Analytical studies of isothermal sphere collapse

Evangelia Ntormousi, Patrick Hennebelle, Chris Matzner, Matthias Gritschneder

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Outline

- Motivation
- Collapse solutions to the equations of hydrodynamics
- Applying a linear perturbation to a collapse flow solution
- Two methods for calculating the dispersion relation for the perturbation
- Outlook of this project

What is the typical outcome of star formation?

- Stars typically form in binary systems. But how does a binary system form? Can it come from fragmentation of a core as it collapses? More generally, how many stars does one core produce? What determines the result of core collapse?

- Although a lot of observational and numerical work has been done to explore this question, the initial conditions for core collapse and the processes that connect these initial conditions to the outcome of the collapse are not well understood.

- We want to study the collapse of a core analytically, by imposing a linear perturbation to a self-similar collapse solution. This will enable us to pinpoint the processes that could lead to core fragmentation at the initial phases of the collapse.

- Ideally, we would like to study various equilibrium solutions, having varying intrinsic stability.

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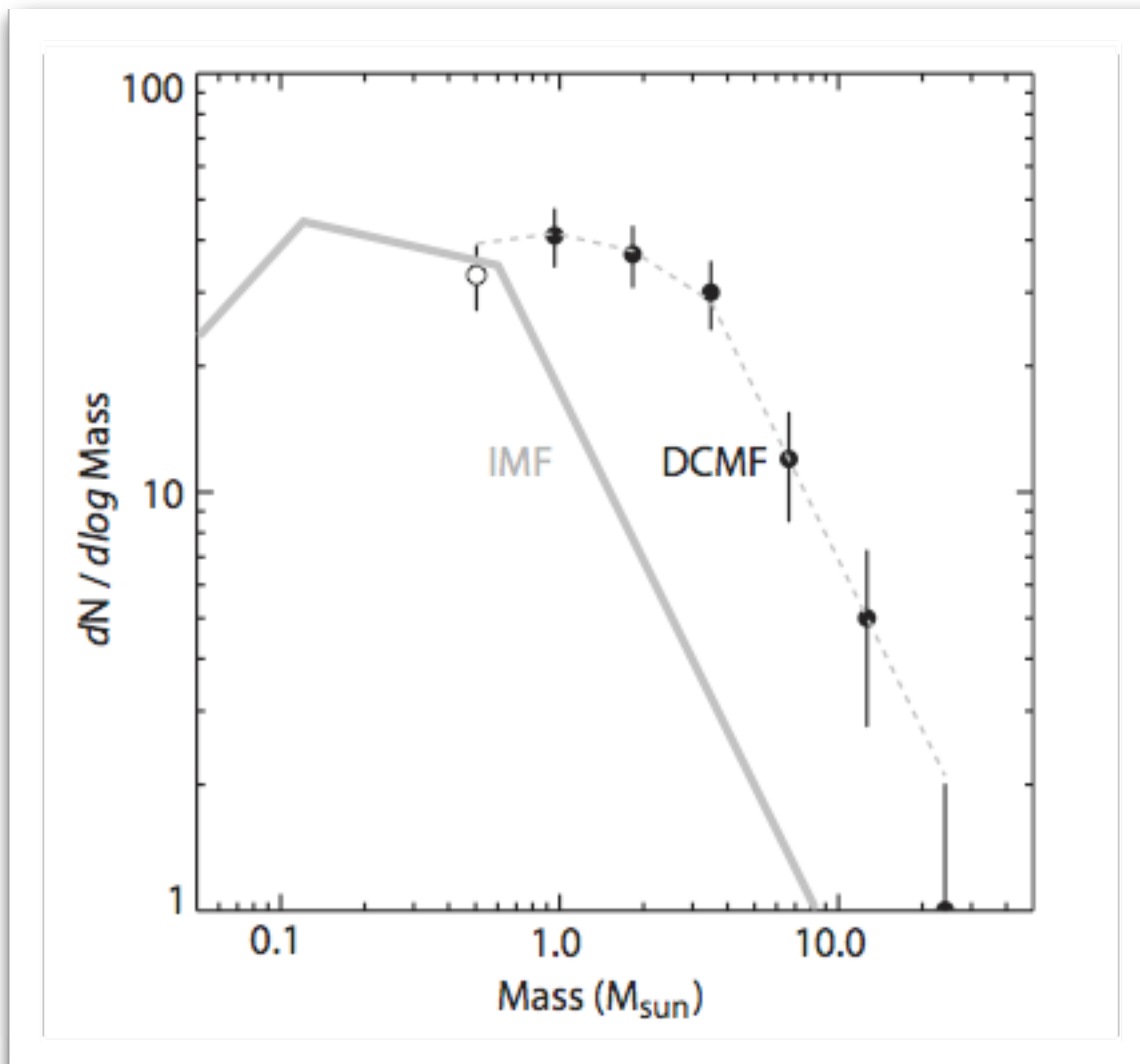

This relates to the origin of the initial mass function: Does one core give one star? If it fragments, how is the mass distributed?

