Stoked Dynamos

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Project Question

- Can a nondynamo be supplied with magnetic field and made to look like a dynamo?

Outline

- Dynamo characteristics and governing equations
- Classification of dynamo action
- Magnetic feeding (stoking)
- Preliminary Results
- Speculation and Future Work

Dynamo Properties

- Dynamo action involves the amplification of magnetic fields and the maintenance of fields against dissipation
- Invoked to explain the origin of magnetic fields in the Universe and their persistence over cosmological timescales
- Amplification may be treated as a kinematic problem, whereas field maintenance requires solving the fully nonlinear MHD problem

$$\left(\partial_t - R_e{}^{-1}\nabla^2\right)\vec{U} + \vec{U}\cdot\nabla\vec{U} = -\nabla p + \vec{J}\times\vec{B} + \vec{F}$$ momentum equation

$$\left(\partial_t - R_m{}^{-1}\nabla^2\right)\vec{B} = \nabla\times\left(\vec{U}\times\vec{B}\right)$$ induction equation

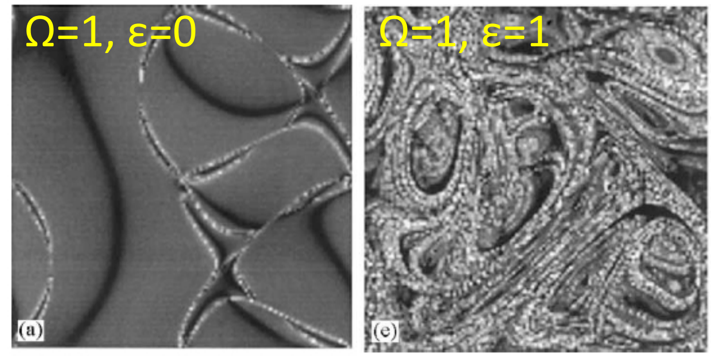
$$\nabla\cdot\vec{B} = \nabla\cdot\vec{U} = 0$$ divergence-free, incompressibility

- Transition to nonlinear regime when Lorentz forces become sufficient to react back on the flow

Classification of Dynamo Action

- Linear and nondynamo action of a modified ABC flow investigated by Brummell et al. 1998, 2001

$$\vec{U}_0(\vec{x},t) = \left[\sin(z+\varepsilon\sin\Omega t)+\cos(y+\varepsilon\sin\Omega t)\right],$$
$$\left[\sin(x+\varepsilon\sin\Omega t)+\cos(x+\varepsilon\sin\Omega t)\right],$$
$$\left[\sin(y+\varepsilon\sin\Omega t)+\cos(x+\varepsilon\sin\Omega t)\right]$$



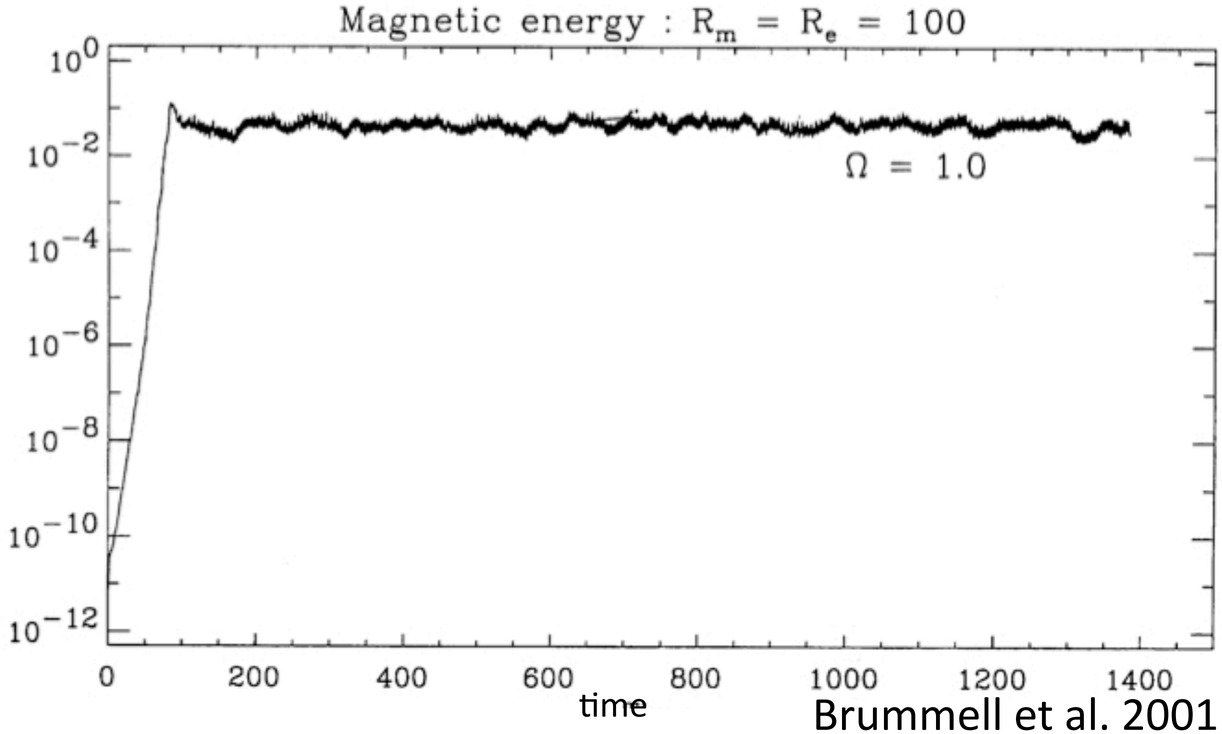
- Choose amplitude ε=1 so that chaotic streamlines occupy most of the volume, vary driving frequency Ω
- Forcing term of the momentum equation defined so as to drive U_0:

$$\vec{F}_0(\vec{x},t) = \left(\partial_t - R_e^{-1}\nabla^2\right)\vec{U}_0(x,t)$$

- Initial conditions: $U(0)=U_0(0)$, weak random seed field

Dynamo Solution: $\Omega = 1.0$

- Clear kinematic phase of exponential growth of magnetic energy, followed by saturation and settling down of solution into a statistically steady state

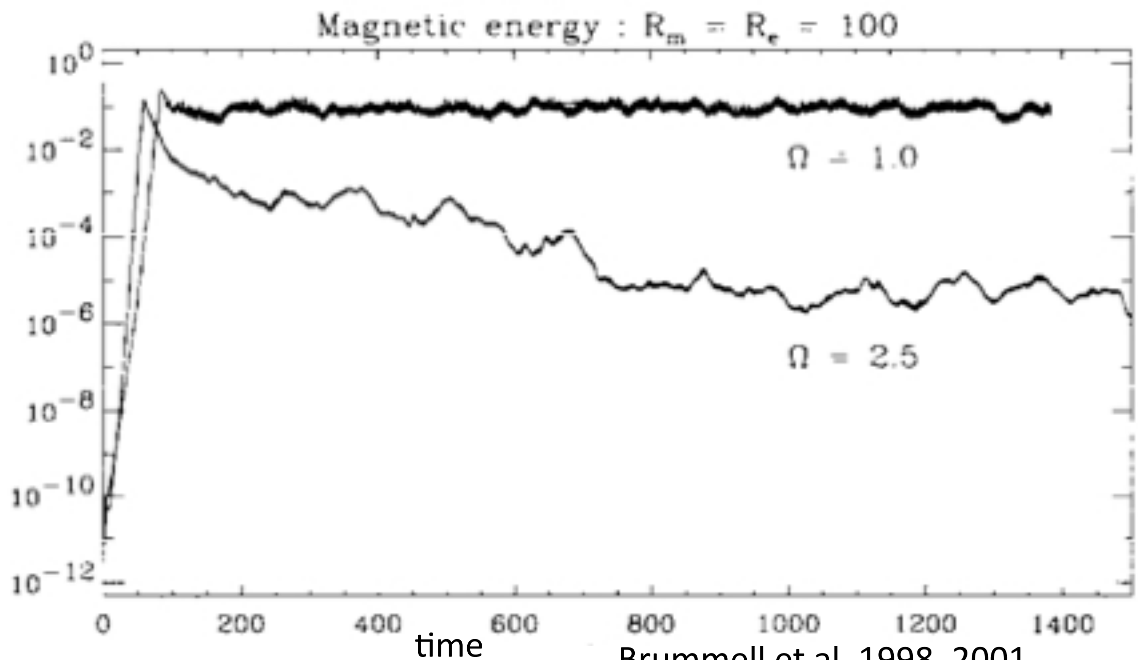


Brummell et al. 2001

- This is the traditional view of the operation of a dynamo

Nonlinear Nondynamo Solution: $\Omega = 2.5$

- Observe initial kinematic amplification
- But instead of saturation see decay in the nonlinear regime

