Chemical separation and compositionally-driven convection in neutron star oceans

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Medin & Cumming 2010 PRE 81, 036107
Medin & Cumming 2011 ApJ 730, 97

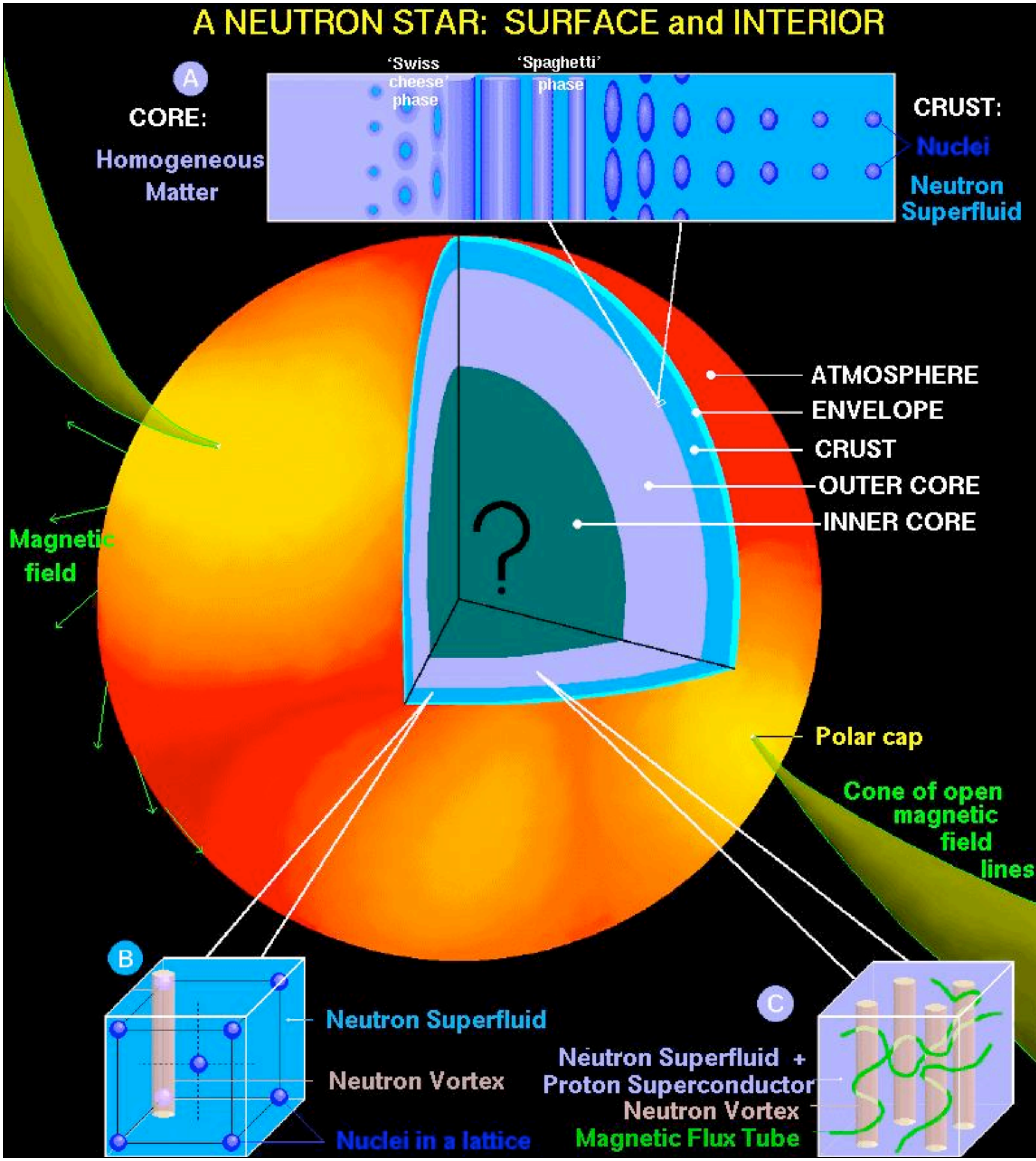
Basic idea

- When a mixture freezes, in general the solid formed has a different composition than the liquid. For example, heavier elements preferentially go into the solid, light elements are left behind in the liquid.

- This chemical separation leads to mixing, e.g. light elements rise buoyantly resulting in convection, or heavier solid particles can “snow”.

Several interesting examples in astrophysics

- *Earth's core*: compositionally-driven convection as inner core freezes from outer core fluid, important for driving dynamo
- *Ganymede*: compositionally-driven convection plays a role in driving dynamo
- *Giant planets*: He droplets rain out, gravitational energy release could explain luminosity of Saturn, deplete Ne in Jupiter's atmosphere
- *White dwarf cooling*: CO separation->delayed cooling, formation of a Ne core -> Type Ia ignition
- *Neutron star oceans*: enrichment of light elements and heating, formation of two-phase solid in the crust