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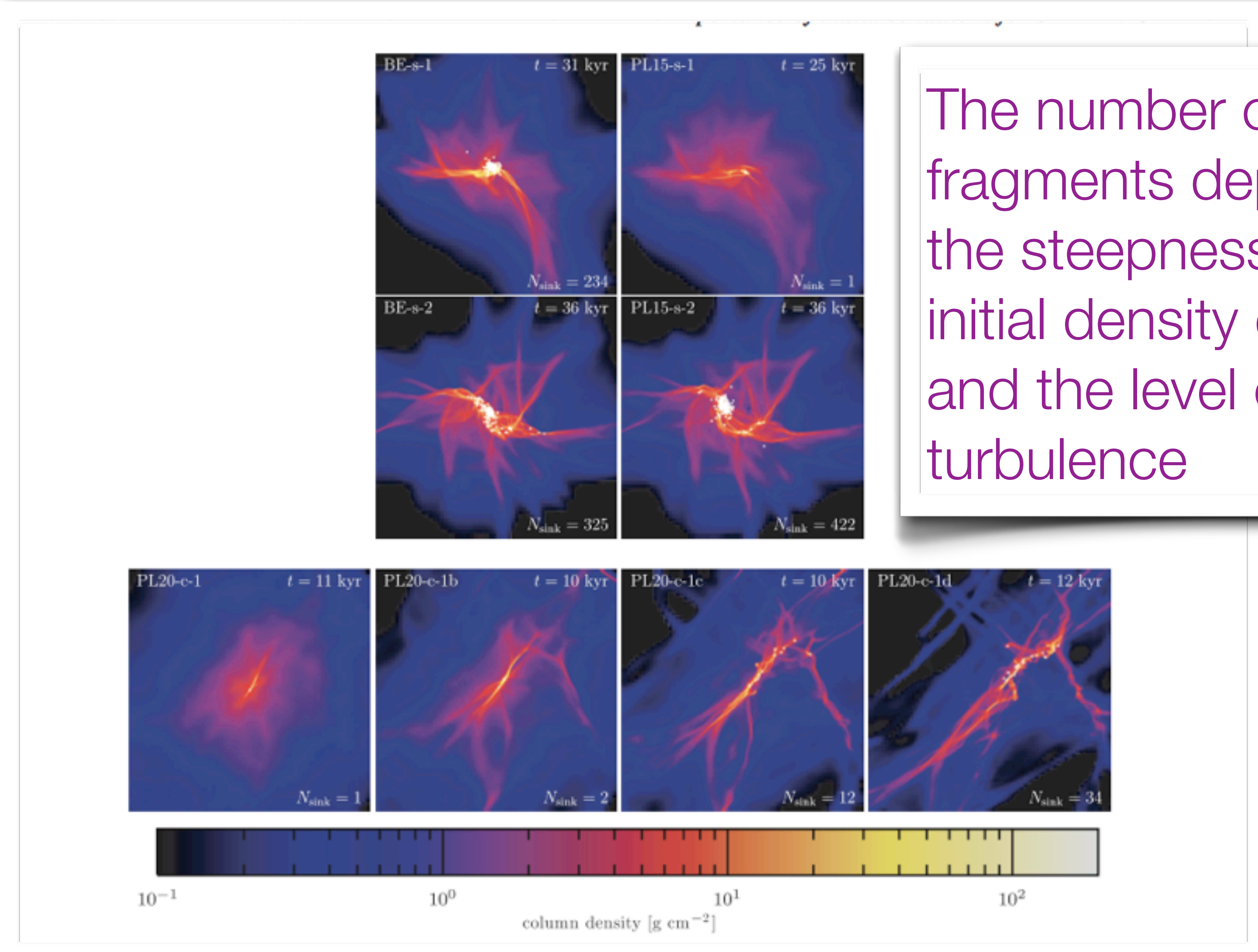

The number of fragments depends on the steepness of the initial density distribution and the level of internal turbulence

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- Althou... to explore this questi... that connect these i... understood.
- We wa... linear perturb... to pinpoint the proces... s of the collaps...

Observations of Young Stellar Objects (ie Maury et al. 2010) suggest that Class 0 (younger) objects show less multiplicity than Class I/II (more evolved) objects. This suggests that fragmentation should occur at the early stages of the collapse

- Ideally, we would like to study various equilibrium solutions, having varying intrinsic stability.

