Searching for Radiative Hydrodynamic Instabilities in Massive Star Envelopes

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Introduction

- Basic fluid equations (Blaes & Socrates 2003):

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}) = 0 \, ,$$

$$\cancel{\rho\left(\frac{\partial \boldsymbol{v}}{\partial t} + \boldsymbol{v} \cdot \nabla v\right)} = -\nabla p + \rho \boldsymbol{g} + \cancel{\frac{1}{4\pi}(\nabla \times \boldsymbol{B}) \times \boldsymbol{B}} + \frac{\kappa_F \rho}{c} \boldsymbol{F} \, ,$$

$$\frac{\partial u}{\partial t} + \boldsymbol{v} \cdot \nabla u + \gamma u \nabla \cdot \boldsymbol{v} = \kappa_J \rho c E - \kappa_{\mathrm{P}} \rho c a T_g^4 - \kappa_{\mathrm{T}} \rho c \left(\frac{4 k_{\mathrm{B}} T_g}{m_e c^2} - \frac{h \bar{\nu}}{m_e c^2}\right) E \, ,$$

$$\frac{\partial E}{\partial t} + \boldsymbol{v} \cdot \nabla E + \frac{4}{3} E \nabla \cdot \boldsymbol{v} = \cancel{-\nabla \cdot \boldsymbol{F}} - \kappa_J \rho c E + \kappa_{\mathrm{P}} \rho c a T_g^4 + \kappa_{\mathrm{T}} \rho c \left(\frac{4 k_{\mathrm{B}} T_g}{m_e c^2} - \frac{h \bar{\nu}}{m_e c^2}\right) E \, ,$$

$$0 = -\frac{1}{3} \nabla E - \frac{\kappa_F \rho}{c} \boldsymbol{F} \, ,$$

Full Dispersion Relation

- Too long to be written on one slide…
  - Equation (49) in Blaes & Socrates (2003)

Short-Wavelength Limit with $T_g = T_r$

- Acoustic wave modes:

$$\omega = \pm k c_i - i\frac{\kappa_{\mathrm{F}}}{2cc_i}\left(1+\frac{3p}{4E}\right)\left[\left(\frac{4E}{3}+p\right)c_{\mathrm{i}} \mp (\hat{\boldsymbol{k}}\cdot\boldsymbol{F})\Theta_{\rho}\right] + \mathcal{O}(k^{-1}).$$

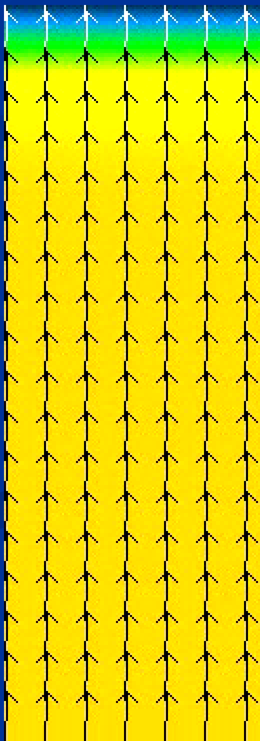
- An order-of-magnitude instability criterion:

$$F\Theta_{\rho} \gtrsim \max[E, p]c_i$$

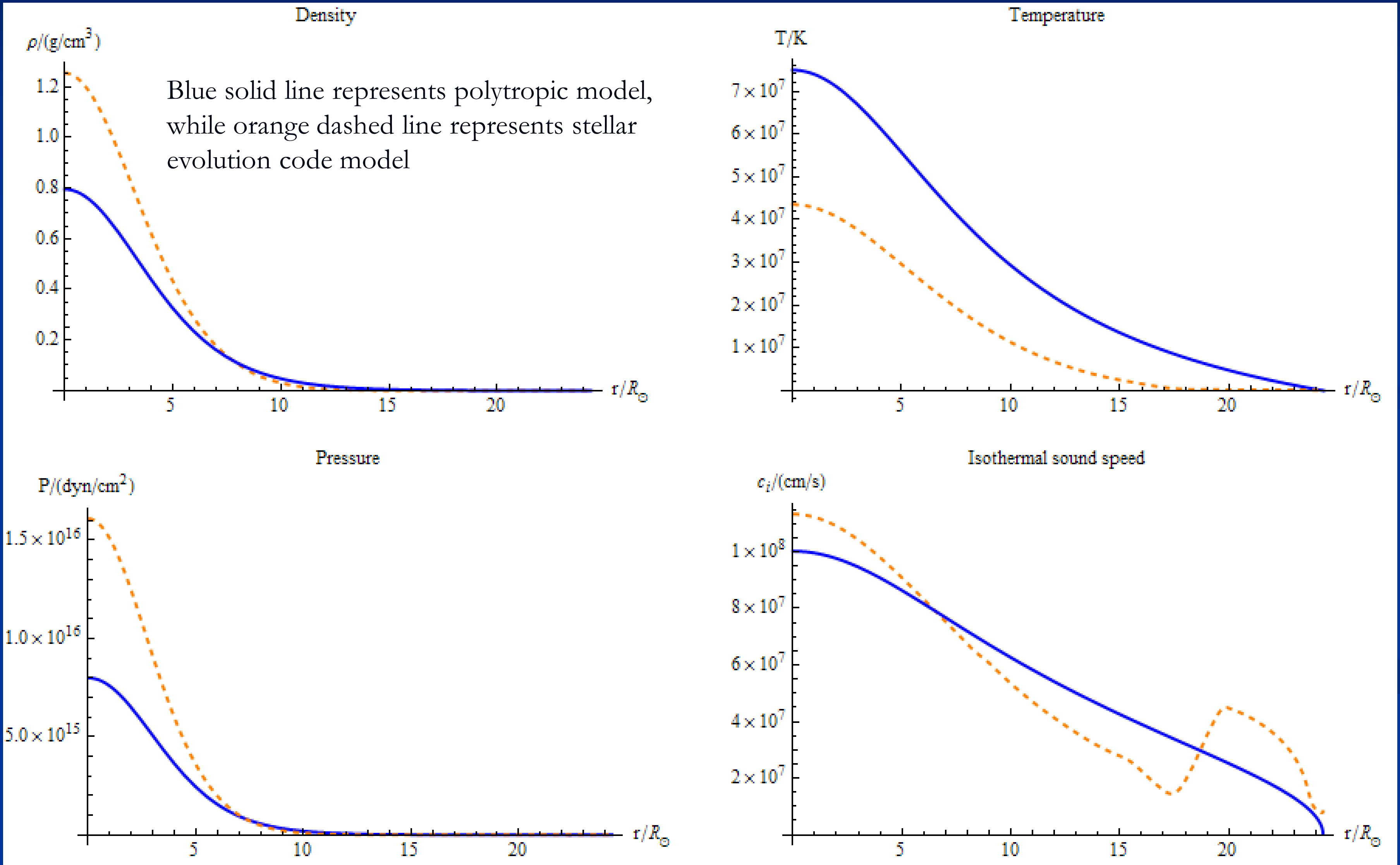
$$F \sim Ec/\tau_F$$

$$\tau_F \lesssim \left(\frac{c}{c_i}\right)\Theta_{\rho}$$

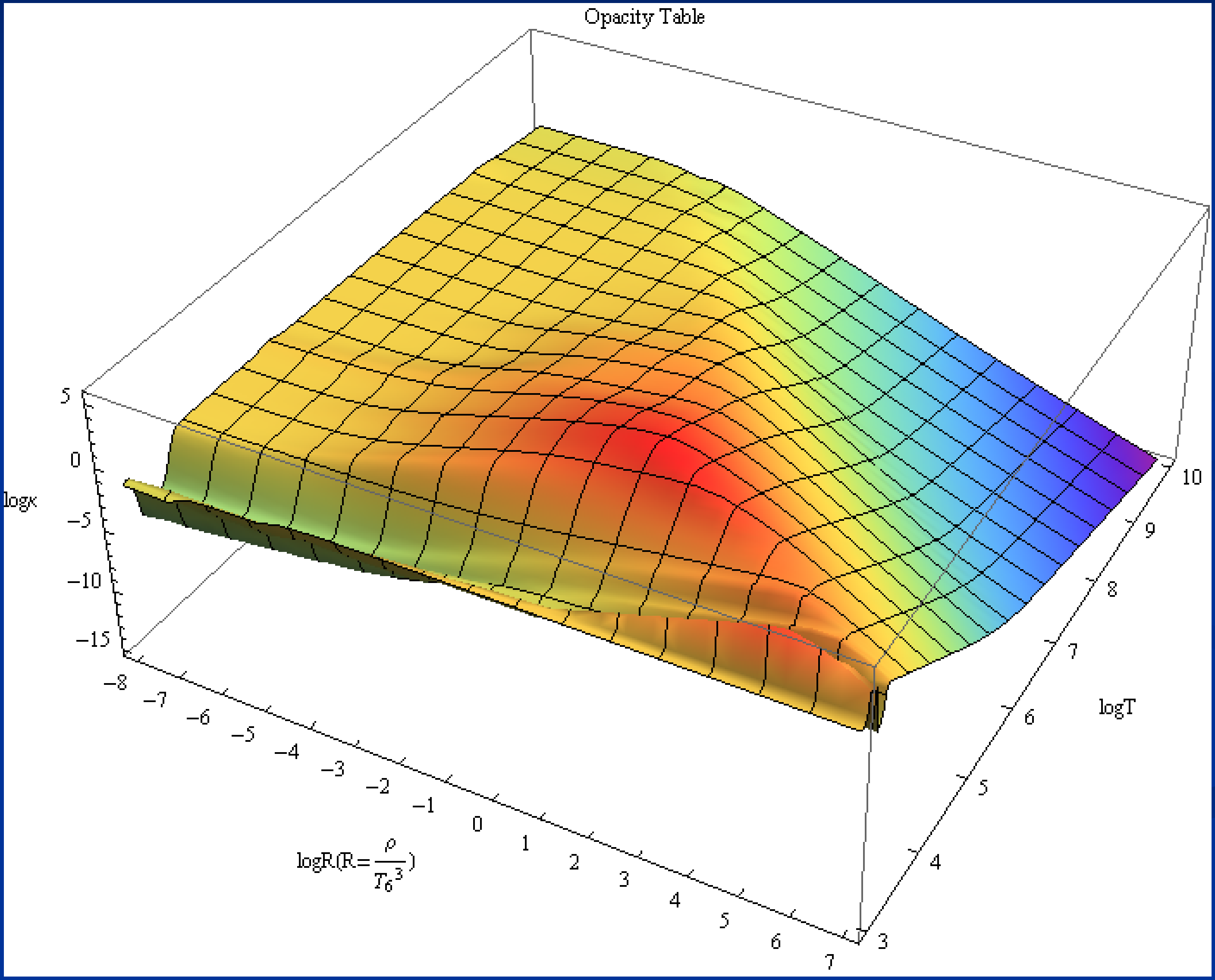
$$\tau_F \lesssim \left(\frac{c}{c_i}\right)\left(\frac{E}{p}\right)\Theta_{\rho}$$