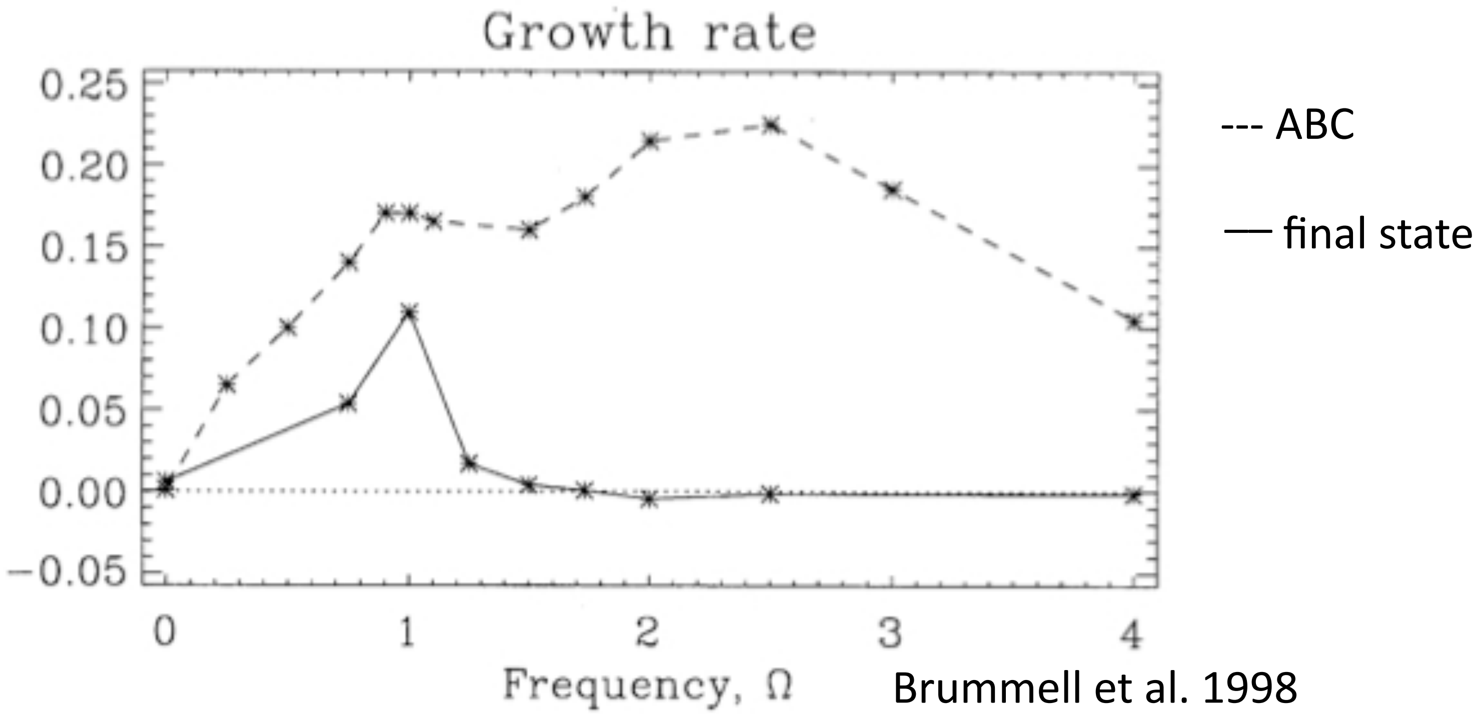


Brummell et al. 1998, 2001

- Ultimately the $\Omega = 2.5$ flow is not a dynamo, since the magnetic field is not maintained for all time
- System eventually relaxes into a pure hydrodynamic state that is not the original flow

Comparison of Initial and Final Flow States

- Kinematic growth rates obtained using final flow state as initial conditions are smaller than those obtained using the ABC initial conditions with the same forcing

