Turbulent transport in stably stratified atsmospheres

F. Lignières

Observatoire Midi-Pyrénées

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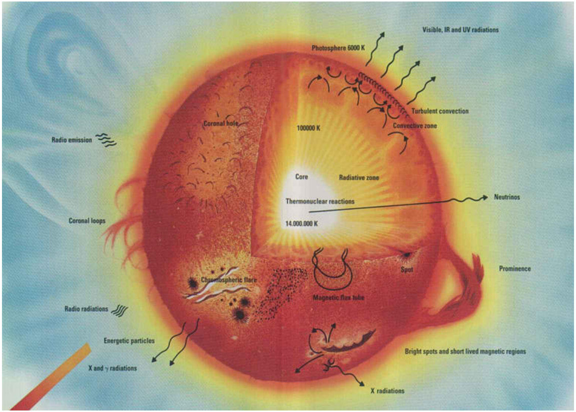
Turbulent transport in stably stratified atmospheres

- Transport in the ocean, in planetary atmospheres
- Transport of chemical elements/angular momentum in stellar radiative zones
- ...



Outline

- Some remarks on the mixing length theory
- The G.I. Taylor diffusion theory
- Vertical transport of chemical elements in homogeneous stably stratified turbulence
- Vertical transport of chemical elements in stellar radiative zones

The mixing length theory

Based on an analogy with molecular diffusion in perfect gas

- Small departure from thermodynamic equilibrium $\Rightarrow \quad \nu = \frac{1}{3}\lambda v_{th}$
- Prandtl's assumption $\quad \nu_T = \ell_m v = \ell_m^2 |\frac{dU}{dz}|$

But ...

- while $\lambda \ll L$, there is no such scale separation in general in turbulence : for example, in free shear flows, $\ell \sim L$.
- the theory is incomplete : $\nu_T = \ell_m v$ but ℓ_m and v are unknown.

Does it work anyway ?

Yes, for one-dimensional free shear flows $U(z)$

$$\ell_m = cL$$

where $0.07 \leq c \leq 0.18$ depending on the type of shear flows

For this one length scale L and one time scale $1/\frac{dU}{dz}$ problem, this success can be attributed to dimensional analysis :

$$\ell \sim L \qquad \sqrt{< u^2 >}/\ell \sim \frac{dU}{dz}$$

$$| < uw > | \sim \sqrt{< u^2 >}\sqrt{< w^2 >} \sim < u^2 > \sim |L^2 \left(\frac{dU}{dz}\right)^2 | \sim \nu_T |\frac{dU}{dz}|$$

But ... if the fluid is stably stratified

the buoyancy force introduces another time scale $1/N$

$$\Rightarrow \quad c = f(N/\frac{dU}{dz}) \quad \text{with } f \text{ unknown}$$

G.I. Taylor's turbulent diffusion model

Vertical displacement of single particule

$$\frac{dz}{dt} = W(t) \quad \Rightarrow \quad z - z_0 = \int_0^t W(s)ds$$

$$(z - z_0)W(t) = W(t)\int_0^t W(s)ds$$

$$\frac{1}{2}\frac{d(z-z_0)^2}{dt} = \int_0^t W(t)W(s)ds$$

Ensemble average over many particules $< .. >$

$$\frac{1}{2}\frac{d < (z-z_0)^2 >}{dt} = \int_0^t < W(t)W(s) > ds$$

The mean displacement depends on the Lagrangian autocorrelation :

$$R_{t,s} = \frac{< W(t)W(s) >}{< W(t)^2 >^{1/2} < W(s)^2 >^{1/2}}$$

G.I. Taylor's turbulent diffusion II

Statistical stationarity

$$\frac{< W(t)W(s) >}{< W(t)^2 >^{\frac{1}{2}} < W(s)^2 >^{\frac{1}{2}}} = \frac{< W(t)W(t+s-t) >}{< W^2 >} = R_{s-t}$$

$$\frac{1}{2}\frac{d < (z-z_0)^2 >}{dt} = < W^2 > \int_0^t R_{s-t} ds =< W^2 > \int_0^t R_u du$$

$$< (z-z_0)^2 > (T) = 2 < W^2 > \int_0^T \left(\int_0^t R_u du \right) dt$$

The Lagrangian correlation time τ_L

$$\text{As } \tau_L = \lim_{t \to \infty} \int_0^t R_u du \text{ is finite} \quad \Rightarrow$$

$$< (z-z_0)^2 > (T) = 2 < W^2 > \tau_L T \quad \text{for } T \gg \tau_L$$

Brownian like diffusion with $D =< W^2 > \tau_L$

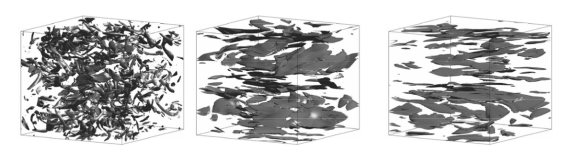
The two successive transport regimes predicted by the Langevin-type model of Pearson et al. (1983)

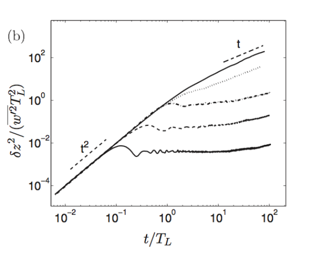
$$< \delta z^2 >= \frac{< w^2 >}{N^2} + c\frac{< w^2 >}{N} t$$

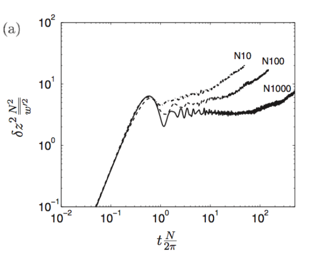
- Adiabatic regime : suppression of the vertical dispersion by the buoyancy force $< \delta z^2 >= \frac{<w^2>}{N^2}$
- Diabatic regime : vertical diffusion induced by thermal diffusivity $< \delta z^2 >= c\frac{<w^2>}{N} t$

According to Lindborg & Fedina (2008), $D = \epsilon_P N^2$, where ϵ_P is the rate of potential energy dissipation through molecular diffusion.

Forced turbulence for different stratification (van Aartrijk et al. 2008)







Vertical transport of chemical elements in stellar radiative zones

Observational constraints

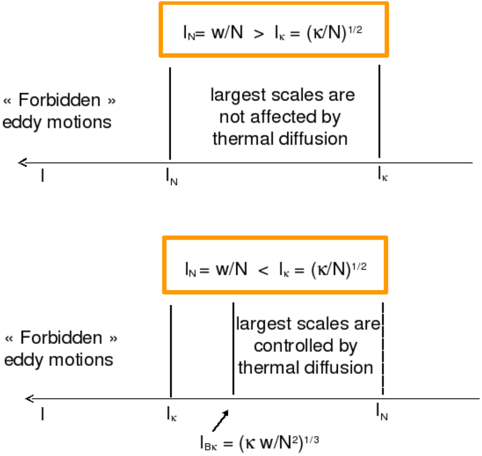
- Abundance anomalies in the sun, Am-Fm stars can be accounted for by very small effective diffusivities (Michaud & Zahn 1998) e.g. $D = 2000 cm^2/s$ in the solar radiative zone

  $\Rightarrow$ Small length scales and/or small velocity scales

The dynamical role of thermal diffusion

- reduces the amplitude of the buoyancy force
- dissipate kinetic energy
- changes the equilibrium level of fluid elements

Two different regimes depending on the amplitude of vertical velocities (Lignières et al. 2005)



$D = \ell_{B\kappa} w < \kappa$ as expected from the observations