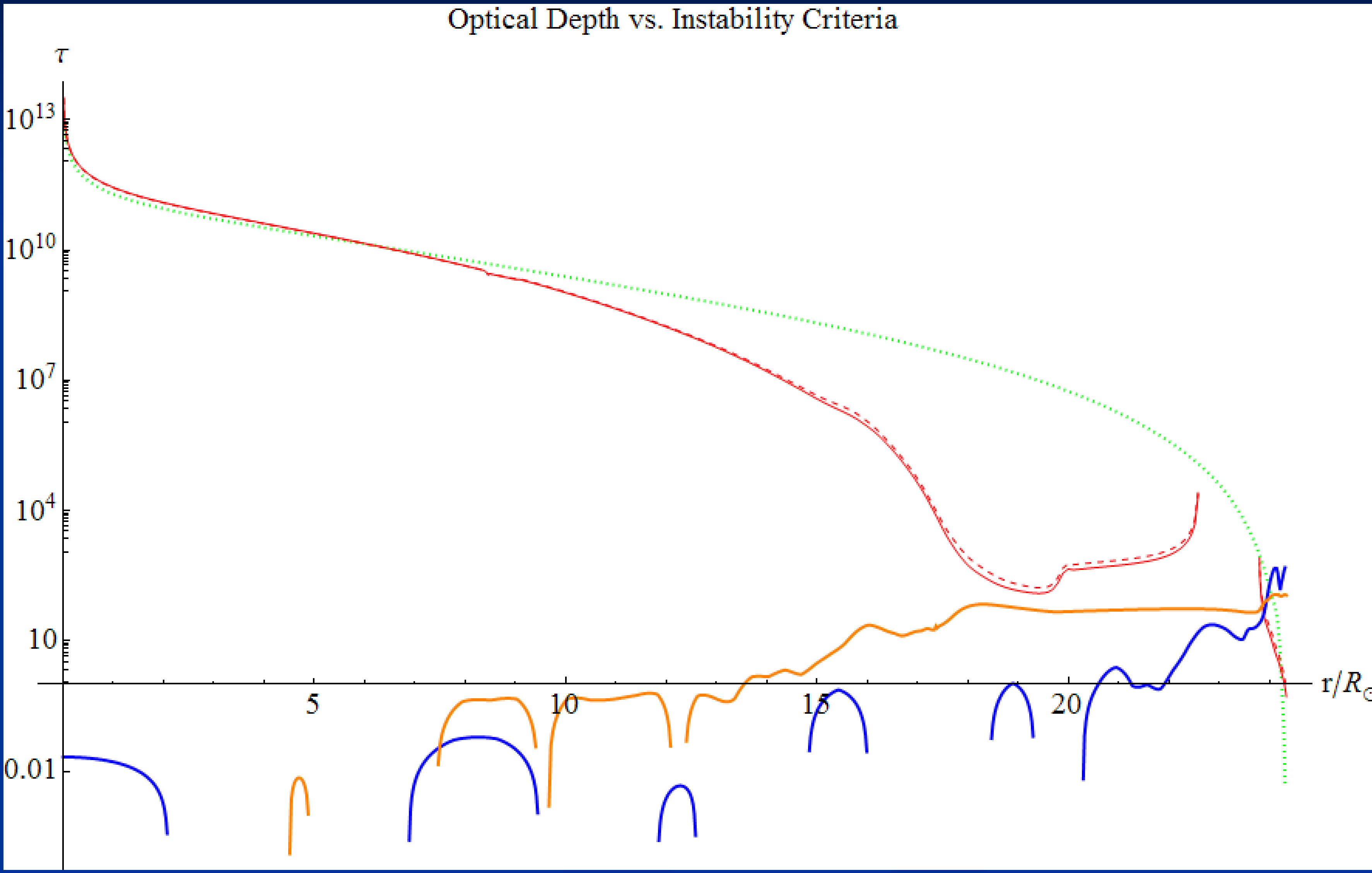


n = 3 Polytropic Stellar Model



Opacity Table for solar metallicity abundance





Optical Depth vs. Instability Criteria
τ
10^13
10^10
10^7
10^4
10
0.01
5
10
15
20
r/R⊙