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Analytical collapse solutions:
The Shu self-similar solutions (Shu 1977)

Mass conservation: $$\frac{\partial M}{\partial t} + u\frac{\partial M}{\partial r} = 0\ ,\quad \frac{\partial M}{\partial r} = 4\pi r^2 \rho$$

Force equation: $$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} = -\frac{\alpha}{\rho}\frac{\partial \rho}{\partial r} - \frac{GM}{r^2}$$

Choosing the similarity variable: $$x = r/\alpha t$$

We look for solutions of the form:

$$\rho(r,t) = \frac{\alpha(x)}{4\pi G t^2} \qquad M(r,t) = \frac{\alpha^3 t}{G} m(x) \qquad u(r,t) = \alpha u(x)$$

So the fluid equations become:

$$\left[(x-u)^2 - 1\right]\frac{1}{\alpha}\frac{d\alpha}{dx} = \left[\alpha - \frac{2}{x}(x-u)\right](x-u)$$

$$\left[(x-u)^2 - 1\right]\frac{du}{dx} = \left[\alpha(x-u) - \frac{2}{x}\right](x-u)$$

$$m = x^2\alpha(x-u)$$

Solving these equations gives a family of collapse solutions

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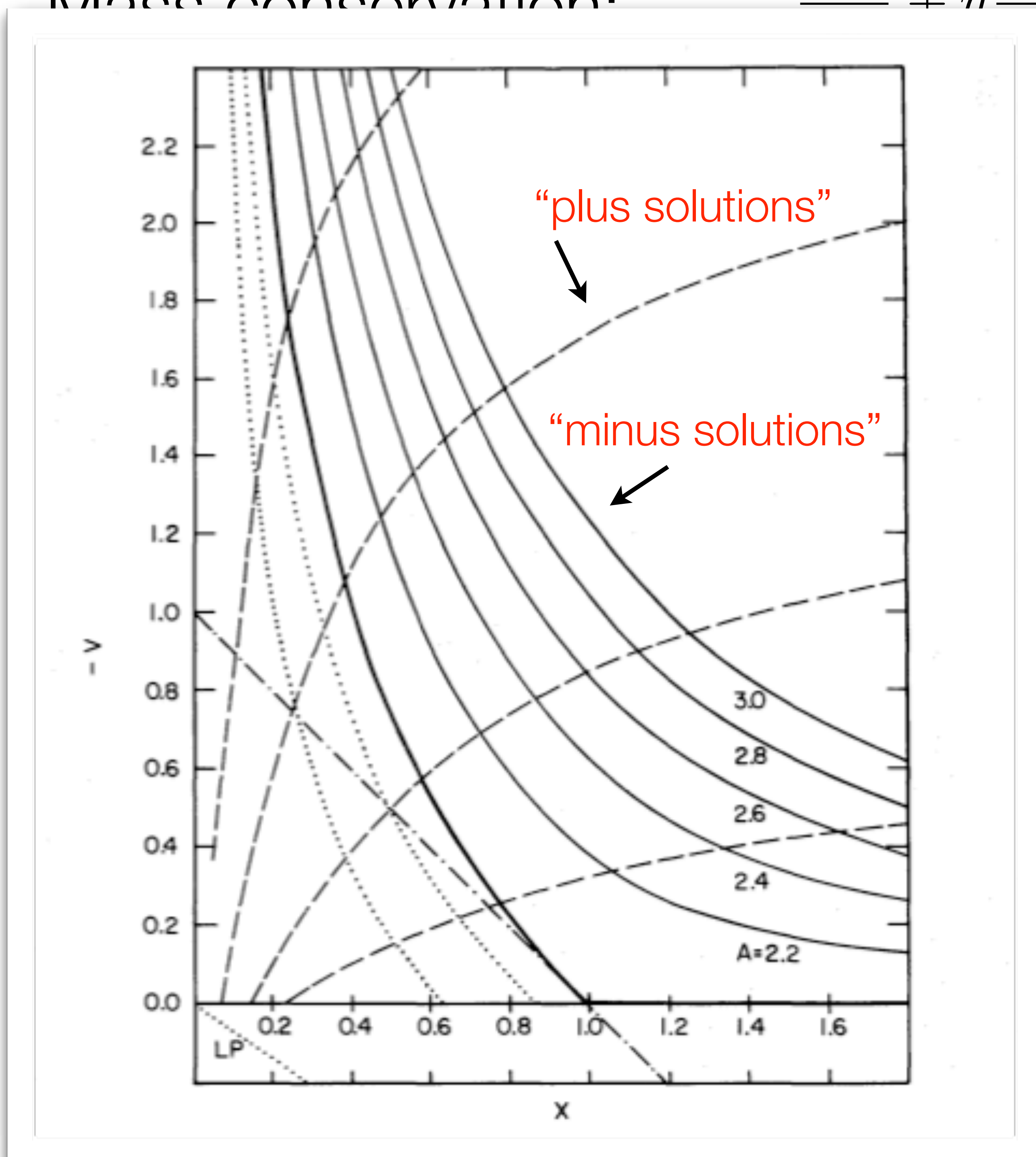


