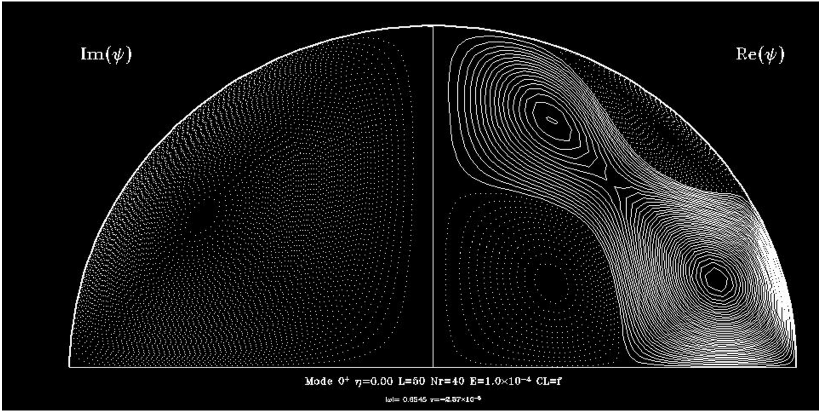


FIG.: A regular inertial mode.

Gravity modes

The Brunt-Väisälä frequency

- This is the natural oscillation frequency of a fluid particle moved away from its equilibrium place. It shows the strength of the buoyancy force :

$$N^2 = \frac{g}{T}\left(\frac{dT}{dr} - \frac{dT_{ad}}{dr}\right) = g\left(\frac{1}{\gamma}\frac{dP}{dr} - \frac{d\rho}{dr}\right)$$

Gravity modes

Using the Boussinesq approximation :

$$\left\{ \begin{array}{l} \dfrac{\partial \delta \vec{v}}{\partial t} = -\dfrac{1}{\rho}\vec{\nabla}\delta p + \dfrac{\delta\rho}{\rho}\vec{g} \\[2ex] \dfrac{\partial \delta T}{\partial t} + \delta\vec{v}\cdot\vec{\nabla}T_0 = 0 \\[2ex] \vec{\nabla}\cdot\delta\vec{v} = 0 \end{array} \right. \tag{2}$$

with $\frac{\delta\rho}{\rho} = -\alpha\delta T$.

Gravity modes

Consider time-periodic disturbances. The pressure fluctuations verify :

$$\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} - \left(\frac{\omega^2}{N^2 - \omega^2}\right) \frac{\partial^2 P}{\partial z^2} = 0$$

Again the Poincaré equation shows up ! Gravity modes are also low frequency modes, namely $\omega < N$.

Gravito-inertial modes

When the fluid is rotating AND stably stratified, gravity and inertial modes combine into gravito-inertial modes. Examples :

Gravito-inertial modes

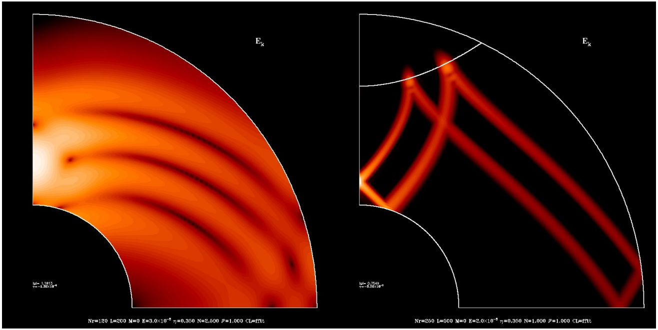


FIG.: A regular (left) and singular (right) gravito-inertial mode.