$$M \gtrsim 1.2\ M_\odot$$
$$R \approx 10\ \mathrm{km}$$

outer layers
atmosphere, ocean,
crust

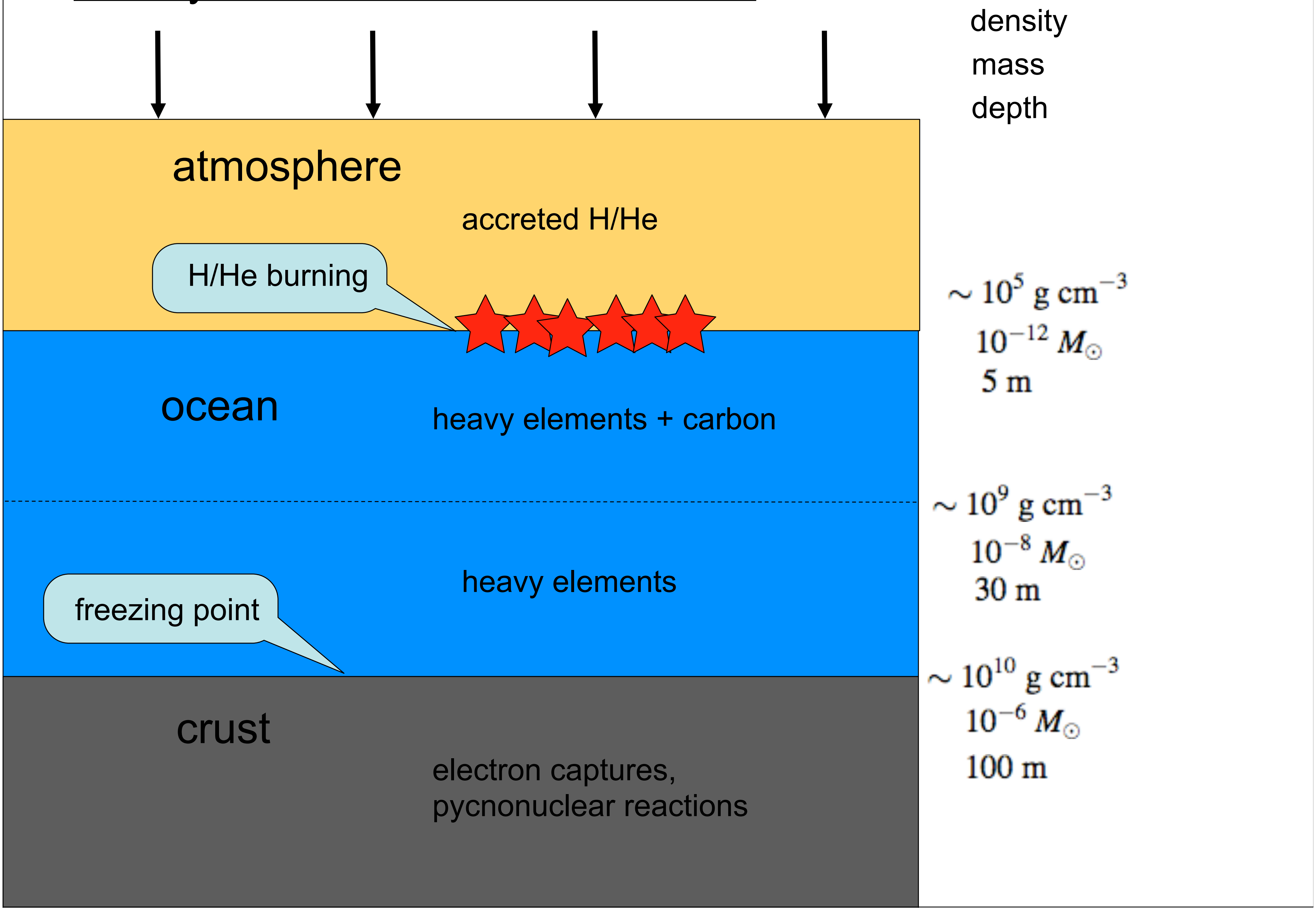
$$\rho < 10^{14}\ \mathrm{g\ cm^{-3}}$$

core
matter above
nuclear density

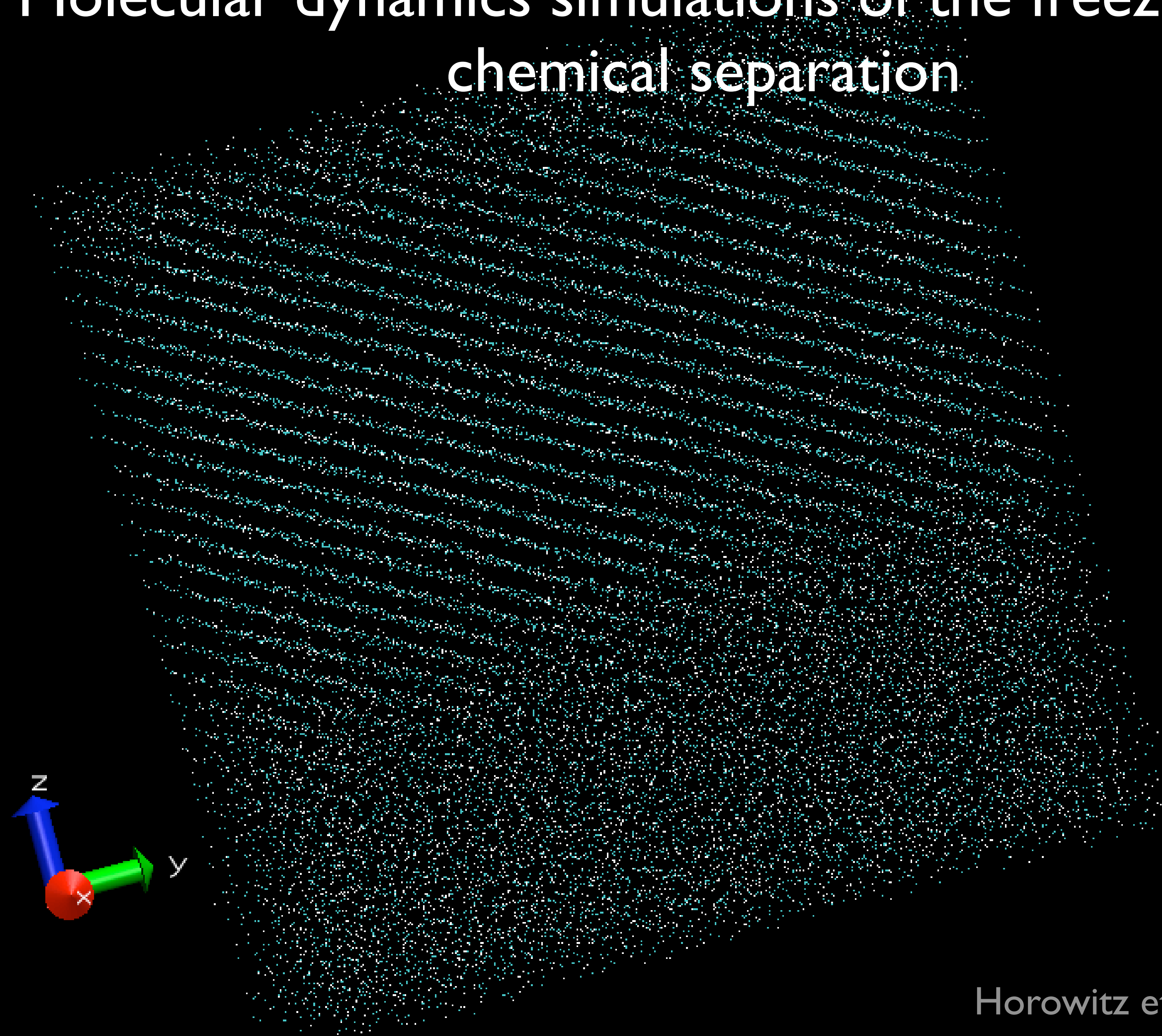
$$\rho \sim 10^{14}\ \mathrm{to}\ > 10^{15}\ \mathrm{g\ cm^{-3}}$$

figure from Dany Page

History of an accreted fluid element



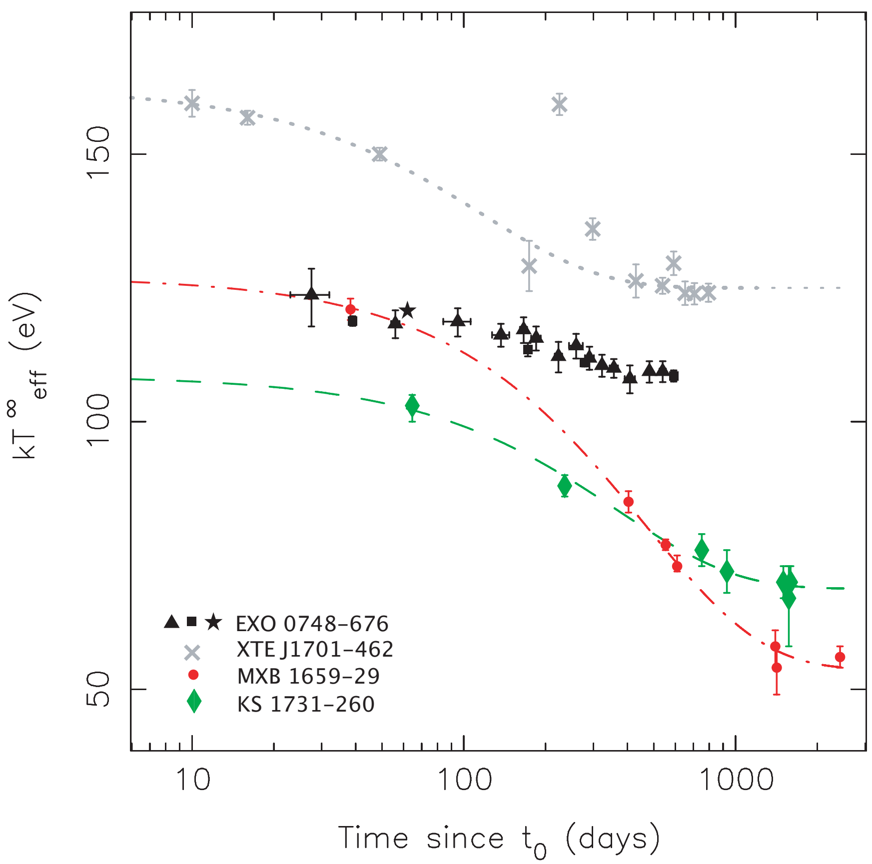
Molecular dynamics simulations of the freezing show chemical separation



Horowitz et al. (2007)

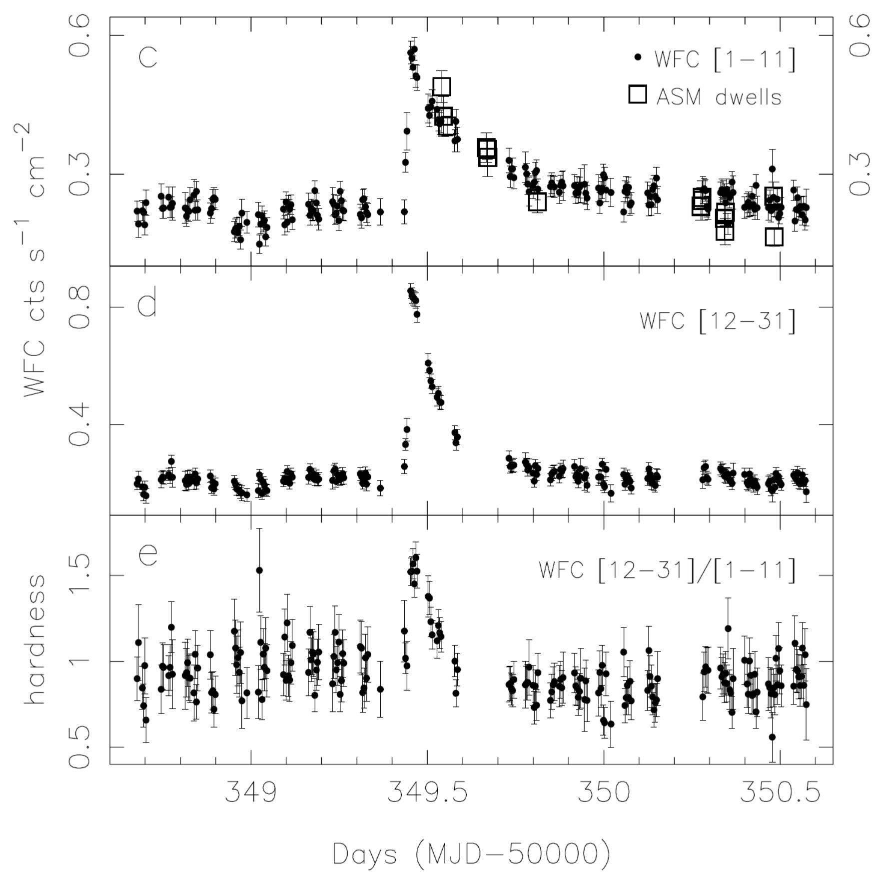
There are several observables that can tell us about the neutron star ocean

- Cooling after accretion outbursts



Degenaar et al. (2010)

- Thermonuclear flashes



Kuulkers et al. (2002)

This talk

Discuss the basic physics of

- Freezing and chemical separation
- Compositionally-driven convection

with recent results on neutron star oceans and white dwarf interiors along the way

Physics of the ocean and outer crust

- Matter is fully-ionized. Electrons are degenerate and form a uniform neutralizing background. Ions interact with a screened Coulomb potential.

- Key parameter:

$$\Gamma = \frac{Z^2 e^2}{a k_B T} \qquad \frac{4\pi}{3} a^3 n_i = 1$$

$\Gamma \lesssim 1$	gas
$\Gamma \gtrsim 1$	liquid
$\Gamma > 173$	solid

- The value 173 comes from Monte Carlo or molecular dynamics simulations of the (classical) one-component plasma (OCP)

Free energies of liquid and solid OCP

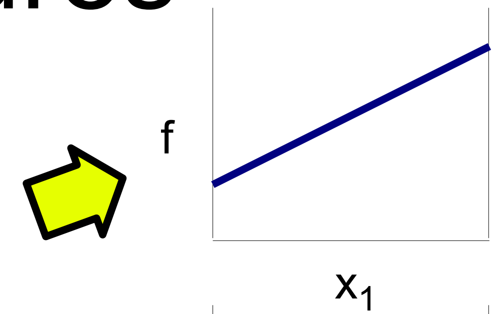
$$f_l^{\mathrm{OCP}}(\Gamma) \equiv \frac{F_l^{\mathrm{OCP}}}{Nk_BT} = -0.899\,172\Gamma + 1.8645\Gamma^{0.323\,01} - 0.2748\,\ln(\Gamma) - 1.4019.$$

$$\frac{F_s^{\mathrm{OCP}}}{Nk_BT} = -0.895\,929\Gamma + 1.5\,\ln(\Gamma) - 1.1703 - \frac{10.84}{\Gamma} - \frac{176.4}{\Gamma^2} - \frac{5.980 \times 10^4}{\Gamma^3}.$$