Solutions which have zero velocities at the beginning of the collapse have the asymptotic form:

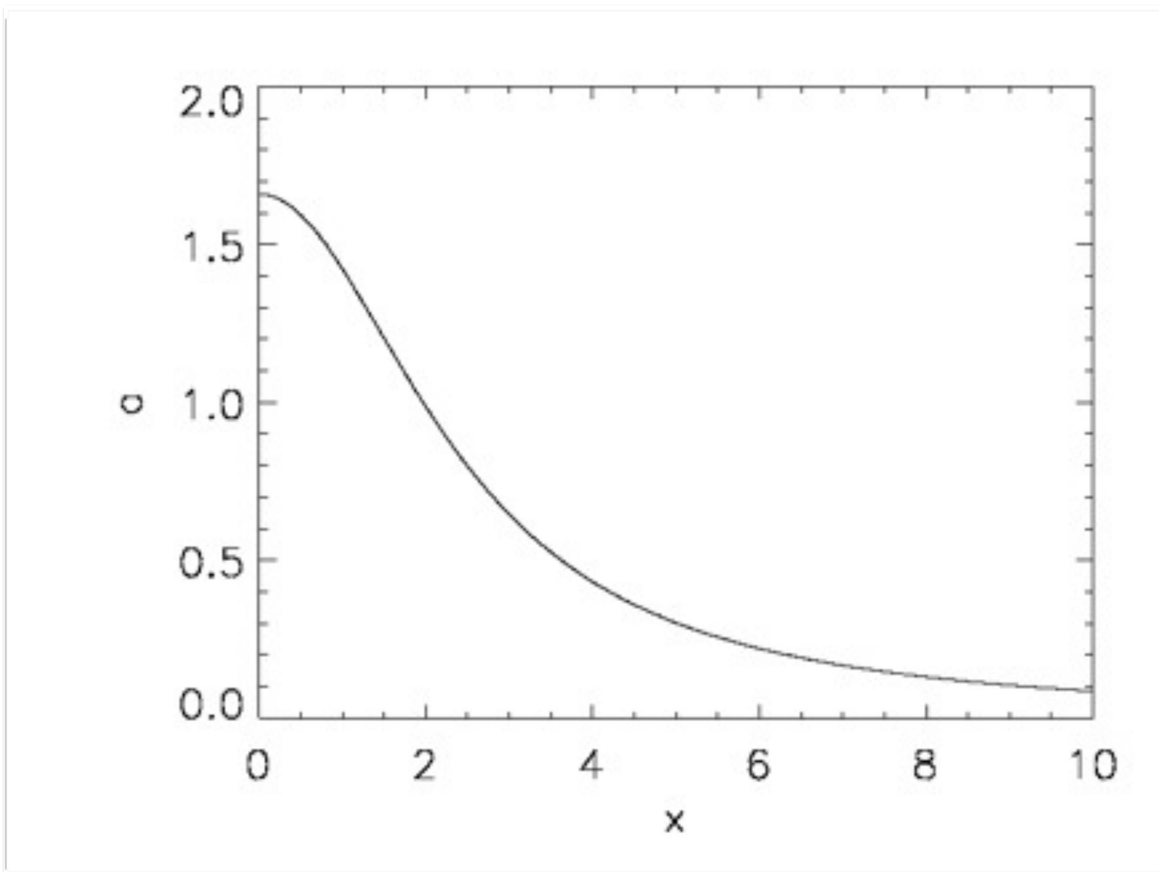
$\alpha \sim A/x^2 \qquad u \sim -(A-2)/x$