Rossby waves

They are a sub-class of inertial modes. Namey, inertial modes that meet an additional constraint. Two examples :

- In the Earth atmosphere all inertial modes with $v_z \ll v_x, v_y$
- In stars r-modes invented by Papaloizou et Pringle (1978) where $v_r = 0$

$$v_\theta = Ar^m(\sin\theta)^{m-1}\sin(m\phi + 2\Omega t/(m+1))$$

$$v_\phi = Ar^m(\sin\theta)^{m-1}\cos\theta\cos(m\phi + 2\Omega t/(m+1))$$

Outline

The condition for the existence of a hydrostatic solution

An introduction to baroclinicity

The condition for the existence of a hydrostatic solution

$$
\begin{aligned}
-\vec{\nabla}P - \rho\vec{\nabla}\Phi &= \vec{0} \quad \text{Mechanical equilibrium}\\
\vec{\nabla}\cdot(\chi\vec{\nabla}T) &= 0 \quad \text{Thermal equilibrium}\\
f(P,\rho,T) &= 0 \quad \text{Equation of state}
\end{aligned}
$$

$\vec{\nabla}\Phi$ is prescribed or $\Delta\Phi = 4\pi G\rho - 2\Omega^2$.

Baroclinicity

The mechanical equilibrium is possible if

$$\vec{\nabla} \times \left(\frac{1}{\rho}\vec{\nabla}P\right) = \vec{0} \Longleftrightarrow \vec{\nabla}\rho \wedge \vec{\nabla}P = \vec{0}$$

i.e. the gradients of P and ρ are parallel.

Baroclinicity

But

$$\frac{\partial Eos}{\partial P}dP + \frac{\partial Eos}{\partial \rho}d\rho + \frac{\partial Eos}{\partial T}dT = 0$$

If isobars and isotherms are the same, then on such surfaces

$$\frac{\partial Eos}{\partial T}dT = 0$$

thus

- either $\frac{\partial Eos}{\partial T} = 0$ i.e. $f(P, \rho) = 0$ and the fluid is barotropic
- or $dT = 0$ and isobars are also isotherms (and equipotentials).

Baroclinicity

Generally, $\frac{\partial f}{\partial T} \neq 0$ and isotherms are fixed by thermal equilibrium. They are usually different from isobars.

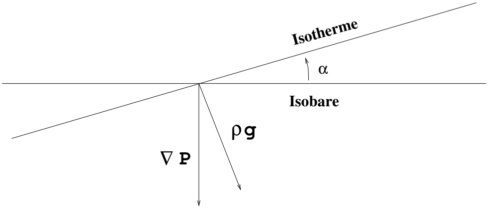
Therefore, usually, on an isobar $dT \neq 0$...

Baroclinicity

Generally, $\frac{\partial f}{\partial T} \neq 0$ and isotherms are fixed by thermal equilibrium. They are usually different from isobars.

Therefore, usually, on an isobar $dT \neq 0$...

Baroclinicity



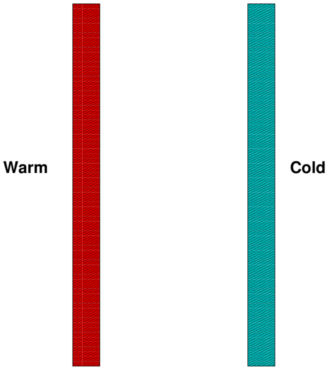
The baroclinic torque

The equation of vorticity

$$\frac{\partial \vec{\omega}}{\partial t} + \vec{\nabla} \times (\vec{v} \cdot \vec{\nabla} \vec{v}) = \frac{1}{\rho^2} \vec{\nabla} \rho \wedge \vec{\nabla} P$$

shows that baroclinicity is giving rise to a torque.

Example : the double-paned window



Double-paned windows

Temperature field

We assume the panes to be infinite and separated by d. The warm one is at $x = -d/2$ and the cold one at $x = d/2$. A steady state for the temperature field implies :

$$\delta T(x) = -(T_c - T_f)x/d$$

$$\delta\rho = \frac{\alpha(T_c - T_f)\rho_0}{d}x$$

Double-paned windows
Velocity field

Let's consider a quasi-incompressible fluid with small amplitude motions.

$$\begin{cases} -\vec{\nabla}\delta P + \delta\rho\vec{g} + \mu\Delta\vec{v} = 0 \\ \vec{v}\cdot\vec{\nabla}T_{eq} = \kappa\Delta\delta T \\ \vec{\nabla}\cdot\vec{v} = 0 \\ \dfrac{\delta\rho}{\rho} = -\alpha\delta T \end{cases} \tag{3}$$

In our case $\vec{\nabla}T_{eq} = \vec{0}$ since equibrium is possible only if T is constant.