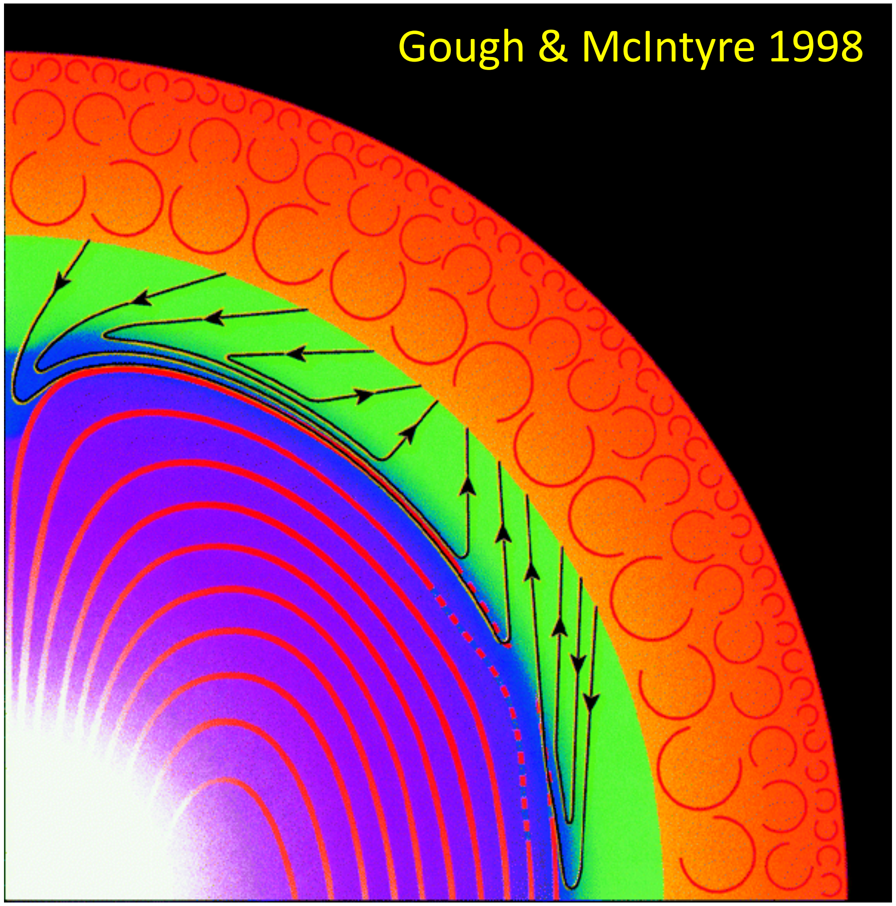


Brummell et al. 1998

- A particular forcing has multiple solutions, not all of which are necessarily dynamos

Magnetically-fed (Stoked) Dynamos?

- What is a stellar dynamo? (Cattaneo, Hughes, & Weiss 1991)
- Magnetic cycles could be produced by an oscillating field, as well as a dynamo
- Stochastic forcing of induction equation
  (Farrell & Ioannou 1999)
- Is the Solar dynamo self-sustaining?
- Or does it rely on being fed by field dredged up by an upwelling tachocline flow?
  (Gough 2007)



Magnetic Feeding

- Introduce magnetic field into the computational box by forcing the induction equation to drive a given field profile in the absence of induction

$$\left(\partial_t - R_m^{-1}\nabla^2\right)\vec{B} = \nabla \times \left(\vec{U} \times \vec{B}\right) + \vec{F}_B$$

where $\vec{F}_{B0}\left(\vec{x},t\right) = \left(\partial_t - R_m^{-1}\nabla^2\right)\vec{B}_0\left(\vec{x},t\right)$

- The forcing field should be added in such a way that it provides zero net flux, will not diffuse too fast, and such that the fluid does the driving
- Use the following simple functional form

$$\vec{B}_0\left(\vec{x},t\right) = B_0 \sin\omega t \sin kz\left[\sin ky, 0, 0\right]$$

Numerical Simulation Details

- Solve the MHD equations in a triply periodic box of size 2π using pseudospectral methods
- Resolution of 96^3 Fourier modes
- Re = Rm = 100

- Forcing of momentum equation using modified ABC flow (Brummell et al. 1998, 2001) with $\varepsilon = 1$ and varying Ω
- Forcing of induction equation using our chosen field profile, with ω = 1e-4, $k = 1$, and varying B_0
- Ran on Grape (UCSC), Pleiades (UCSC), and Kraken (NICS)

Three Numerical Experiments

What is the effect of supplying the following systems with magnetic field?