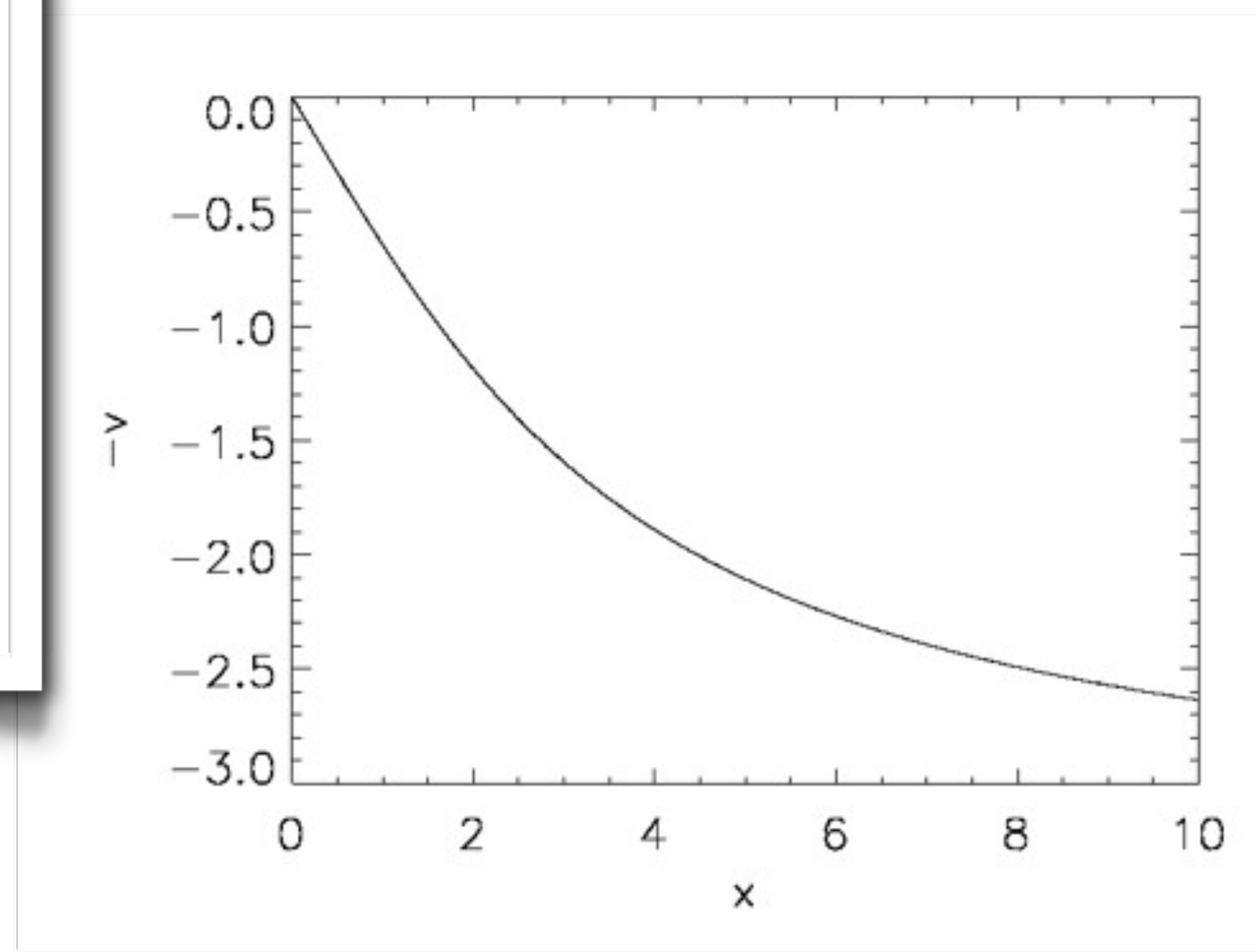
as $\quad x \rightarrow \infty$

Solving these equations gives a family of collapse solutions

Analytical collapse solutions:
The Larson-Penston flow

The Larson-Penston flow (Larson 1969, Penston 1969a) is a special case of the Shu solutions and belongs to the class of solutions which have critical points.





The critical point is located at x=2.33

Applying a linear perturbation to an equilibrium collapse solution: exponentially growing modes only

The equations of hydrodynamics in spherical coordinates:

$$\frac{\partial\rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\rho u_r\right) + \frac{1}{rsin\theta}\frac{\partial}{\partial\theta}\left(sin\theta\rho u_\theta\right) + \frac{1}{rsin\theta}\frac{\partial}{\partial\phi}\left(\rho u_\phi\right) = 0$$

$$\frac{\partial u_r}{\partial t} + u_r\frac{\partial u_r}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_r}{\partial\theta} + \frac{u_\phi}{rsin\theta}\frac{\partial u_r}{\partial\phi} - \frac{u_\phi^2 + u_\theta^2}{r} = -\frac{1}{\rho}\frac{\partial\rho}{\partial r} - \frac{\partial\Psi}{\partial r}$$