Double-paned windows
Velocity field

The velocity field must satisfy :

$$\mu\Delta\vec{v} - \vec{\nabla}\delta P = \frac{\alpha(T_c - T_f)\rho_0 g}{d} x\vec{e}_z \qquad \text{and} \qquad \vec{\nabla}\cdot\vec{v} = 0$$

The baroclinic torque is balanced by the viscous torque :

$$\mu\Delta\vec{\omega} = \frac{\alpha(T_c - T_f)\rho_0 g}{d}\vec{e}_x \wedge \vec{e}_z$$

Double-paned windows

The flow

The equations are easily solved for the velocity field and give :

$$v_z(x) = \frac{\alpha(T_c - T_f)\rho_0 g}{24\mu d} x(d^2 - 4x^2) \tag{4}$$

Double-paned windows
Illustration

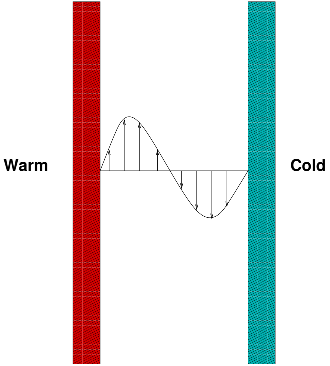


FIG.: The flow within a double-paned windows.

Outline

Second example : the thermal wind

We consider now a baroclinic state of a rotating fluid. Velocities are still weak enough for the problem to be linear.

$$2\vec{\Omega} \wedge \vec{v} = -\frac{1}{\rho_0}\vec{\nabla}\delta P - \alpha\delta T\vec{g}$$

In this case the baroclinic torque is balance by the torque coming from the Coriolis force :

$$\vec{\nabla} \times (2\vec{\Omega} \wedge \vec{v}) = \vec{\nabla} \times (\alpha g\delta T\vec{e}_z)$$

Second example : the thermal wind

We consider now a baroclinic state of a rotating fluid. Velocities are still weak enough for the problem to be linear.

$$2\vec{\Omega} \wedge \vec{v} = -\frac{1}{\rho_0}\vec{\nabla}\delta P - \alpha\delta T\vec{g}$$

In this case the baroclinic torque is balance by the torque coming from the Coriolis force :

$$\vec{\nabla} \times (2\vec{\Omega} \wedge \vec{v}) = \vec{\nabla} \times (\alpha g\delta T\vec{e}_z)$$

The thermal wind

Note that the solution of the thermal wind are degenerate : Any geostrophic flow can be added, because

$$\vec{\nabla} \times (2\vec{\Omega} \wedge \vec{v}_{\rm geo}) = \vec{0}$$

Viscosity removes this degeneracy.

The thermal wind

Example of a specific solution

Let the temperature field be given by :

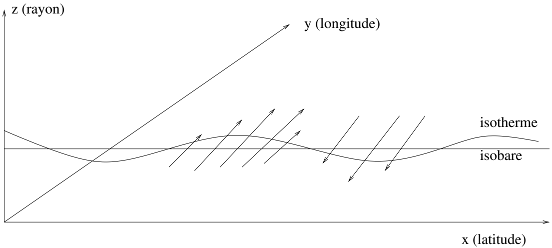
$$\delta T = \delta T_0 \sin(kx)$$

The solution of the flow equations gives

$$\vec{v}_{\rm th} = \left(\frac{\alpha g \delta T_0}{2\Omega}\right) zk \cos kx \vec{e}_y$$

In an inertial frame the distribution of angular has changed because of the baroclinic torque.

The thermal wind



Outline

Outline

Von Zeipel theorem 1924

Von Zeipel shows that mechanical equilibrium is not compatible with radiative equilibrium in a rigidly rotating (non-magnetic) star. If it were true then we would find a paradox !