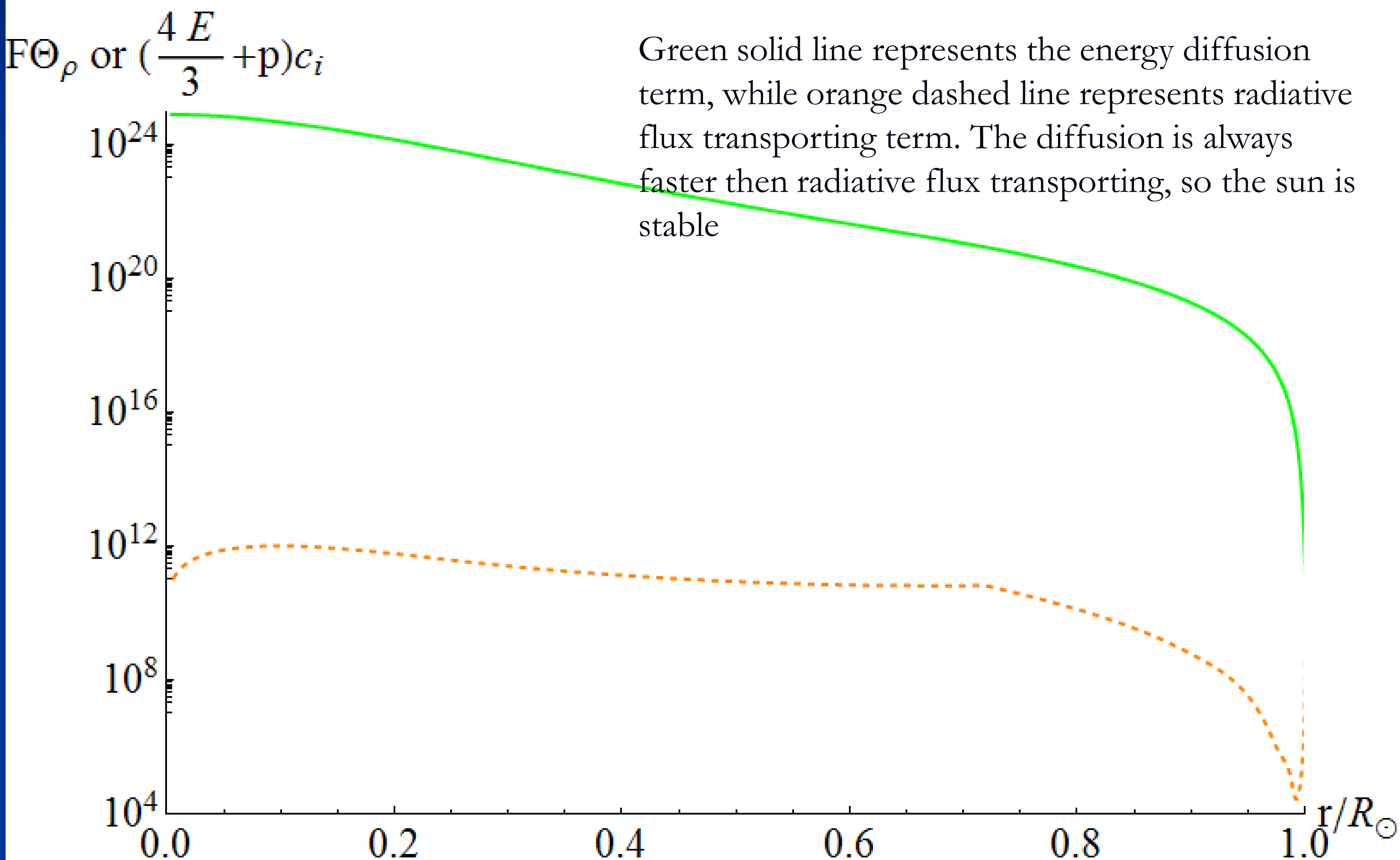
Acoustic Wave Instability

$$\omega = \pm k c_i - i \frac{\kappa_{\mathrm{F}}}{2 c c_i} \left(1 + \frac{3p}{4E}\right) \left[\left(\frac{4E}{3} + p \right) c_{\mathrm{i}} \mp (\hat{\boldsymbol{k}} \cdot \boldsymbol{F}) \Theta_\rho \right] + \mathcal{O}(k^{-1}).$$

where the upper sign represents the upward-propagating wave, the lower sign represents the downward-propagating wave.

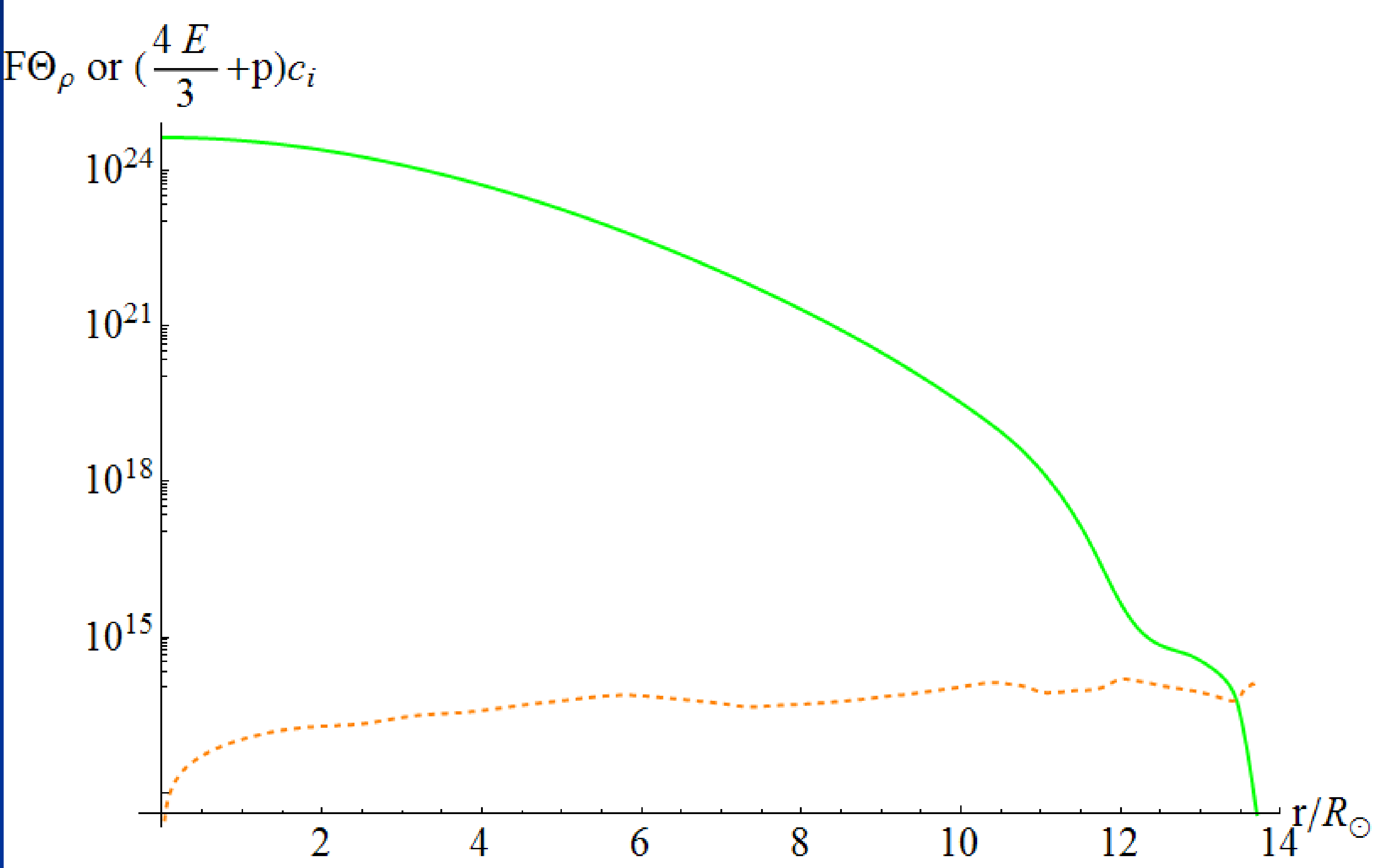
- If the imaginary part is positive, then the upward-propagating acoustic waves are unstable.