$$\frac{\partial u_\theta}{\partial t} + u_r\frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_\theta}{\partial\theta} + \frac{u_\phi}{rsin\theta}\frac{\partial u_\theta}{\partial\phi} + \frac{u_r u_\theta}{r} - \frac{u_\phi^2}{rtan\theta} = -\frac{1}{r\rho}\frac{\partial\rho}{\partial\theta} - \frac{\partial\Psi}{\partial\theta}$$

$$\frac{\partial u_\phi}{\partial t} + u_r\frac{\partial u_\phi}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_\phi}{\partial\theta} + \frac{u_\phi}{rsin\theta}\frac{\partial u_\phi}{\partial\phi} + \frac{u_r u_\phi}{r} = -\frac{1}{r\rho sin\theta}\frac{\partial\rho}{\partial\phi} - \frac{\partial\Psi}{\partial\phi}$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right)\Psi + \frac{1}{r^2 sin\theta}\frac{\partial}{\partial\theta}\left(sin\theta\frac{\partial}{\partial\theta}\right)\Psi + \frac{1}{r^2 sin^2\theta}\frac{\partial^2\Psi}{\partial\phi^2} = \rho$$

Applying a linear perturbation to an equilibrium collapse solution: exponentially growing modes only

The equations of hydrodynamics in spherical coordinates:

We want to apply a perturbation of the form:

$$\rho(r,t) = \frac{R(x)}{t^2} + t^{\sigma-2} r_1(x) Y_l^m = \rho_0 + \delta\rho$$

$$u_r(r,t) = u(x) + t^{\sigma} u_{1r}(x) Y_l^m = u_{r0} + \delta u_r$$

$$u_\theta(r,t) = \frac{1}{l+1} t^{\sigma} u_{1\theta}(x) \frac{\partial Y_l^m}{\partial \theta} = \delta u_\theta$$

$$u_\phi(r,t) = \frac{1}{l+1} \frac{1}{sin\theta} t^{\sigma} u_{1\theta}(x) \frac{\partial Y_l^m}{\partial \phi} = \delta u_\phi$$

$$\Psi(r,t) = \Psi_0(x) + t^{\sigma} \Phi_1(x) Y_l^m = \Psi_0 + \delta\Phi$$

Applying a linear perturbation to an equilibrium collapse solution: exponentially growing modes only

The equations of hydrodynamics in spherical coordinates:

We want to apply a perturbation of the form:

Finally, after some manipulation, the perturbed equations take the form:

$$[(u-x)^2-1]\frac{dr_1}{dx}+\left[\left(\sigma-2+\frac{2u}{x}+\frac{du}{dx}\right)(u-x)+\frac{1}{R}\frac{dR}{dx}\right]r_1+$$

$$\left[\left(\frac{2R}{x}+\frac{dR}{dx}\right)(u-x)-R\left(\sigma+\frac{du}{dx}\right)\right]u_{1r}-\frac{lR}{x}(u-x)u_{1\theta}-R\frac{d\Phi}{dx}=0$$