Let $\Phi = \phi_g + \phi_c$ be the full potential

$$\Delta\phi_g = 4\pi G\rho \quad \text{and} \quad \phi_c = -\frac{1}{2}\Omega^2 r^2 \sin^2\theta$$

Von Zeipel theorem 1924

Von Zeipel shows that mechanical equilibrium is not compatible with radiative equilibrium in a rigidly rotating (non-magnetic) star.
If it were true then we would find a paradox !
Let $\Phi = \phi_g + \phi_c$ be the full potential

$$\Delta\phi_g = 4\pi G\rho \quad \text{and} \quad \phi_c = -\frac{1}{2}\Omega^2 r^2 \sin^2\theta$$

Von Zeipel theorem 1924

If a static equilibrium exists, then

$$P \equiv P(\Phi), \quad \rho \equiv \rho(\Phi), \quad T \equiv T(\Phi)$$

i.e. isobars, isochores, isotherms and equipotentials are identical.
Radiative balance without heat sources demands :

$$\vec{\nabla} \cdot (\chi(\Phi)\vec{\nabla} T(\Phi)) = 0$$

$$\Longleftrightarrow \chi T' \Delta\Phi + (T" + \chi' T')(\vec{\nabla}\Phi)^2 = 0$$

Von Zeipel theorem 1924

Thus

$$\underbrace{\chi(\Phi)T'(\Phi)(4\pi G\rho(\Phi)-2\Omega^2)}_{\mathrm{Cst}}+$$
$$\underbrace{(T''(\Phi)+\chi'(\Phi)T'(\Phi))}_{\mathrm{Cst}}\underbrace{(\vec{\nabla}\Phi)^2}_{\mathrm{notCst}}=0$$

$\vec{\nabla}\Phi$ is just the *effective gravity* not constant on an equipotential ($\vec{g}_{\mathrm{pole}} \neq \vec{g}_{\mathrm{eq}}$).

Von Zeipel theorem 1924

Von Zeipel rightly concludes that the static solution is impossible. In 1925 Vogt and Eddington, independently, suggest the existence of a meridional circulation...

Outline

Les errements d'Eddington-Sweet et de la littérature

Sweet 1950 tried to translated the ideas of Vogt and Eddington into an expression of the flows but did not fully understood these ideas...

Here is the kind of reasoning that can be found in the literature since that time :

Since $\vec{\nabla}\cdot(\chi\vec{\nabla}T)\neq 0$ and since we are looking for a steady solution, the flow is such that :

$$\rho c_v(\vec{v}\cdot\vec{\nabla}T)=\vec{\nabla}\cdot(\chi\vec{\nabla}T)$$

Les errements d'Eddington-Sweet et de la littérature

Sweet 1950 tried to translated the ideas of Vogt and Eddington into an expression of the flows but did not fully understood these ideas...
Here is the kind of reasoning that can be found in the literature since that time :
Since $\vec{\nabla}\cdot(\chi\vec{\nabla}T)\neq 0$ and since we are looking for a steady solution, the flow is such that :

$$\rho c_v(\vec{v}\cdot\vec{\nabla}T) = \vec{\nabla}\cdot(\chi\vec{\nabla}T)$$

which leads to the radial component of the meridional flow :

$$\rho c_v v_r \frac{\partial T}{\partial r} \simeq \vec{\nabla} \cdot (\chi \vec{\nabla} T)$$

while v_θ is derived from mass conservation.

$$\vec{\nabla} \cdot (\rho \vec{v}) = \frac{1}{r^2} \frac{\partial}{\partial r}(r^2 \rho v_r) + \frac{1}{r \sin\theta} \frac{\partial}{\partial \theta}(\sin\theta \rho v_\theta) = 0$$

The problem

Such a velocity field is not a solution of the momentum equation :
Angular momentum is not conserved.
Actually, the role of viscosity is crucial and that was already percieved by Eddington and worked out by Randers 1941.
However, viscosity is weak and its efficiency is counter-intuitive....

The problem

Such a velocity field is not a solution of the momentum equation :
Angular momentum is not conserved.
Actually, the role of viscosity is crucial and that was already percieved by Eddington and worked out by Randers 1941.
However, viscosity is weak and its efficiency is counter-intuitive....