One Solar Mass Check

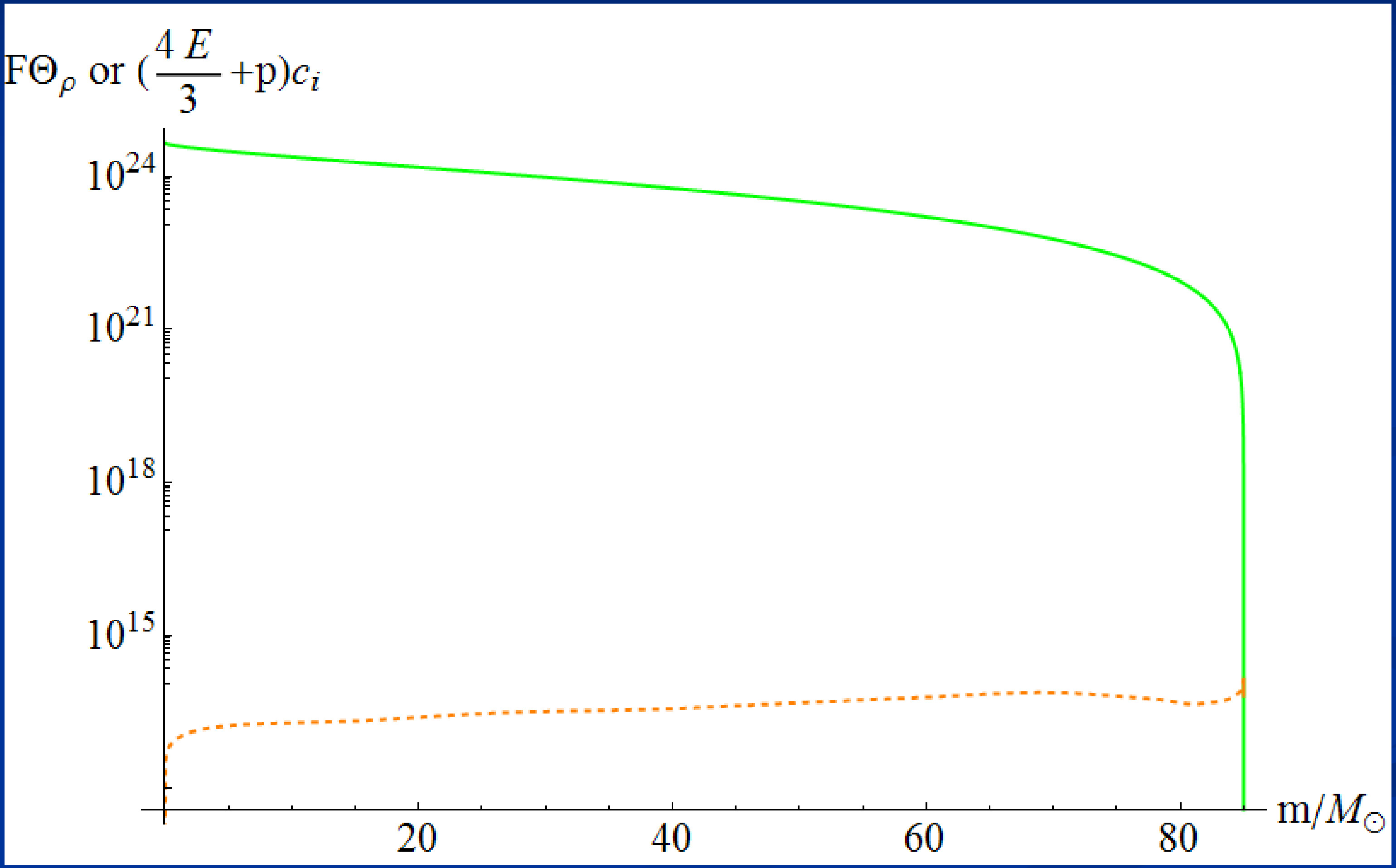


Green solid line represents the energy diffusion term, while orange dashed line represents radiative flux transporting term. The diffusion is always faster then radiative flux transporting, so the sun is stable