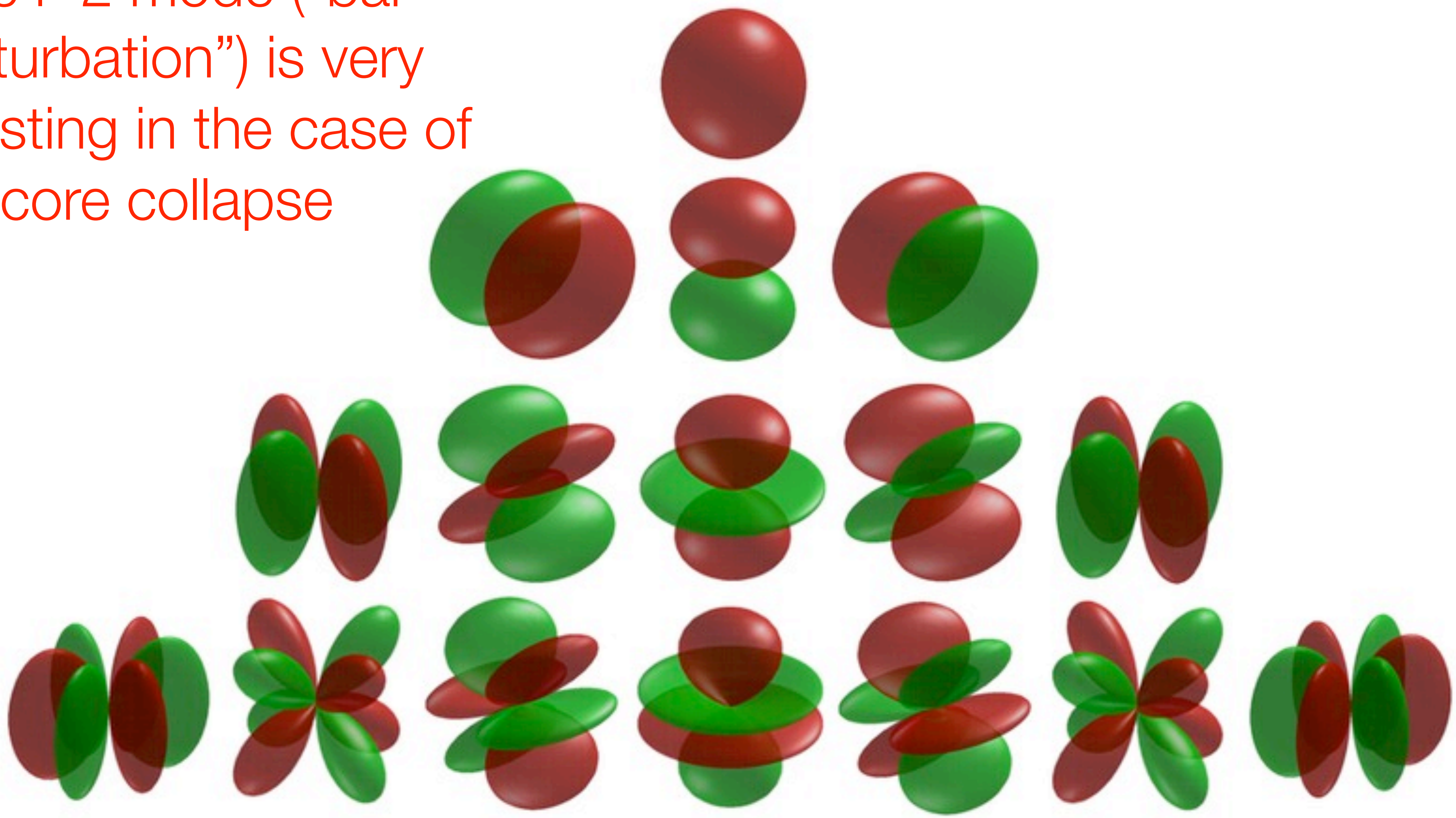
$$(u-x)\frac{du_{1r}}{dx}+\frac{1}{R}\frac{dr_1}{dx}-\frac{1}{R^2}\frac{dR}{dx}r_1+\left(\sigma+\frac{du}{dx}\right)u_{1r}+\frac{d\Phi_1}{dx}=0$$

$$(u-x)\frac{du_{1\theta}}{dx}+\left(\frac{u}{x}+\sigma\right)u_{1\theta}+(l+1)\frac{1}{xR}r_1+\frac{l+1}{x}\Phi_1=0$$

$$\frac{d^2\Phi_1}{dx^2}+\frac{2}{x}\frac{d\Phi_1}{dx}-\frac{l(l+1)}{x^2}\Phi_1=r_1$$

Perturbation with spherical harmonics

The l=2 mode ("bar perturbation") is very interesting in the case of core collapse



l=0

l=1

l=2

l=3

Looking for a dispersion relation by a “shooting” method

- We start with the asymptotic form of the perturbation near x=0, replacing the Larson-Penston solution in our equations:

$$r_1 = ax^l, u_{1r} = bx^{l-1}, u_{1\theta} = b\frac{l+1}{l}x^{l-1}, \Phi_1 = \left(-\frac{a}{\rho_0} - b(\sigma + 1 - \frac{l}{3})\right)x^l$$

- We get two independent solutions for (a,b)=(0,1) and (1,0) and integrate them to the critical point. Skipping the critical point, we integrate to infinity, looking for σ such that it minimizes the value of the potential at infinity (This condition required by the asymptotic form of the solution at infinity).

Applying a linear perturbation to an equilibrium collapse solution: full perturbation, including oscillatory modes

In this case, the perturbation has its full form:

$$\rho(r,t) = \frac{R}{t^2} + Y_l^m t^{\sigma-2}\left[r_1(x)cos(\omega lnt) + r_2(x)sin(\omega lnt)\right] = \rho_0 + \delta\rho$$

$$u_r(r,t) = u(x) + Y_l^m t^{\sigma}\left[u_{1r}(x)cos(\omega lnt) + u_{2r}(x)sin(\omega lnt)\right] = u_{r0} + \delta u_r$$

$$u_\theta(r,t) = \frac{1}{l+1}\frac{\partial Y_l^m}{\partial\theta}t^{\sigma}\left[u_{1\theta}cos(\omega lnt) + u_{2\theta}sin(\omega lnt)\right] = \delta u_\theta$$

$$u_\phi(r,t) = \frac{1}{l+1}\frac{1}{sin\theta}\frac{\partial Y_l^m}{\partial\phi}t^{\sigma}\left[u_{1\phi}cos(\omega lnt) + u_{2\phi}sin(\omega lnt)\right] = \delta u_\phi$$

$$\Psi(r,t) = \Psi_0(x) + Y_l^m t^{\sigma}\left[\Phi_1 cos(\omega lnt) + \Phi_2 sin(\omega lnt)\right] = \delta\Psi$$