Some landmarks of the literature

Mestel (1961) avoids the problem with magnetic field !
But usually, the Sweet solution is reproduced :

- Rose 1998 *Advance Stellar Physics*
- Kippenhan and Weigert 1990 *Stellar structure and evolution* note the angular momentum problem but does not go further.
- Schatzman et Praderie 1985 *Les étoiles*

The solutions with no circulation

Some authors (Schwarzschild 1947, Baker and Kippenhahn 1959, Roxburgh 1964) noticed that if the differential rotation was correctly adjusted no circulation was necessary to solve the problem.
But these solutions were thought to be unrealistic, probably because their origin could not be identified or thought to be unstable.

Busse 1981 : "Do Eddington-Sweet Circulations exist ?"

Poser la question c'est y répondre ...
Some obvious remarks

- Angular momentum conservation stops any meridional circulation if no other force comes into play.
- If the mechanical balance is fixed and if radiave balance is not insured, the temperature field changes.

Initial conditions

In fact the question is that of initial conditions : one starts from a thermally unbalanced set-up. So

- either we converge to a universal solution after the damping of a transient
- or the transient is very long...

Studying the transient with a simplified model in cartesian geometry

We follow Busse 1981 (GAFD 17, p215) :

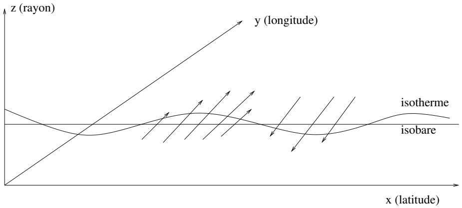


FIG.: The baroclinic set-up

In this set-up the steady-state is the thermal wind solution. Busse wonders about the way this solution arises from arbitrary conditions.

What is the transient flow and which time scale controls its evolution ?

In this set-up the steady-state is the thermal wind solution.
Busse wonders about the way this solution arises from arbitrary conditions.
What is the transient flow and which time scale controls its evolution ?

Busse's solution

The answer given by Busse's model is :
if

$$\vec{v} = \left\{ v(x,z)\vec{e}_y + \vec{\nabla} \times \left(\Psi(x,z)\vec{e}_y\right) \right\} e^{\lambda t}$$

(namely, axisymmetric solutions) ; $\lambda = -\sigma + i\omega$, then

$$\left[(\omega^2 + 2i\omega\sigma)\Delta - 4\Omega^2 \frac{\partial^2}{\partial z^2} - N^2 \frac{\partial^2}{\partial x^2} \left(1 - \frac{i\kappa}{\omega}\Delta \right) \right] \Psi = 0$$

whose solution are gravito-inertial modes

More explicitely

$$\begin{aligned}
\Psi &= \cos(mkx)\sin\left(n\pi(z/d+1/2)\right) \\
V &= \frac{2\Omega n\pi}{i\omega}\cos(mkx)\cos\left(n\pi(z/d+1/2)\right) \\
\theta &= \frac{\beta mk}{i\omega\rho_0}\sin(mkx)\sin\left(n\pi(z/d+1/2)\right) \\
\omega^2 &= \frac{4\Omega^2 n^2 + N^2(mkd/\pi)}{n^2+(kd/\pi)^2} \\
\sigma &= \kappa k^2 m^2 \frac{N^2}{2\omega^2}
\end{aligned}$$

Baroclinic modes

$$
\begin{aligned}
\theta &= \sin(mkx)\sin\left(n\pi(z/d+1/2)\right) \\
V &= \frac{mk\rho\alpha gd}{2\Omega k\pi}\cos(mkx)\cos\left(n\pi(z/d+1/2)\right) \\
\Psi &= \mathcal{O}(\kappa/d^2) \\
\omega^2 &= 0 \\
\sigma &= \frac{\kappa}{d^2}\;\frac{n^2\pi^2+m^2k^2}{1+N^2/4\Omega^2\left(mk/n\pi\right)^2}
\end{aligned}
$$