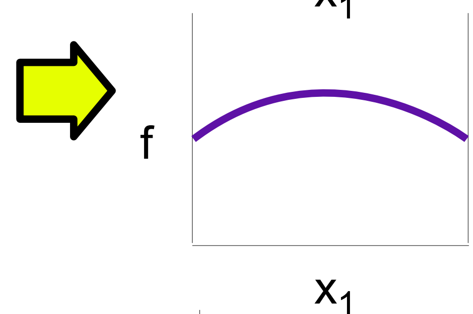
taken from DeWitt & Slattery (2003), Dubin (1990)

Two-component plasmas: Homogeneous mixtures

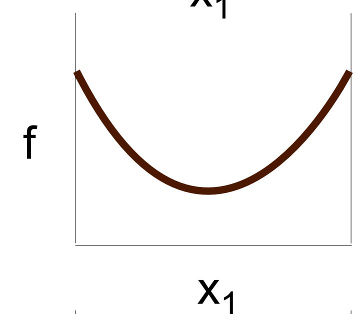
The free energy: $f = f_{LM} + \Delta f - \Delta s$



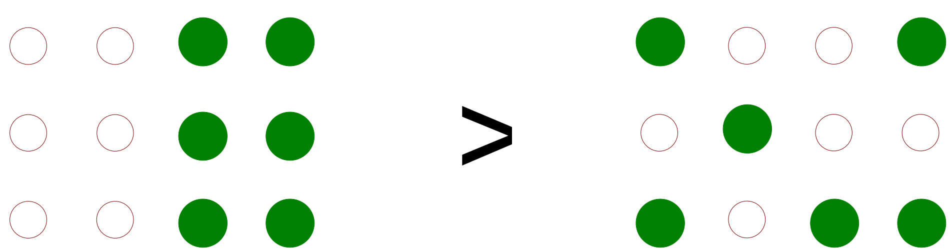
- **Linear mixing rule**: $f_{LM} = x_1 f_1(\Gamma_1) + x_2 f_2(\Gamma_2)$

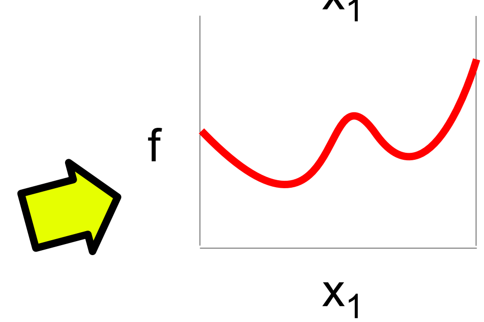


- **Enthalpy of mixing**: $\Delta f \propto x_1 x_2 \Gamma_1$
  - e.g., $\Delta f = x_1 x_2 \{u_{12} - ½[u_{11} + u_{22}]\}$
  - Ogata et al. 93, DeWitt & Slattery 03



- **Entropy of mixing**: $-\Delta s = x_1 \ln \{x_1 \frac{Z_1}{\langle Z \rangle}\} + x_2 \ln \{x_2 \frac{Z_2}{\langle Z \rangle}\}$





- **Total**

Two-component plasmas: Heterogeneous mixtures

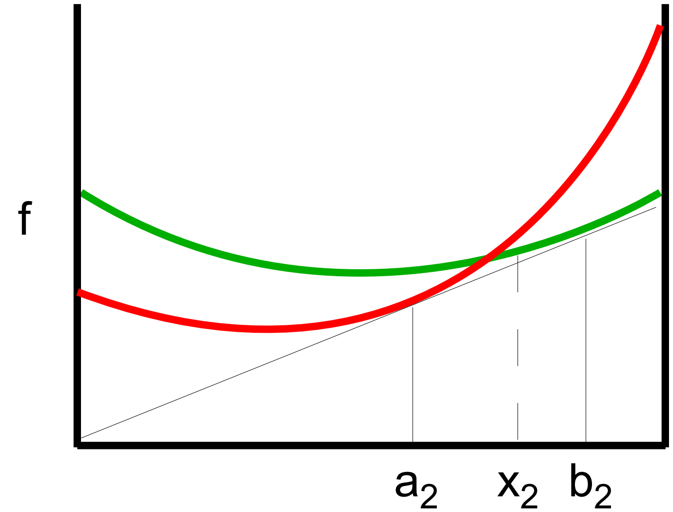
- For homogeneous mixtures $\{x_1,x_2\}$, $\{a_1,a_2\}$, and $\{b_1,b_2\}$: if

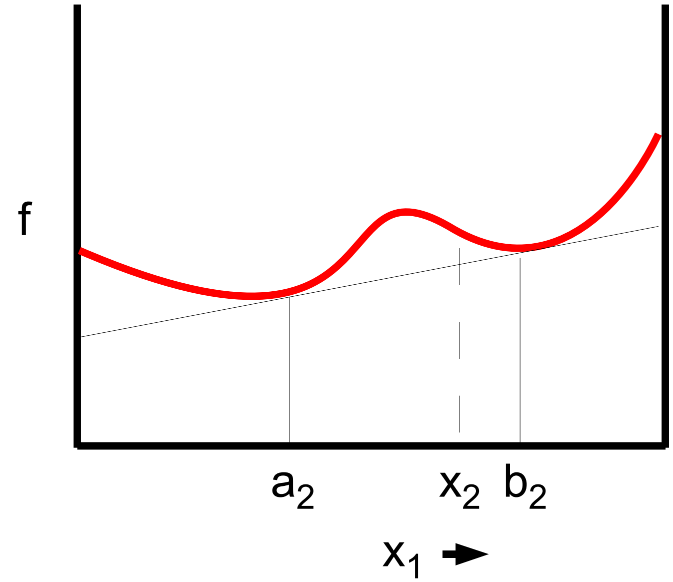
  $Aa_1+(1-A)b_1 = x_1$

  $Aa_2+(1-A)b_2 = x_2$

  $Af_a+(1-A)f_b < f_x$

  then **in equilibrium** a heterogeneous mixture will form
- "Double tangent construction"





For binary mixtures, the type of phase diagram depends on the charge ratio

Segretain & Chabrier (1993)

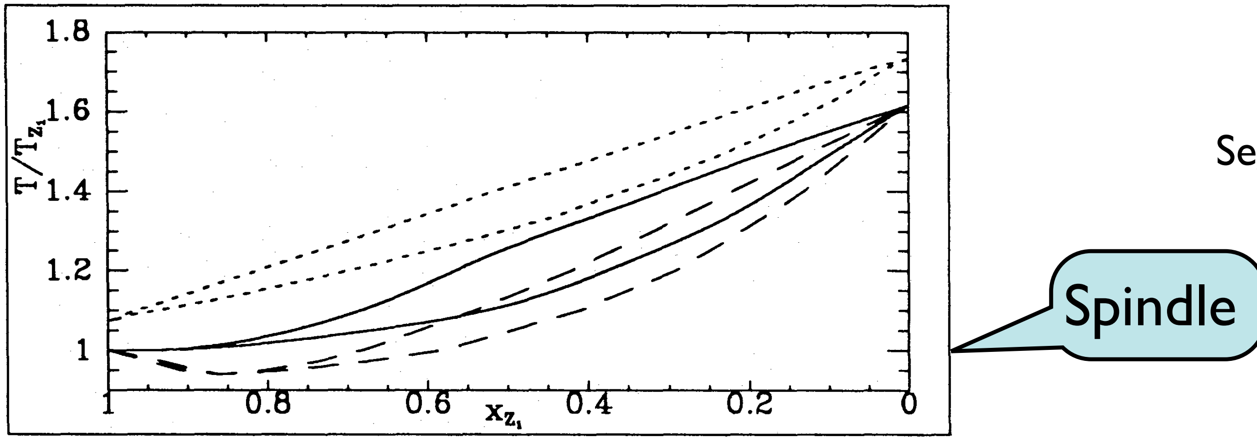


Fig. 3. Fluid-solid phase diagram for a BIM with charge ratio $Z_1/Z_2 = 0.75$. $T_{Z_1} = Z_1^{5/3} \frac{\Gamma_e}{\Gamma_c} T$ is the freezing temperature of the pure Z_1 element ($\Gamma_c = 178$). The short-dashed line and the long-dashed line show respectively the diagrams of Barrat et al. (1988) (where $\Gamma_c = 160$) and Iyetomi et al. (1989)

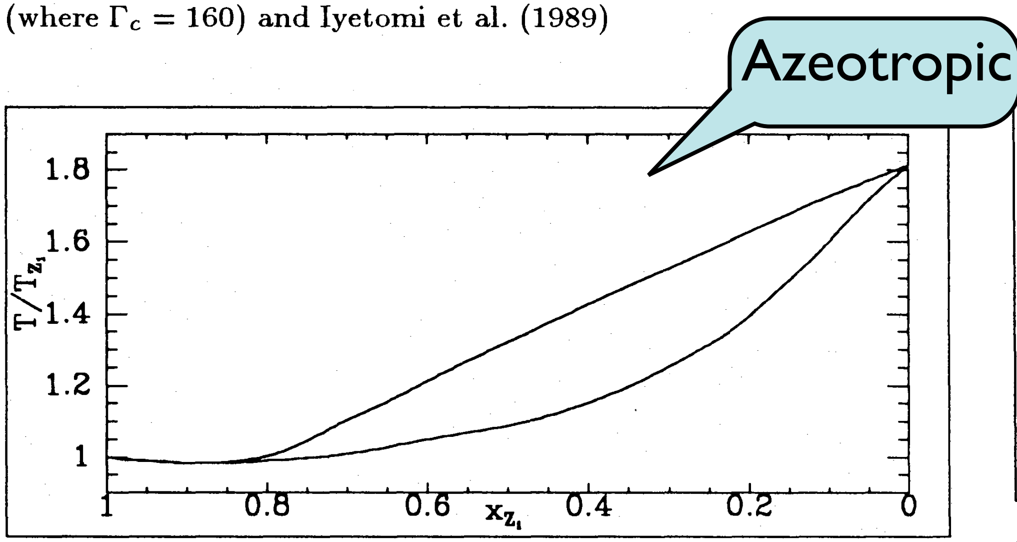


Fig. 4. Same as Fig. 3 for a charge ratio $Z_1/Z_2 = 0.70$. The coordinates of the azeotrope point are $T_a = 0.97\, T_{Z_1}$ and $x_a = 0.87$