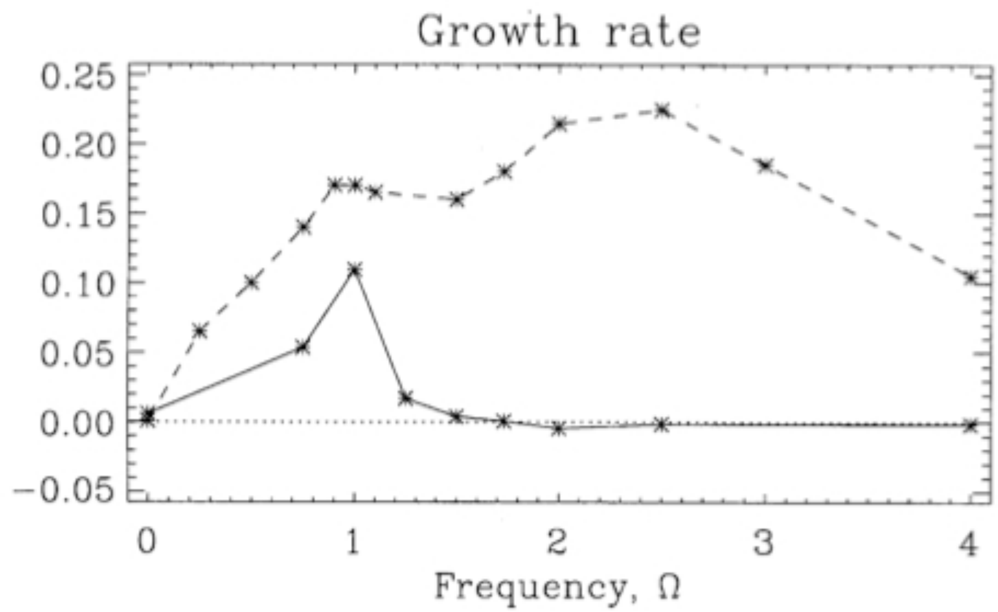
1) marginal kinematic dynamos and nondynamos

2) nonlinear nondynamos

3) traditional dynamos

1) Forcing of Marginal Kinematic Dynamos

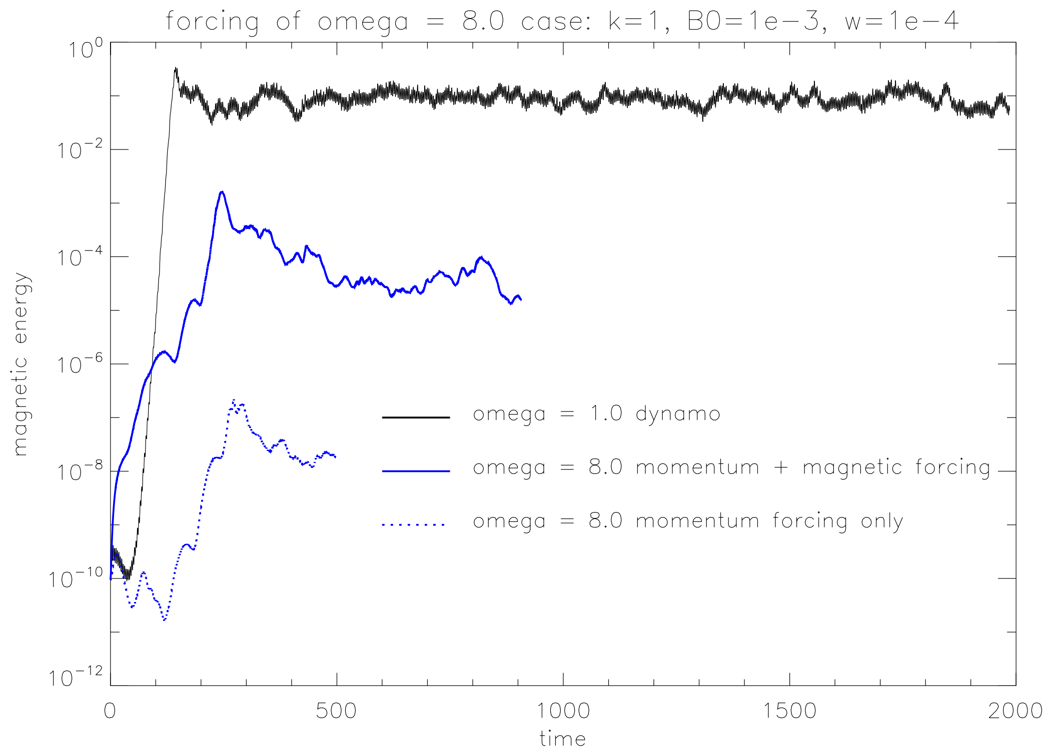
- Find the value of Ω at which the kinematic growth rate is negative



- What is the effect of magnetic forcing either side of this value of Ω?