Comments

which need to be completed by the geostrophic mode and purely thermal disturbances.
The completeness of the Fourier basis insures the unicity of the decomposition of the initial condition.
The evolution of the transient is thus controled by least damped modes, i.e. the large-scale ones.
To get a spherical model, Busse writes the equivalences :

$$d \sim R, \qquad k \sim 2/R$$

Thus two time-scales characterize the damping of the transient :

$$\sigma_{GI} \sim \frac{4\kappa}{R^2} \qquad \sigma_{\rm baro} \sim \frac{(\pi^2+4)\pi^2}{4}\eta\frac{\kappa}{R^2}$$

where $\eta = \frac{4\Omega^2}{N^2} < 1$.
These time-scale are well-known :

- $T_{KH} = \frac{R^2}{\kappa}$ Thermal diffusion time (i.e. Kelvin-Helmoltz time)
- $T_{ED} = T_{KH}/\eta$ Eddington-Sweet time

For rapid rotators

$$\eta \lesssim 1$$

and a steady-state is reach after a KH time.
For slow rotators, $\eta \ll 1$, (e.g. the Sun with $\eta \sim 10^{-4}$), baroclinic modes are vanishing very slowly, therefore initial conditions are crucial.

Meridional circulation ?

During the transient can we face a meridional circulation ?

- Yes thanks to gravito-inertial modes, but they disappear after a few T_{KH}.
- if $\eta \ll 1$, baroclinic modes are present but their meridional circulation is very weak :

$$(v_r, v_\theta) \sim \frac{\kappa}{\Omega R^2} v_\phi \ll v_\phi$$

Ekman layers

- To complete the picture Busse considers the Ekman layers as other possible sources of circulation.
- He finds that they are still weaker.
- He concludes with a discussion of the stability of the baroclinic flow.

To summarize, Busse shows that a self-consistent model of a steady state of a stably stratified rotating fluid works in the following way :

Ekman layers

- To complete the picture Busse considers the Ekman layers as other possible sources of circulation.
- He finds that they are still weaker.
- He concludes with a discussion of the stability of the baroclinic flow.

To summarize, Busse shows that a self-consistent model of a steady state of a stably stratified rotating fluid works in the following way :

Ekman layers

- To complete the picture Busse considers the Ekman layers as other possible sources of circulation.
- He finds that they are still weaker.
- He concludes with a discussion of the stability of the baroclinic flow.

To summarize, Busse shows that a self-consistent model of a steady state of a stably stratified rotating fluid works in the following way :

Ekman layers

- To complete the picture Busse considers the Ekman layers as other possible sources of circulation.
- He finds that they are still weaker.
- He concludes with a discussion of the stability of the baroclinic flow.

To summarize, Busse shows that a self-consistent model of a steady state of a stably stratified rotating fluid works in the following way :

Rotation + Stratification

Baroclinic temperature field

Baroclinic torque (mechanical imbalance)

Thermal wind (differential rotation)

Shear

viscosity

Meridional circulation

Eddington-Sweet ?

The final word of Busse :

"... there are no special limits in which the Eddington-Sweet theory provides a correct solution of the basic equations".
Busse 1982, ApJ, **259**, 759

Outline

Zahn's approach 1992

Zahn (1992) worked out Busse's idea, in the frame work of 1D stellar models.
The main idea : if there is a differential rotation, then it is essentially radial because the turbulence of a stably stratified fluid diffuses more efficiently horizontally than vertically.

The model solves three equations

- Advection-diffusion of angular momentum

$$\frac{1}{5r^2}\frac{\partial}{\partial r}(\rho r^4 U\Omega) + \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^4\nu_h\frac{\partial\Omega}{\partial r}\right) = 0$$

where $v_r = U(r)P_2(\cos\theta)$

- Baroclinicity

$$-\frac{\vec{\nabla}\rho\wedge\vec{\nabla}P}{\rho^2} = \vec{\nabla}\Omega^2\wedge s\vec{e}_s$$