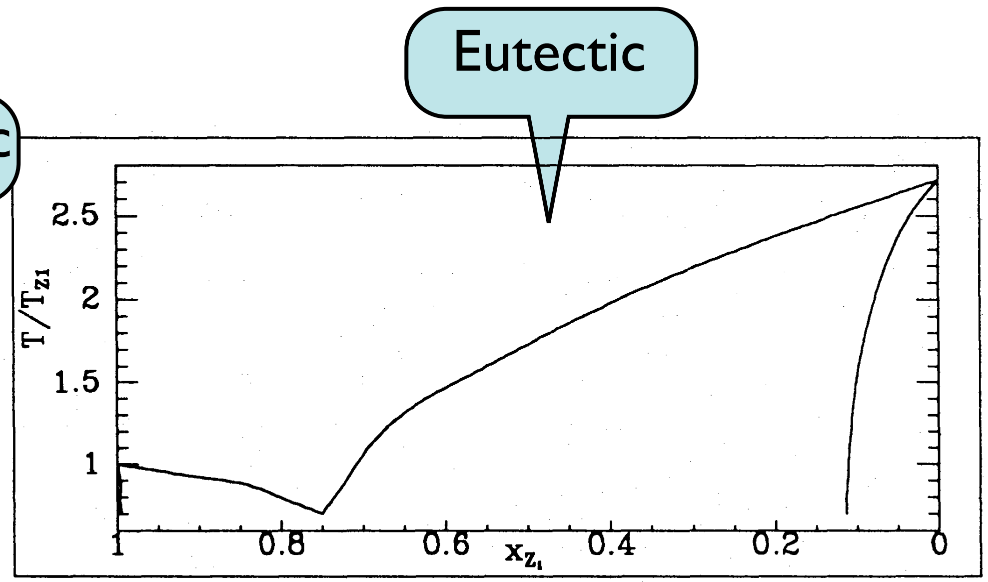
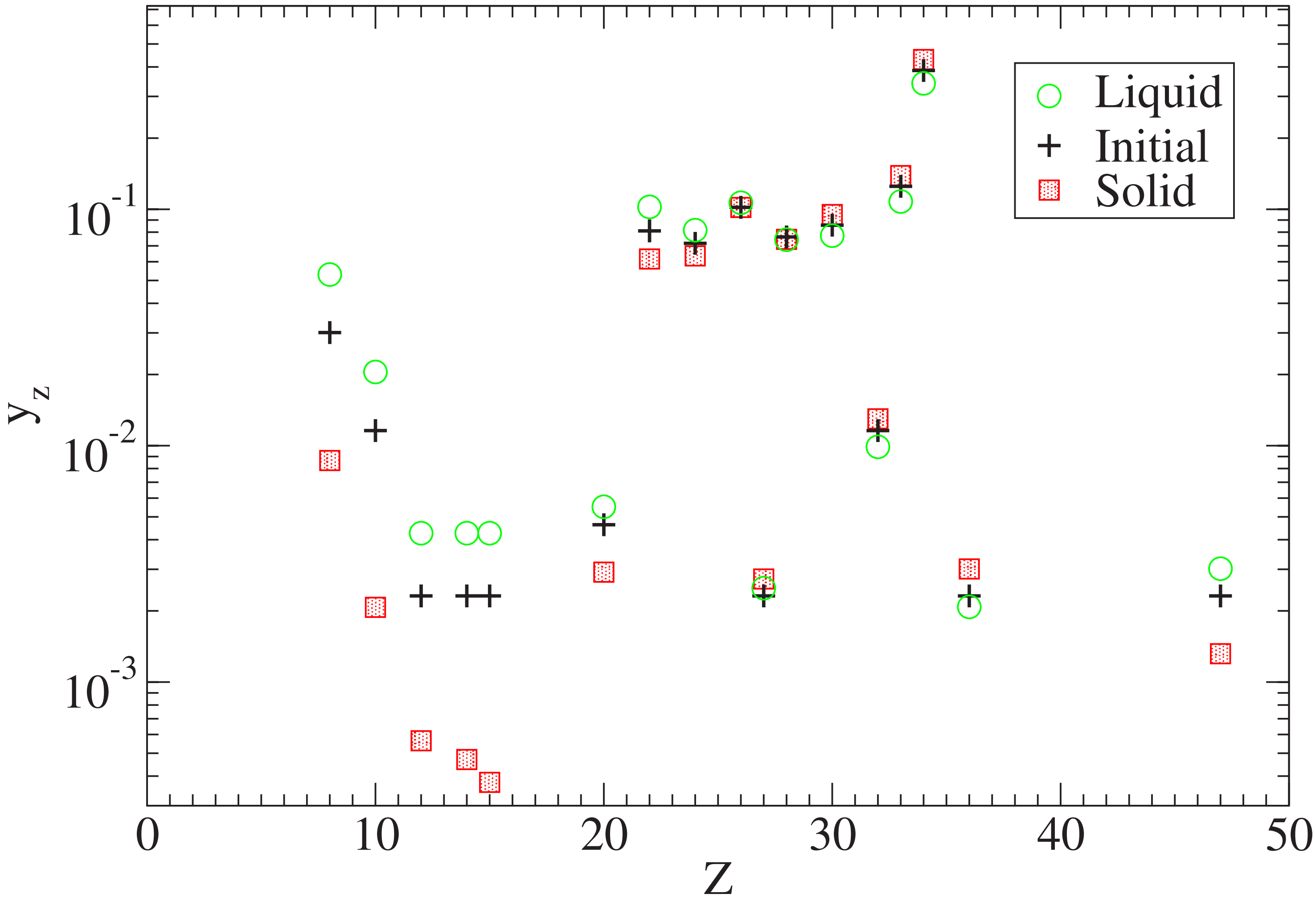


Fig. 5. Same as Fig. 3 for a charge ratio $Z_1/Z_2 = 0.55$. The coordinates of the eutectic point are $T_e = 0.7\, T_{Z_1}$ and $x_e = 0.75$

Horowitz et al. simulated a 17 component plasma



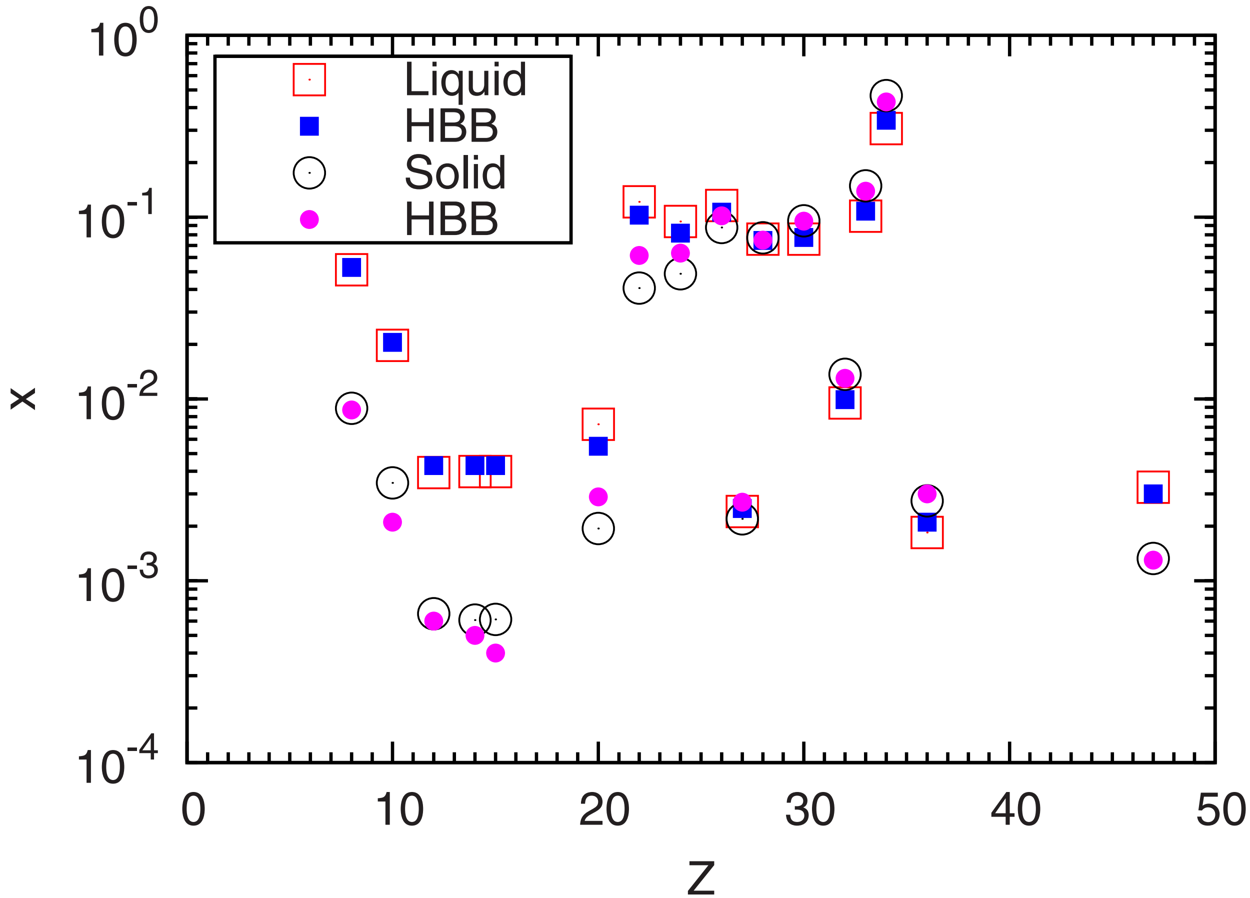
Idea: use fits to the free-energies of one and two component plasmas and extend to higher dimensions

$$f_s^{\mathrm{MCP}}(\Gamma_1, x_1, \ldots, x_{m-1}) \simeq \sum_{i=1}^{m} x_i \left[f_s^{\mathrm{OCP}}(\Gamma_i) + \ln\left(x_i \frac{Z_i}{\langle Z \rangle} \right) \right] + \Delta f_s(\Gamma_1, x_1, \ldots, x_{m-1}) .$$

$$\Delta f_s(\Gamma_1, x_1, \ldots, x_{m-1}) \simeq \sum_{i=1}^{m-1} \sum_{j=i+1}^{m} \Gamma_i x_i x_j \Delta g \left(\frac{x_j}{x_i + x_j}, \frac{Z_j}{Z_i} \right)$$