Magnetic Forcing of the Ω = 8 Case

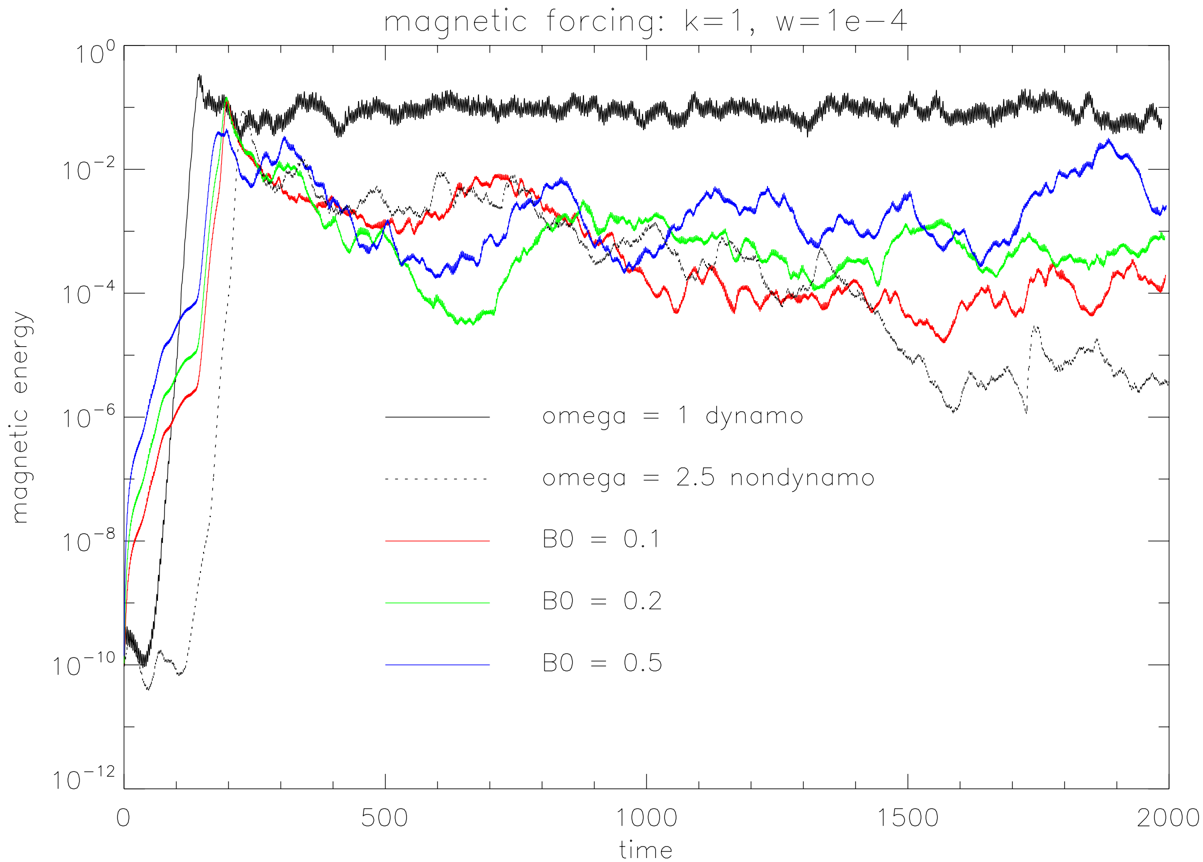
- Still need to find the critical value of Ω with the intention of forcing marginal kinematic dynamos and nondynamos
- For now we have magnetically forced the Ω=8 case