Applying a linear perturbation to an equilibrium collapse solution: full perturbation, including oscillatory modes

And the system of equations we have to solve becomes:

$$(u-x)\frac{dr1}{dx} + R\frac{du_{1r}}{dx} + \left(\frac{2R}{x} + \frac{dR}{dx}\right)u_{1r} + \left(\frac{2u}{x} + \frac{du}{dx} + \sigma - 2\right)r_1 + \omega r_2 - \frac{Rl}{x}u_{1\theta} = 0$$

$$(u-x)\frac{dr_2}{dx} + R\frac{du_{2r}}{dx} + \left(\frac{2R}{x} + \frac{dR}{dx}\right)u_{2r} + \left(\frac{2u}{x} + \frac{du}{dx} + \sigma - 2\right)r_2 - \omega r_1 - \frac{Rl}{x}u_{2\theta} = 0$$

$$\frac{1}{R}\frac{dr_1}{dx} + (u-x)\frac{du_{1r}}{dx} + \left(\sigma + \frac{du}{dx}\right)u_{1r} + \omega u_2 - \frac{1}{R^2}\frac{dR}{dx}r_1 + \Phi_1 = 0$$

$$\frac{1}{R}\frac{dr_2}{dx} + (u-x)\frac{du_{2r}}{dx} + \left(\sigma + \frac{du}{dx}\right)u_{2r} - \omega u_1 - \frac{1}{R^2}\frac{dR}{dx}r_1 + \Phi_2 = 0$$

$$\frac{d^2\Phi_1}{dx^2} + \frac{2}{x}\frac{d\Phi_1}{dx} - \frac{l(l+1)}{x^2}\Phi_1 = r_1$$

$$\frac{d^2\Phi_2}{dx^2} + \frac{2}{x}\frac{d\Phi_2}{dx} - \frac{l(l+1)}{x^2}\Phi_2 = r_2$$

A more general method for finding the eigenvalues

- The "shooting" method used by Hanawa & Matsumoto does not allow for generalization to systems with more degrees of freedom, like the full perturbation we want to implement.

- We then apply a more general method, where we look for the eigenvalue that minimizes the value of the determinant of a matrix composed of five vectors, each representing one solution to the system of equations.

- We start from two independent sets of solutions from x=0 and three from x=infinity and integrate them to the critical point. The determinant of this matrix will be zero when the sigma chosen is an eigenvalue of the system.

- This method is easy to generalize for the full perturbation, and also to apply with different equilibrium solutions.

Project status so far

- We have reproduced the result from Hanawa & Matsumoto (1999), where they find a growth rate of -0.35 for the l=2 perturbation, using their “shooting” method.

- We have derived the full perturbed equations, including the oscillatory modes and their asymptotic behaviors at zero and infinity for the Larson-Penston flow.

- We are now finishing the code which uses the determinant minimization method (still some bugs to be cleaned), so that we can confirm the Hanawa & Matsumoto results and move on to finding the dispersion relation for the full perturbation. This would already be a new result.

Outlook

- Once we have the method for the determinant minimization ready, we will be able to look for unstable solutions for the perturbations of the Larson-Penston flow, including the oscillatory modes as well.

- The next step would be to use the Shu solutions as the state of equilibrium and vary the parameter A to get different initial conditions (more stable vs more unstable initial density and velocity profiles)

- Eventually, we would like to perform the same analysis for a polytropic equation of state, which is more realistic for studying core collapse.

- In addition, we want to simulate the evolution of the perturbed flows using the RAMSES code.

Thank you for your spontaneous applause!

Fragmentation at the Class 0 phase?

- Young Stellar Objects (YSO) are usually classified according to the shape of their Spectral Energy Distribution (SED) and, more specifically according to their spectral index, α:

$$\alpha = \frac{d\log(\lambda F_\lambda)}{d\log(\lambda)}$$

- According to the value of α, YSOs are classified into Class 0, Class I/II and Class III, in a rough evolutionary sequence.

- Searches for multiplicity in Class 0 and Class I/II objects (Maury et al. 2010) have shown that there is a tendency for multiplicity in Class I/II objects, which is not observed for Class 0 objects.

- Class 0 objects, being the initial phase of core collapse, can be thought as the systems our approach is trying to represent.