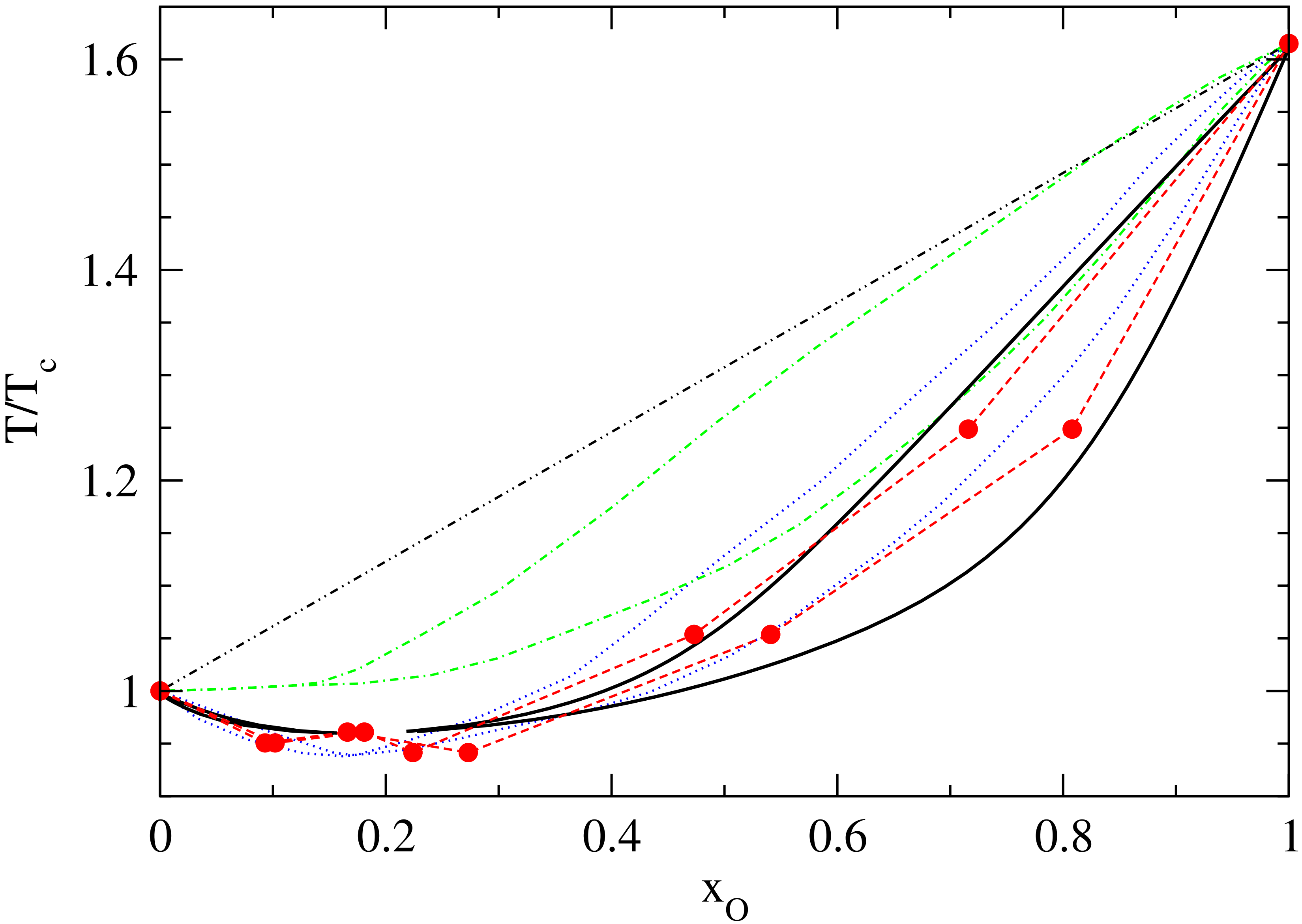
use Ogata et al.'s fitting formula for the deviation from linear mixing, applied between all pairs of nuclei in the mixture

Analytic model reproduces the MD results to 10's of percent



Medin & Cumming (2010)

Another comparison: freezing of a CO mixture



Horowitz et al. (2010)

White dwarf cooling sequence in NGC 6397

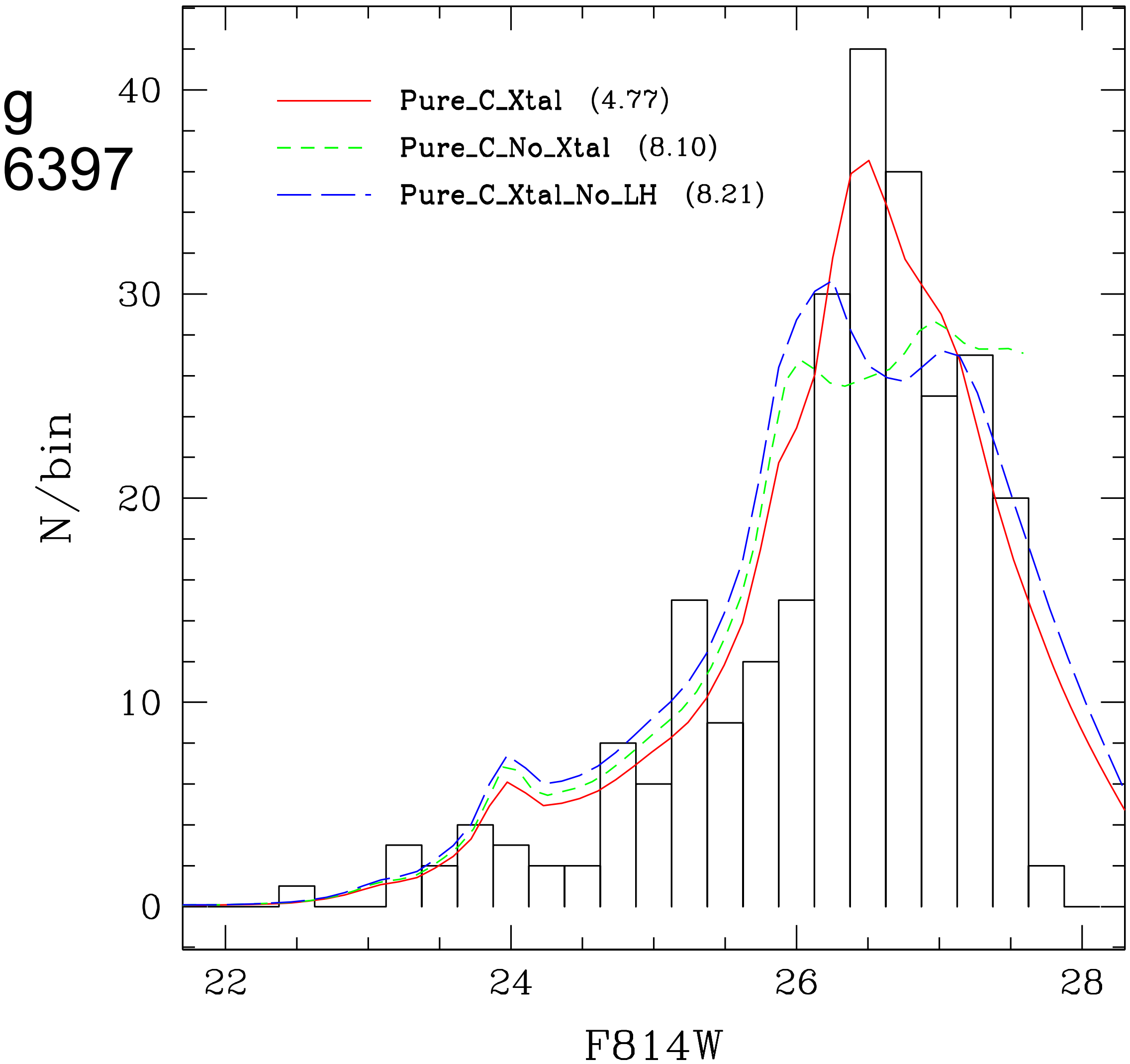
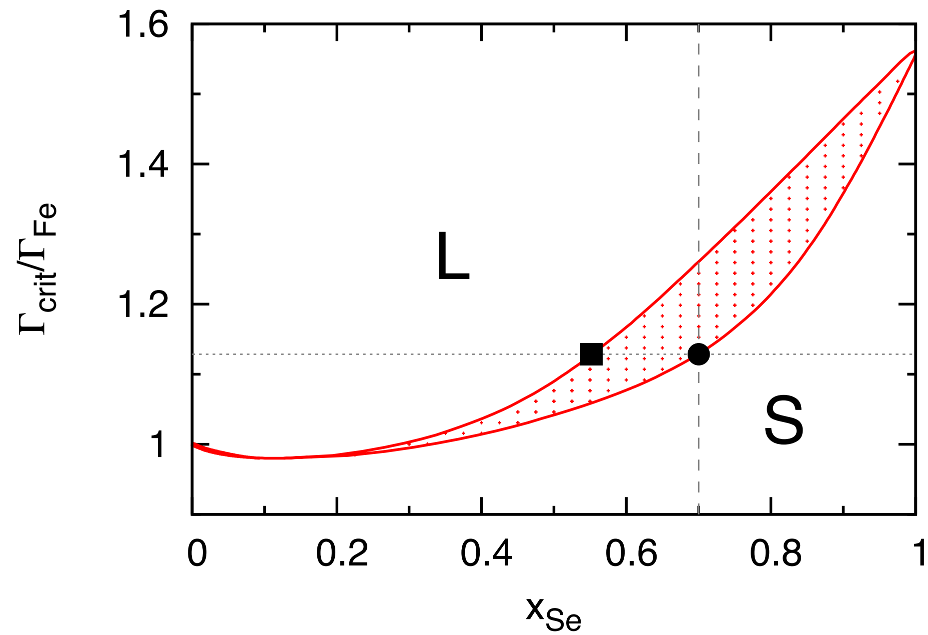