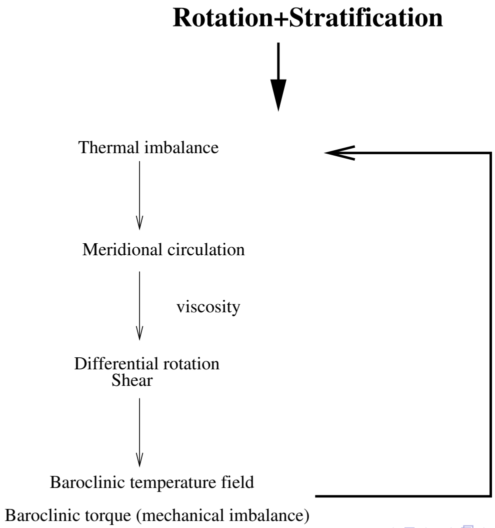
- Thermal imbalance

$$\vec{\nabla}\cdot(\chi\vec{\nabla}T) + \rho\varepsilon = \rho c_v\vec{v}\cdot\vec{\nabla}T$$

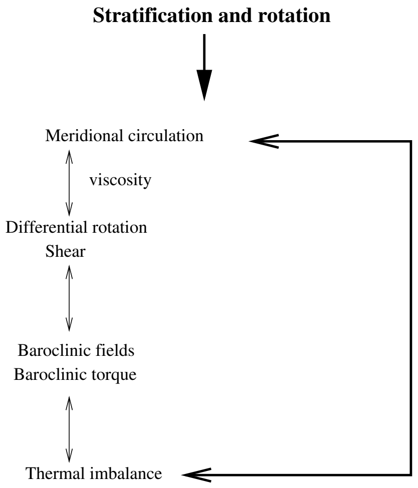
or

$$U(r) = \frac{L}{Mg}\frac{P}{C_p\rho T}\left(\frac{E_\Omega + E_\mu}{\nabla_{\rm ad} - \nabla}\right)$$

A schematic view



or the other way round



Important arrows

Rotation and stratification

↓

Thermal imbalance ←

↓

Meridional circulation

↓ viscosity

Differential rotation
Shear

↓

Baroclinic fields
Baroclinic torque

Something not to do

Some authors are keen on leaving aside the last two branches, reasoning this way :

$$\frac{d\Omega}{dr} = 0 \implies E_\Omega = 2\left(1 - \frac{\Omega^2}{4\pi G\rho} - \frac{\varepsilon}{\varepsilon_m}\right)\frac{\tilde{g}}{\overline{g}} \neq 0$$

$$\implies \quad U(r) \neq 0$$

thus reproducing the inconsistencies of Sweet 1950...

Are slow rotators a good alibi ?

To justify the use of Sweet's reasoning, it is often said that only slow rotators are considered, for

$$t_{\mathrm{ES}} = \frac{t_{\mathrm{KH}}}{\varepsilon} \gg t_{\mathrm{nuc}}$$

For instance, Osaki 1982 writes that the meridional circulation is coming from a very very long transient...
However, the temperature field evolves on a KH time-scale so as to verify $\vec{\nabla} \cdot (\chi \vec{\nabla} T) = 0$.

Baroclinic modes are damped very slowly, but their meridional circulation is very small. Their main effect is to start an appropriate differential rotation.
Finally, the meridional flow in such stars is very much sensitive to the initial conditions, especially to the slightest differential rotation.

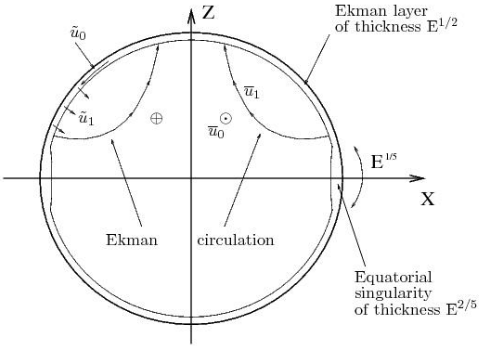
Conclusion

Rotational mixing in slow rotators cannot be discussed without a precise knowledge of their rotation history.

Outline

The spin-down problem

In a sphere



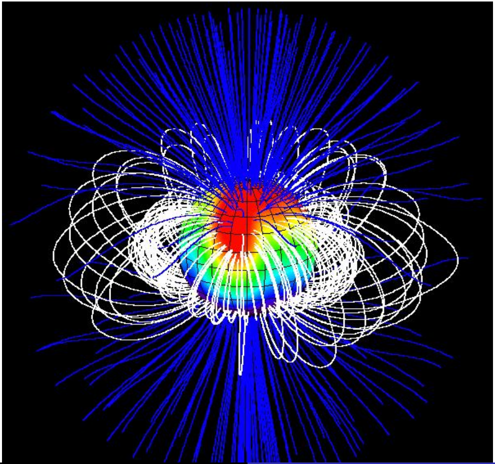
The characteristic time is

$$T \sim (\delta\Omega E^{1/2})^{-1}$$

which is controlled by Ekman circulation : it transfers angular momentum from the walls to the interior.