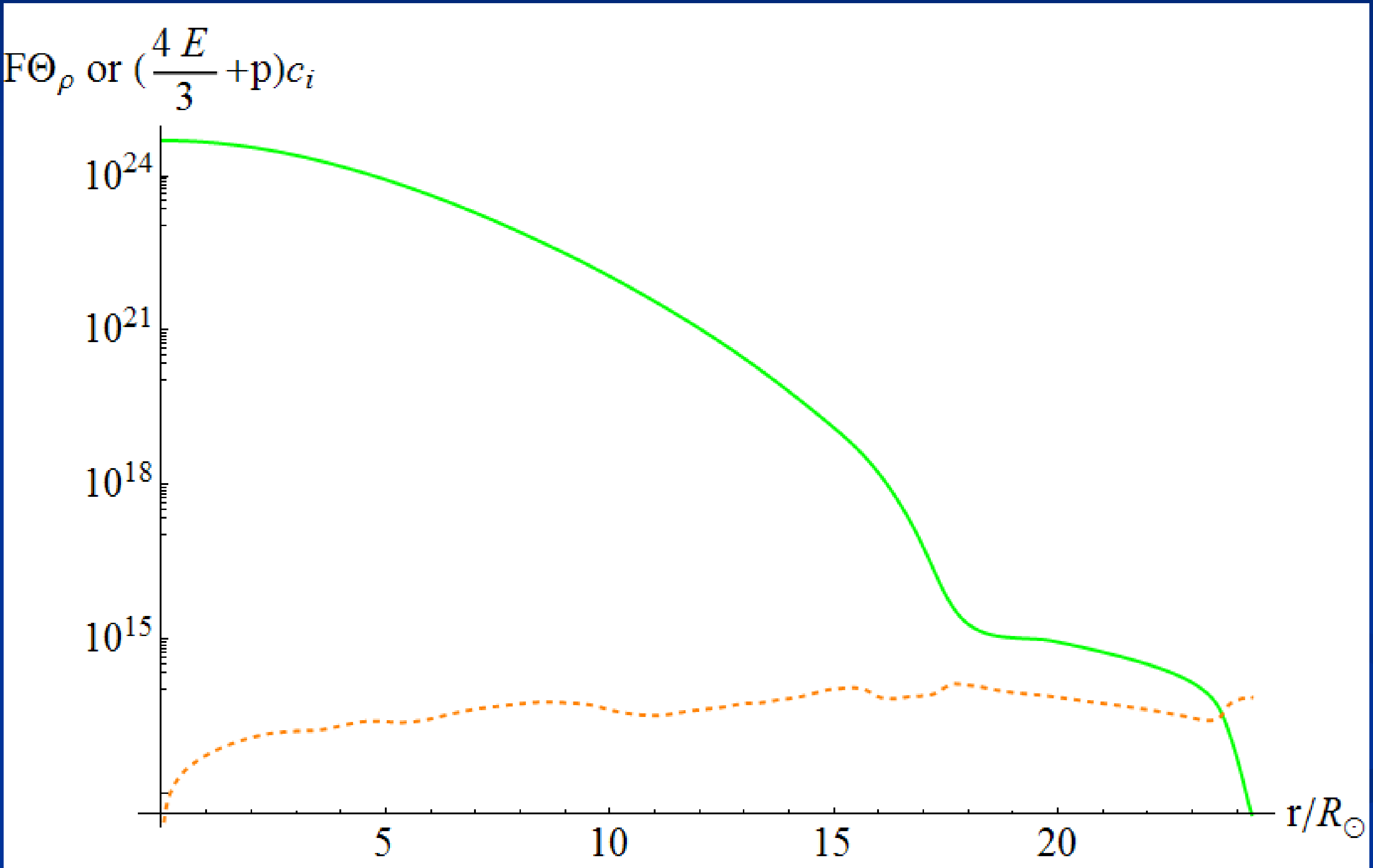
An 85 Solar Mass Star



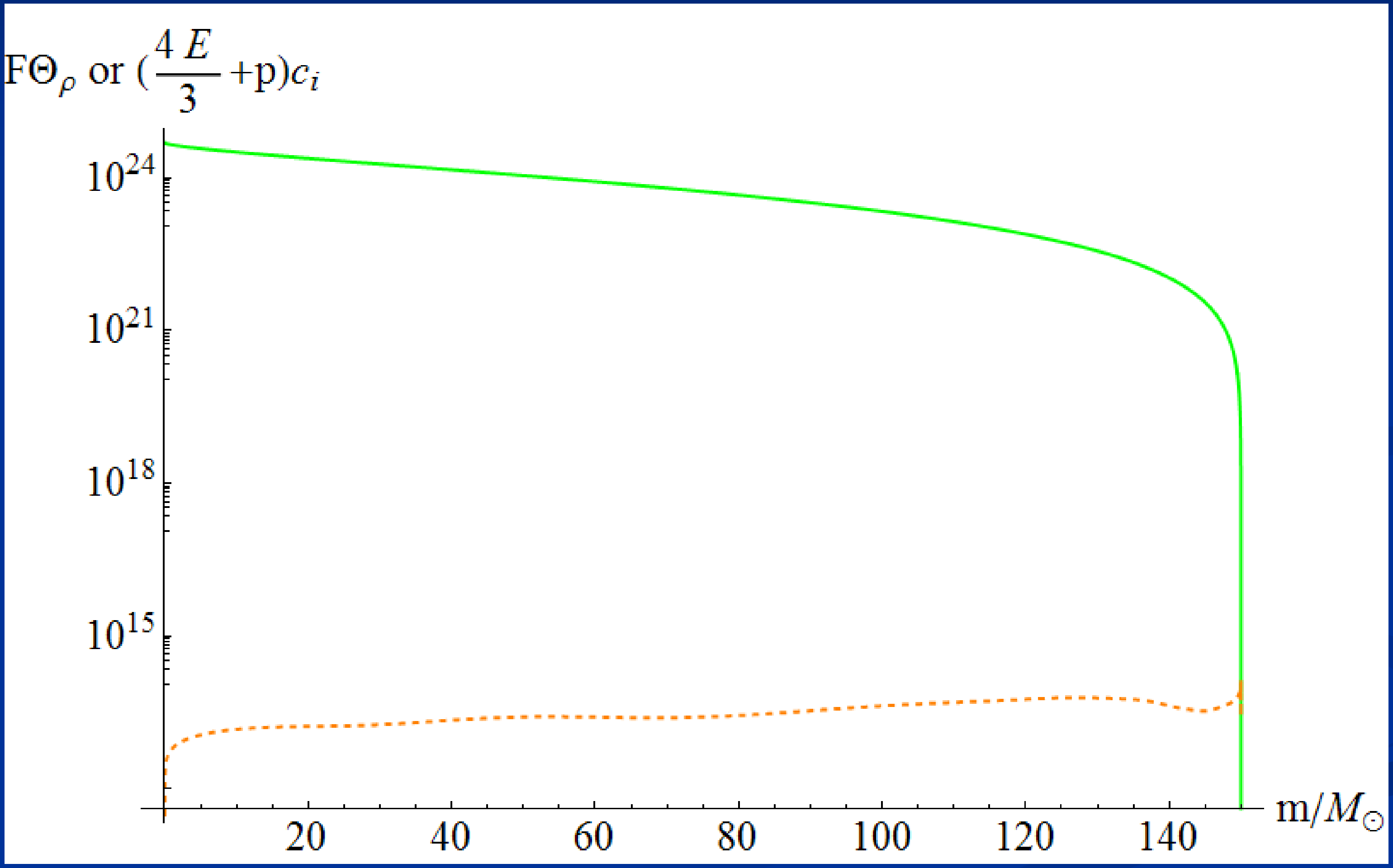
An 85 Solar Mass Star



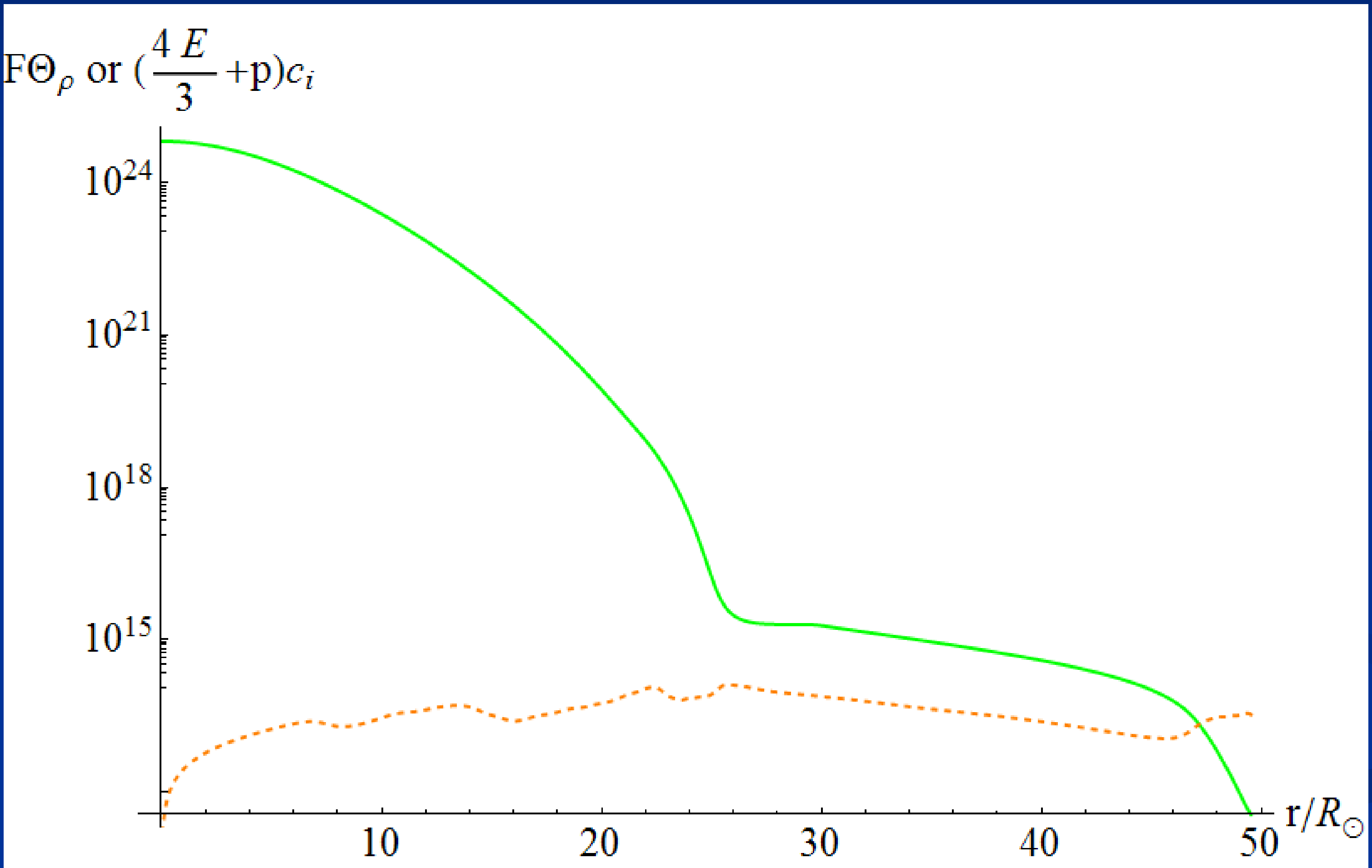
A 150 Solar Mass Star



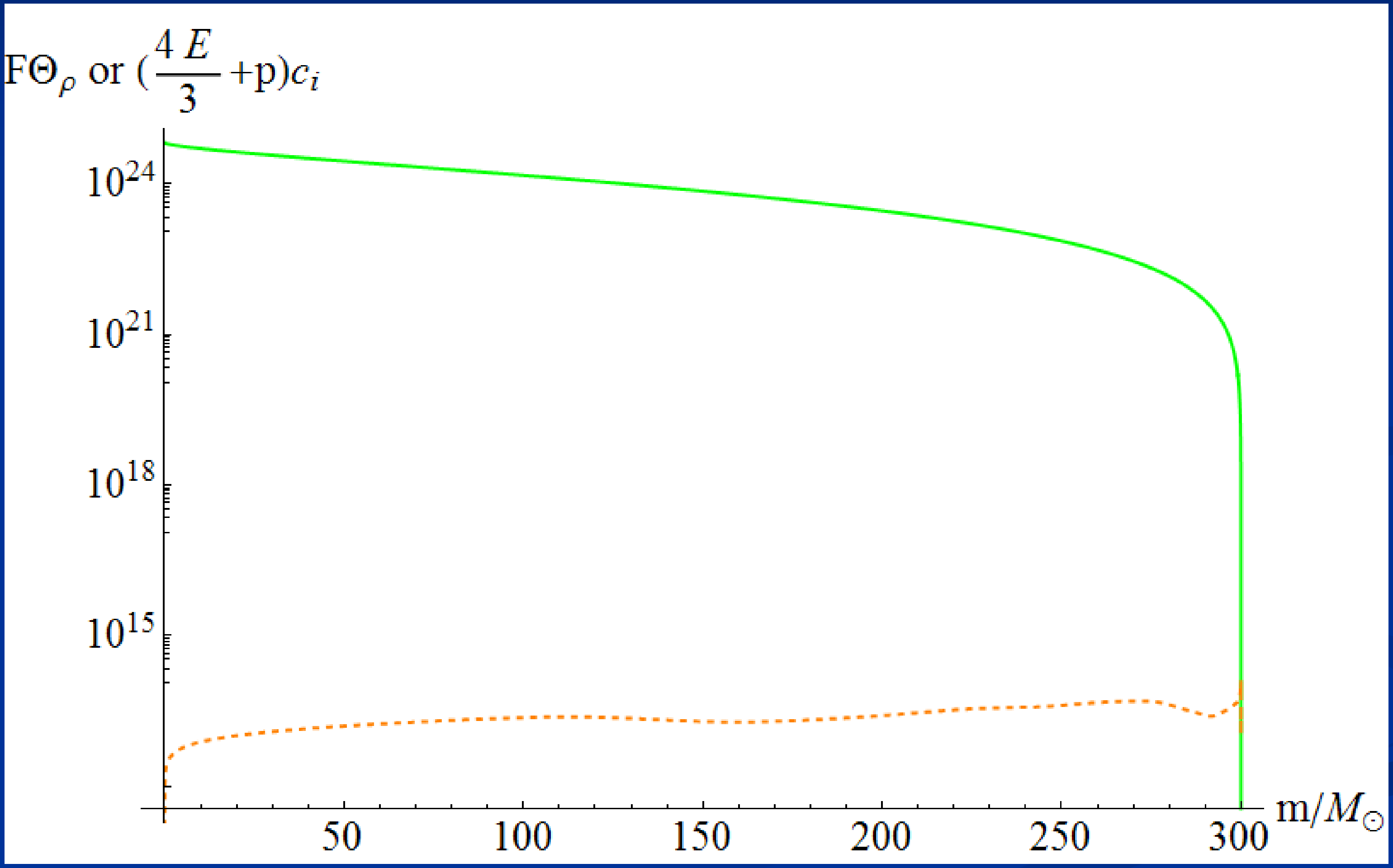
A 150 Solar Mass Star



A 300 Solar Mass Star



A 300 Solar Mass Star



Results

- The sun is stable for radiative hydrodynamic instability. However, even for very massive stars, only a very tiny mass portion of atmosphere would satisfy the instability criteria.
- But the growth rate is fast ($\sim 10^4$ s), which may lead to a catastrophe.
- Magnetic field may be important in this situation, which is neglected by us.

Future Plans

- Consider radiative MHD instability
- Find out cutoff wave number, which breaks down the previous assumptions.
- Compute more stellar models to investigate the instabilities in massive star envelopes.
- How this kind of instability will affect the stellar structure?
- Non-linear perturbations?