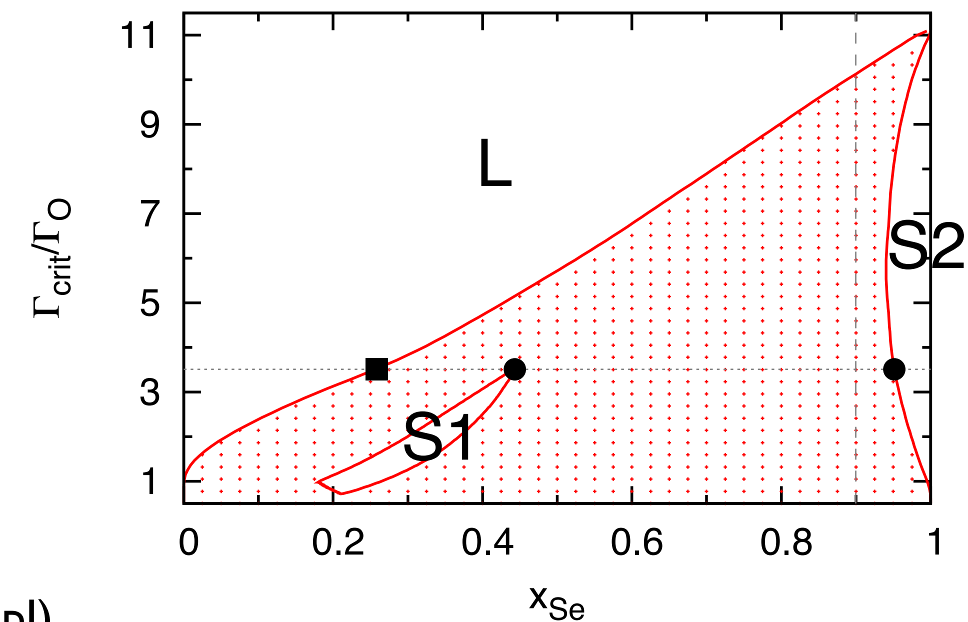
Figure 4. Observed WD LF of NGC 6397 (Richer et al. 2008, histogram) with LFs from theoretical evolutionary sequences of 0.5 $M_{\odot}$ DA models with pure carbon cores (lines): crystallization with (Pure_C_Xtal) and without (Pure_C_Xtal_No_LH) the release of latent heat, and excluding the physics of crystallization altogether (Pure_C_No_Xtal). The normalization of the theoretical curves is chosen to minimize the rms residuals in the neighborhood of the peak, between the magnitudes of 25.1 and 27.7, the faintest value calculated for the no crystallization case. The value of the average residual for each curve is listed in the legend, e.g., it is 4.77 for the "Pure_C_Xtal" case.

Winget et al 2009

Light elements are left behind in the ocean, what is the steady state?



Ocean must be enriched in light elements at the base in order to freeze out the correct mixture



Medin & Cumming (2011, ApJ)

Crystallization of solid particles

Can argue that solid particles rapidly form and "rain out" at the base of the ocean

- Accretion time is long

$$t_{\rm accr} = 3.2\ \frac{y_{12}}{\dot{m}_4}\ {\rm yrs} = 7.2\ \frac{\rho_9^{4/3}}{\dot{m}_4}\left(\frac{Y_e}{0.43}\right)^{4/3}\left(\frac{g_{14}}{2.45}\right)^{-1}\ {\rm yrs}$$

- Nucleation rate is not well-understood, but rapid
- Once nucleated, crystal growth likely diffusion limited

$$\omega_p = \sqrt{4\pi\rho}(Y_e e/m_p) = 1.4\times 10^{19}\rho_9^{1/2}(Y_e/0.43)\ {\rm rad/s}$$

- Estimate sedimentation velocity from Einstein relation

$$v_{\rm sed} \simeq \frac{D}{k_B T}A_s N_s m_p g\frac{\Delta Y_e}{Y_e}$$

- compare to accretion velocity

$$N_{s,\rm crit} = 200\ \rho_9^{-0.6}\left(\frac{T_8}{3}\right)^{-0.45}\ \dot{m}_4^{3/2}\left(\frac{\Delta Y_e/Y_e}{0.01}\right)^{-3/2}$$

Compositional buoyancy and convection

Brunt-Vaisala frequency

$$N^2 = -g\mathscr{A} = -g\left(\frac{d\ln\rho}{dz} - \frac{1}{\Gamma_1}\frac{d\ln p}{dz}\right)$$

Two-component mixture

$$\mathcal{A}H_P = \frac{\chi_T}{\chi_\rho}\left(\nabla - \nabla_{\rm ad}\right) + \frac{\chi_X}{\chi_\rho}\nabla_X$$

Maximum composition gradient that can be supported by thermal buoyancy

$$\nabla_{X,\rm max} = \frac{\chi_T}{\chi_X}\left(\nabla_{\rm ad} - \nabla\right) \approx \frac{\chi_T\nabla_{\rm ad}}{\chi_X}$$

Mixing length theory arguments

Thermal convection

$$v_{\rm conv}^2 \approx gl(\nabla - \nabla_{ad})$$

$$F \approx \rho v c_P T(\nabla - \nabla_{ad}) \approx \rho v^3$$

often the convection is efficient

$$v \ll c_s \qquad \nabla \approx \nabla_{ad}$$

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often the convection is efficient

$$v \ll c_s \qquad \nabla \approx \nabla_{ad}$$

Here we have (assume isothermal)

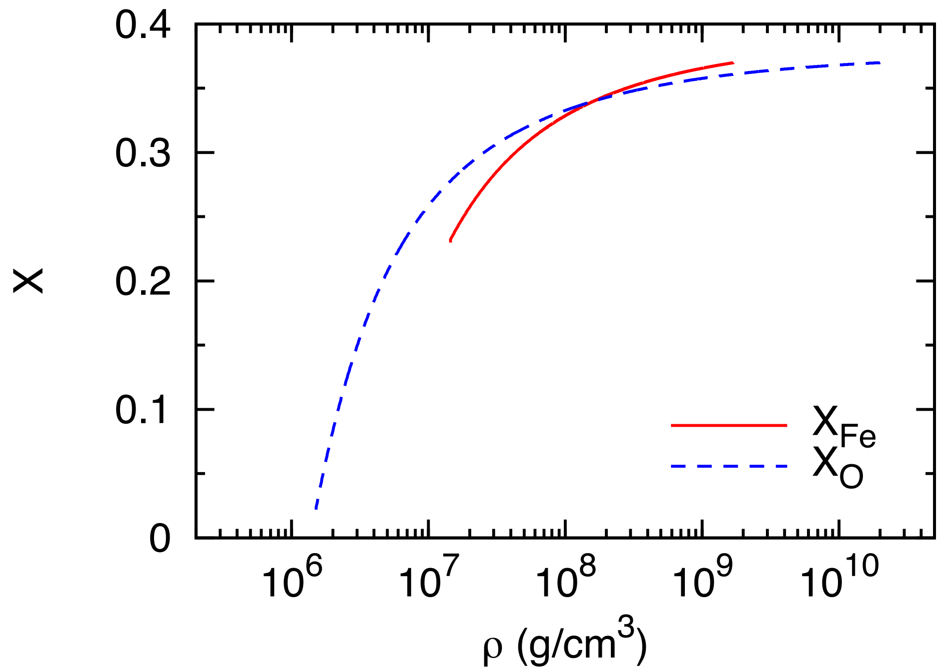
$$v_{\rm conv}^2 \sim g l_m^2 \frac{\chi_X \nabla_X - \chi_T \nabla_{\rm ad}}{H_P \chi_\rho}$$

$$F_X \sim \rho v_{\rm conv} \frac{l_m X \nabla_X}{H_P}$$

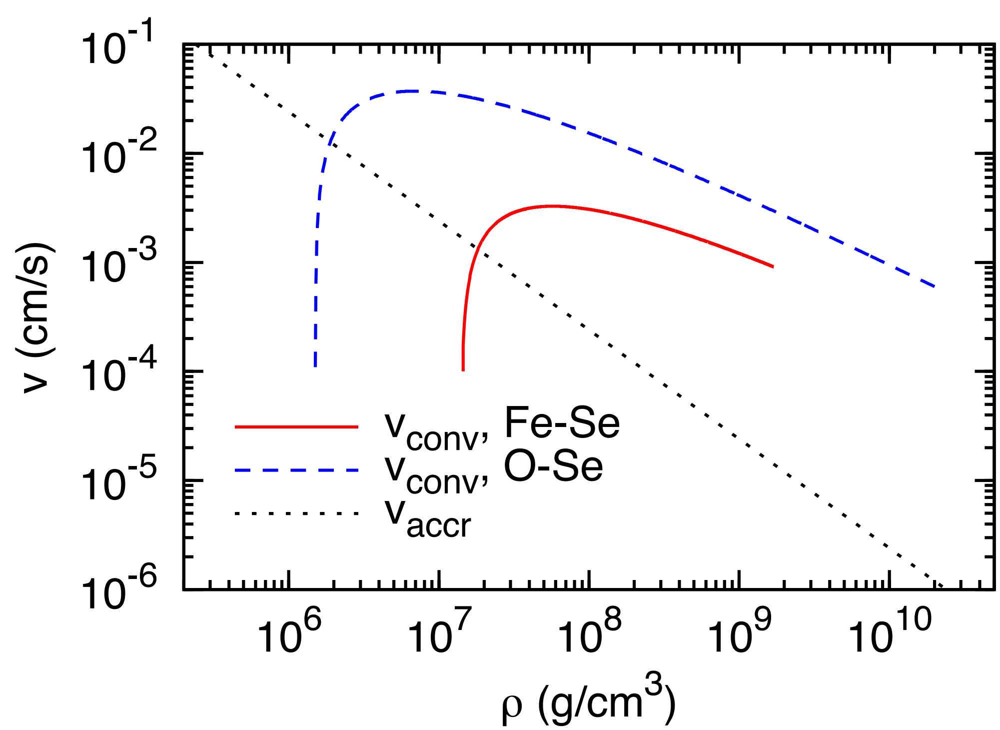
The flux must be $F_X = \dot{m}(X_0 - X_b)$