- See a clear boost in the magnetic energy when apply magnetic forcing

2) Forcing of Nonlinear Nondynamos

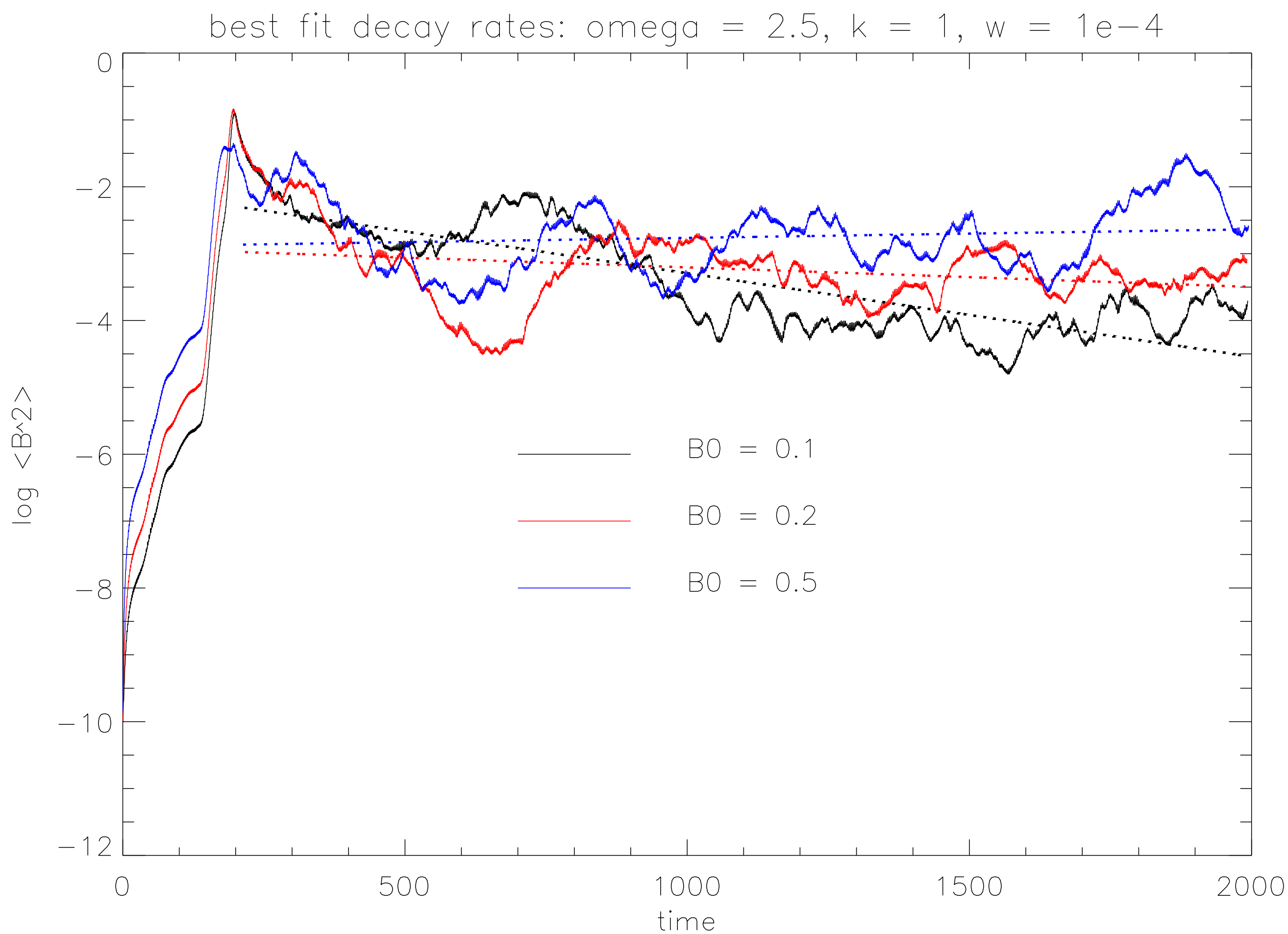
- Magnetic forcing of Ω = 2.5 nonlinear nondynamo with various forcing amplitudes