The case of stars

Mass loss coupled with magnetic fields induces important losses of angular momentum : this is Magnetic Braking.



Magnetic Braking

Ferraro's law

Let's assume the magnetic field be frozen in the star and that everything is in a steady state. Then

$$\vec{\nabla} \times (\vec{v} \wedge \vec{B}) = \vec{0}$$

If the velocity field is just a (differential) rotation, namely

$$\vec{v} = r \sin\theta \Omega(r, \theta) \vec{e}_\varphi$$

and the magnetic field is axisymmetric, $\vec{B} \equiv \vec{B}(r, \theta)$ then

$$(\vec{B} \cdot \vec{\nabla})\Omega = 0$$

which is Ferraro's law, saying that Ω is constant along field lines (Ferraro 1937).

Magnetic Braking
Implication

A new time scale appears in the star's dynamics

$$\frac{L}{\dot{L}}$$

which needs to be compared to the time scale with which angular momentum is carried by flows inside the star, namely $t_{\rm ES}$,

$$t_{\rm ES} < \frac{L}{\dot{L}} \qquad \text{weak wind}$$

$$t_{\rm ES} > \frac{L}{\dot{L}} \qquad \text{strong wind}$$

The weak wind

A torque is exerted on the star but its dynamical reaction is quick : meridional circulation adjusts almost immediatly :

$$\frac{dL}{dt} \sim \rho R^4 \Omega U$$

The angular momentum loss drives the meridional flow, and thus the mixing.

Strong wind

The meridional circulation due to baroclinicity cannot cope with the loss of angular momentum $\Longrightarrow$ the differential rotation increases and shear layers arise. Circulation now looks like Ekman pumping in a stratified fluid, controlled by a turbulent boundary layer...

Some words of conclusions

- The main effect of baroclinicity is differential rotation.
- In a steady state the meridional circulation is weak effect due to viscosity.
- All terms in the flow expression are important...
- The Eddington-Sweet model is never right.
- Using Zahn's model, do not truncate it !

Some words of conclusions

- The main effect of baroclinicity is differential rotation.
- In a steady state the meridional circulation is weak effect due to viscosity.
- All terms in the flow expression are important...
- The Eddington-Sweet model is never right.
- Using Zahn's model, do not truncate it !

Some words of conclusions

- The main effect of baroclinicity is differential rotation.
- In a steady state the meridional circulation is weak effect due to viscosity.
- All terms in the flow expression are important...
- The Eddington-Sweet model is never right.
- Using Zahn's model, do not truncate it !

Some words of conclusions

- The main effect of baroclinicity is differential rotation.
- In a steady state the meridional circulation is weak effect due to viscosity.
- All terms in the flow expression are important...
- The Eddington-Sweet model is never right.
- Using Zahn's model, do not truncate it !

Some words of conclusions

- The main effect of baroclinicity is differential rotation.
- In a steady state the meridional circulation is weak effect due to viscosity.
- All terms in the flow expression are important...
- The Eddington-Sweet model is never right.
- Using Zahn's model, do not truncate it !

A final illustration

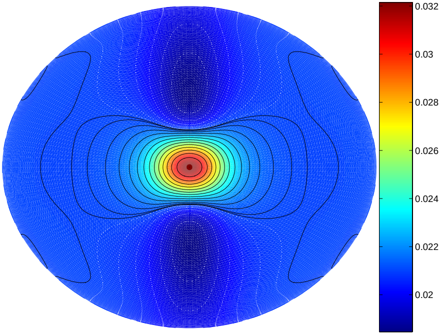


FIG.: A completely radiative star rotating at 80% of the critical velocity (from Rieutord and Espinosa Lara 2009).

END