Only a small super-adiabaticity is needed => $\nabla_X \approx \nabla_{X,\rm max}$

Two-component steady state

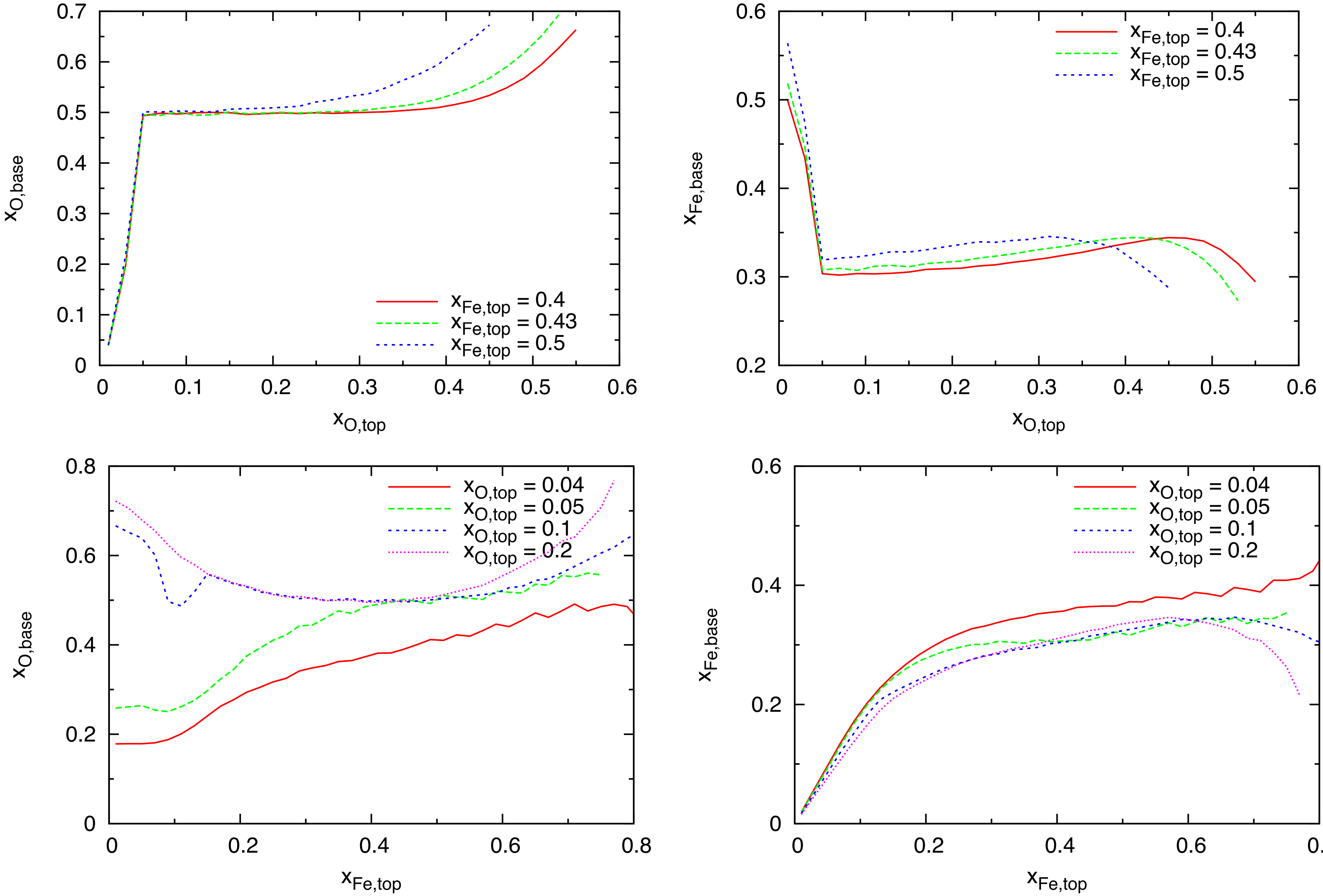


Significant enrichment in light elements. Steady-state composition determined by the phase diagram.



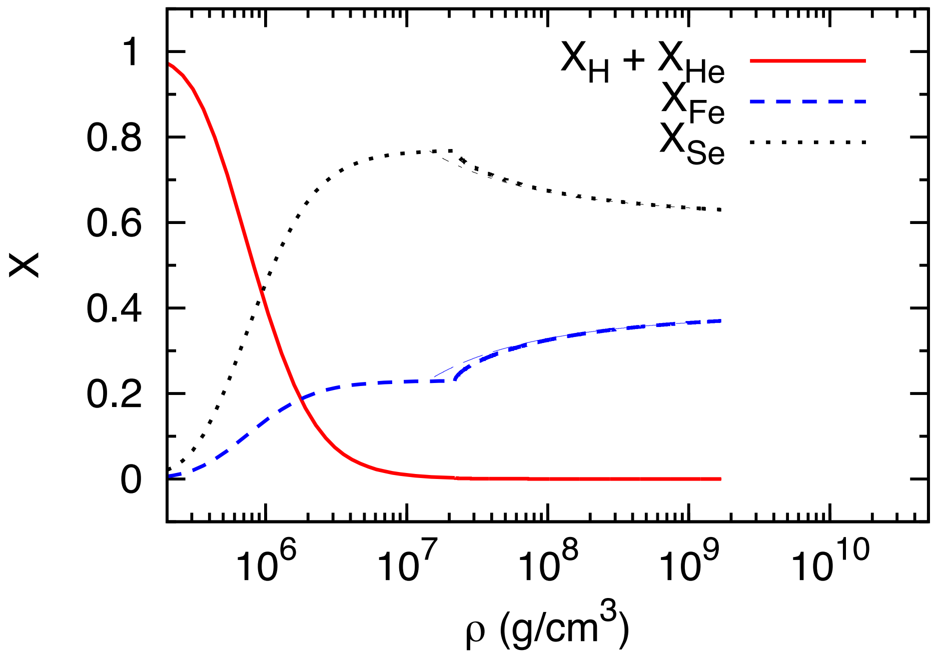
$$\chi_T \approx \left(\frac{T}{P_e}\right)\left(\frac{\partial P_i}{\partial T}\right) \sim \frac{10}{Z}\frac{k_B T}{E_F}$$

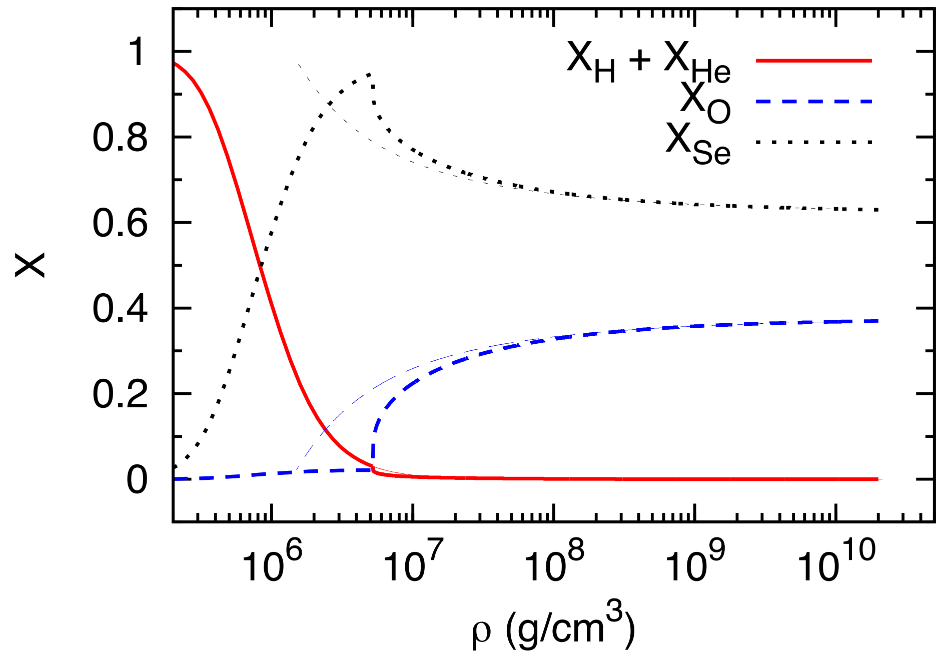
The final ocean composition is insensitive to the incoming composition: O/Fe/Se example



Including burning of H/He to heavy elements

$$\frac{dX_i}{dt} + \mathbf{v}_{\rm accr} \cdot \nabla X_i = -\frac{1}{\rho}\nabla \cdot (\rho \mathbf{v}_{\rm conv} D X_i) + X_i^{\rm burn} X_1 R_{3\alpha}$$