- Increasing the amplitude appears to arrest the decay of magnetic energy

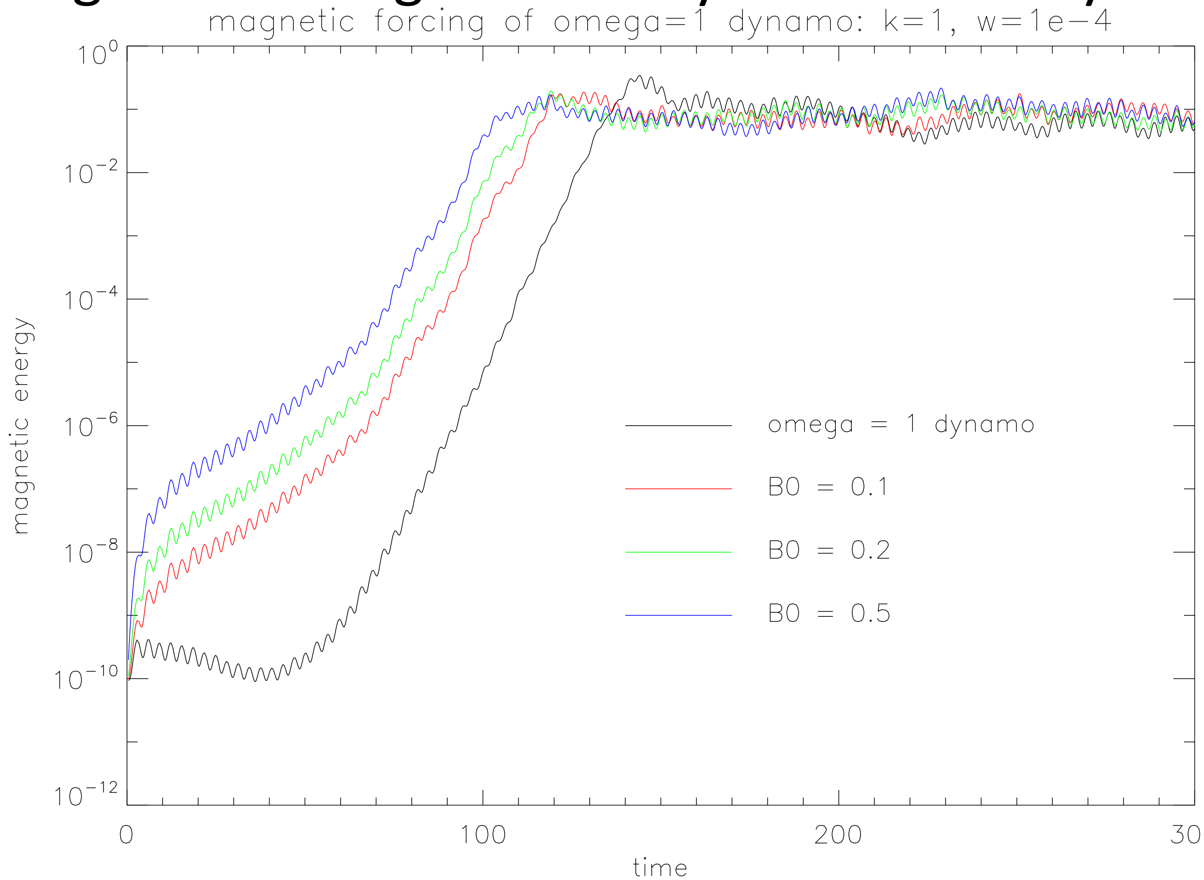
Magnetic Energy Decay Rates

- A more quantitative analysis is pending, but initial results suggest that for $B_0 > 0.2$ magnetic energy may be sustained



3) Magnetic Forcing of Dynamos

- Apply magnetic forcing to Ω = 1 dynamo and vary amplitude



- See some variation early in the kinematic regime, but the forcing appears to make little difference in the nonlinear regime

Speculation and Future Work

1) Forcing of Marginal Kinematic Dynamos

Magnetic forcing of a marginal kinematic dynamo boosts its magnetic energy

Suspect that magnetic forcing of a kinematic nondynamo will yield a positive growth rate

2) <u>Forcing of Nonlinear Nondynamos</u>

With sufficient magnetic forcing it does seem possible to maintain magnetic energy in the nonlinear regime

Magnetic field is only added linearly, but its effects are able to prevent exponential decay of the magnetic energy

We suggest this is due to "dynamo-like" amplification of the forcing field, perhaps by:

- stretching: $\vec{B}_F \cdot \nabla \vec{U}$
- and/or modification of flow by Lorentz force: $\nabla \times \vec{B}_F \times \vec{B}_F$

3) Forcing of Dynamos

Linear Phase

Find variations with forcing amplitude in the kinematic regime, although final kinematic growth rate seems independent of the forcing

Magnetic eigenfunctions?

Nonlinear Phase

Adding magnetic field seems to make no difference to the magnetic energy in the nonlinear regime

Can we detect the signature of the forcing field?