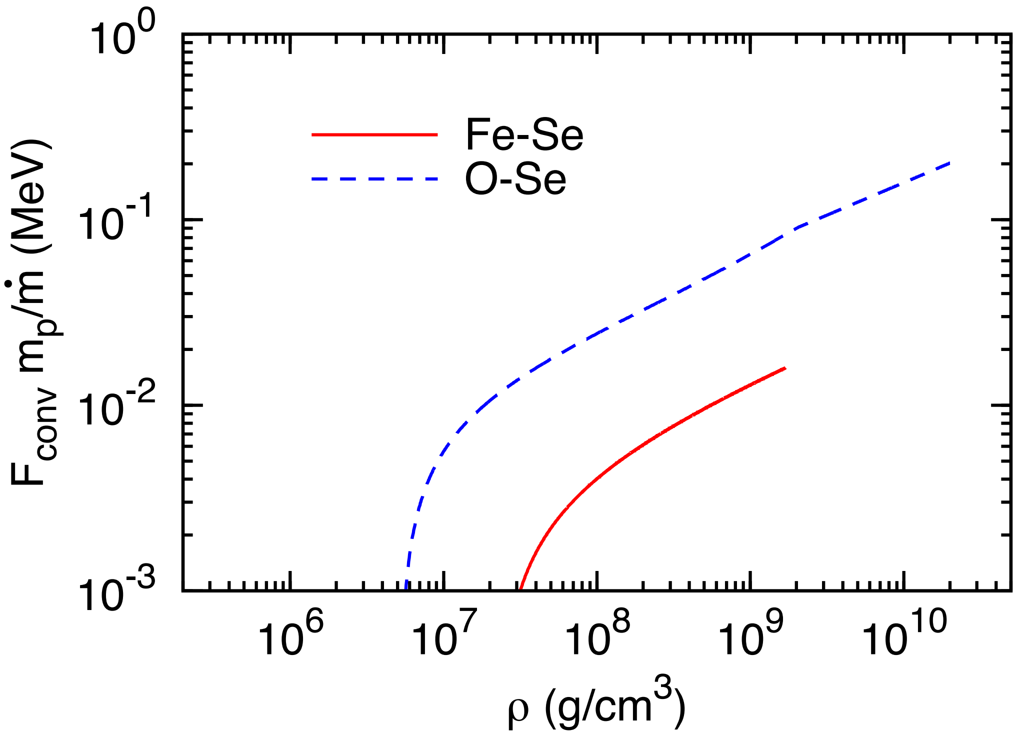


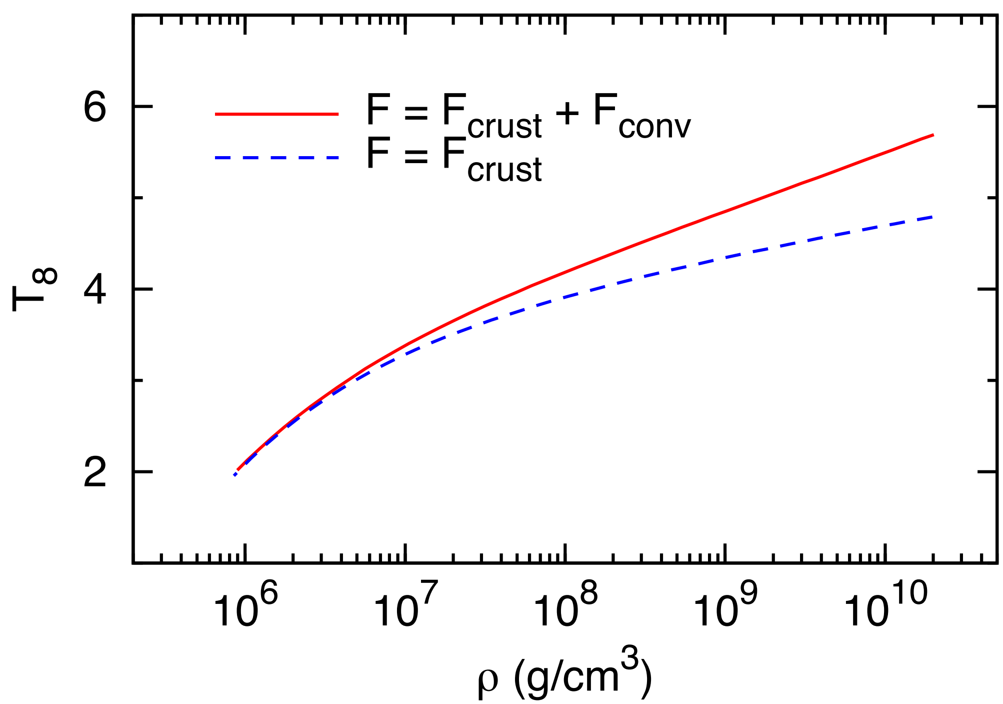
Effect on the thermal profile

This convection transports heat *inwards*

$$\mathbf{F}_{\rm conv} = \frac{\xi}{2}\rho \mathbf{v}_{\rm conv} c_P T(\nabla - \nabla_{\rm ad})$$

In steady state compensated by outwards heat flux carried by thermal conduction => acts to increase the ocean temperature





Convection in Earth's core

Chemical separation at the boundary between the Earth's inner and outer core drives convection

e.g. Stevenson 1981,2003

Interesting differences:

to carry away the latent heat requires $\nabla > \nabla_{ad}$ thermal convection

a convective outer core is possible since $\nabla_{ad} < \nabla_L$

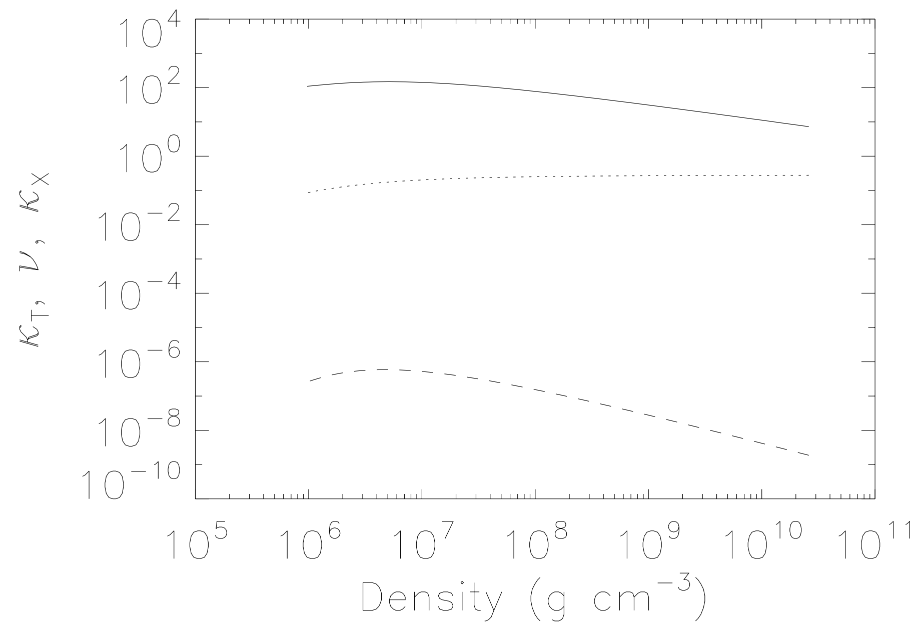
(for a while the opposite was thought to be true, "core paradox" Higgins & Kennedy 1971)
neutron star case has $\nabla_{ad} > \nabla_L$ so the ocean could never be (completely) thermally convective

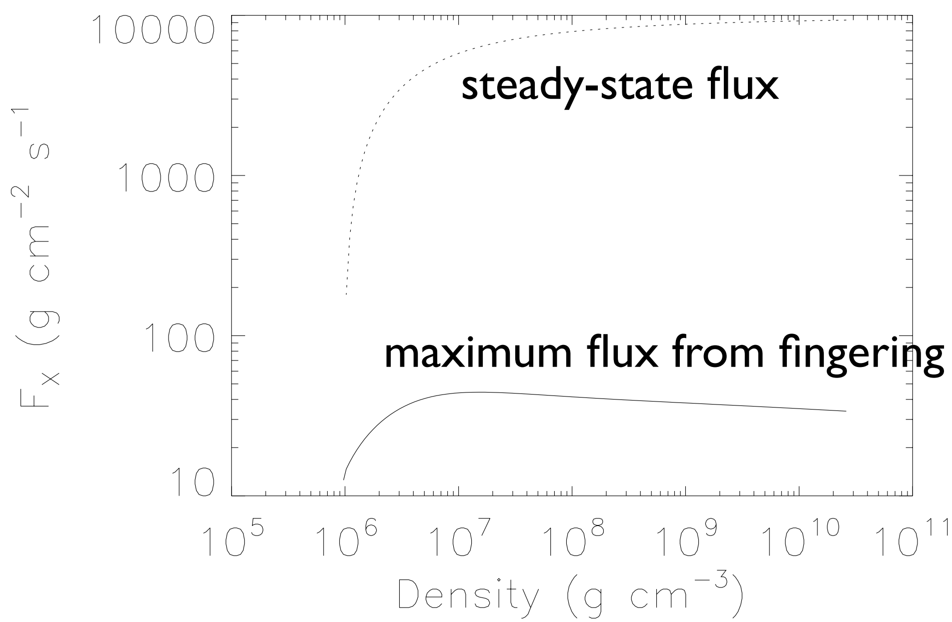
Doubly-diffusive instabilities: fingering convection

If doubly-diffusive salt finger instability was efficient enough the flux of composition could be transported without the ocean becoming fully-convective.

Recent simulations have determined the compositional flux using parameters appropriate for astrophysical environments.
Maximum diffusivity $D \approx 101\sqrt{\nu\kappa_\mu}$

Traxler et al. (2011) Garaud (2011)





$$\mathrm{Pr} = \frac{\nu}{\kappa_T} \sim 10^{-3} - 10^{-1} \qquad \tau = \frac{\kappa_\mu}{\kappa_T} \sim 10^{-9}$$

(interestingly fingering does not transport significant heat flux)

Conclusions

- Chemical separation at the base of the ocean leads to continual mixing *Compositionally-driven convection*

- The convection leads to an enrichment in light elements, and an almost uniform composition over the bulk of the ocean. This final ocean composition is set by the phase diagram.

- The convection transports heat inwards => heating of the ocean as the temperature gradient steepens to balance the inwards flux with an outwards conductive flux. When light elements are present Z~8 this can be ~0.1-0.2 MeV/nucleon

Open issues

• Need to survey the possible enrichment as a function of Z_1/Z_2 for two species, and explore N>2 mixtures

•At high accretion rates, the convective heat flux can only be balanced if $\nabla > \nabla_{ad}$ => steady state not possible. What happens?? What is the time dependent evolution?

•The mixing velocity is extremely slow ~100 x accretion velocity => need to worry about other physics, especially B fields, rotation

• After an accretion outburst, the neutron star cools and the ocean freezes - is there a signature in the lightcurve?