THERMO-COMPOSITIONAL CONVECTION, AND ITS RELEVANCE TO PLANET FORMATION AND EVOLUTION

Pascale Garaud, UC Santa Cruz
Adrienne Traxler, UC Santa Cruz
Giovanni Mirouh, ENS Cachan
Erica Rosenblum, Stony Brooks
Stephan Stellmach, U Muenster
Nic Brummell, UCSC
Timour Radko, NPS Monterey

THE LAST OF THE "MOHEGANS", AND WHAT HAPPENED TO THE OTHERS…

Lecture plan:

- Thermocompositional convection: the basics
- Outstanding questions

Thermo-compositional convection: the basics

- What we expect naively
- Double-diffusive effects
  - The fingering instability (thermohaline convection)
  - The oscillatory instability (semiconvection)
- Why is this relevant to planet formation/evolution?
- Mathematical model and linear stability
- Outstanding questions

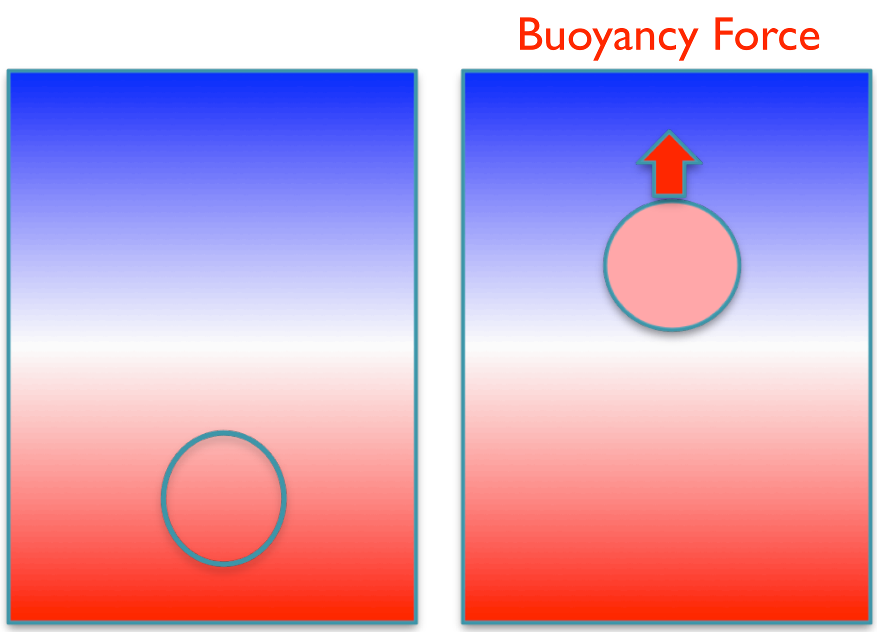
What we expect naively

- Overturning convection is a linear instability of stratified fluids with "top-heavy" density profiles, and occurs whenever $\rho_z > 0$
  - If density depends on temperature only, then we have thermal convection.

Instability criterion:

$$\rho \propto -T \Rightarrow$$

$$\rho_z > 0 \Leftrightarrow T_z < 0$$



What we expect naively

Note that I just swept 2 things under the carpet!

1. Why is $\rho \propto -T$ and what is the proportionality constant ?

In reality $\rho = \rho(T)$ but if density (or temperature) doesn't vary too much in the region considered, we can linearize this to:

$$\rho = \rho(T_0) + \frac{\partial \rho}{\partial T}(T - T_0) = \rho_0 + \frac{\partial \rho}{\partial T}(T - T_0)$$

$$\Rightarrow \tilde{\rho} = \frac{\partial \rho}{\partial T}\tilde{T} \Rightarrow \boxed{\frac{\tilde{\rho}}{\rho_0} = -\alpha \tilde{T}} \quad \text{if} \quad \alpha = -\frac{1}{\rho_0}\frac{\partial \rho}{\partial T}$$

For a perfect gas, $\alpha \approx \dfrac{1}{T_0}$

What we expect naively

- Note that I just swept 2 things under the carpet!

2. What about compressibility ?

In the previous argument, the system is assumed incompressible. In most astrophysical systems, it is not.

The correct criterion for **instability** is

$$\left(\frac{\partial\rho}{\partial p}\right)_{ad} > \left(\frac{\partial\rho}{\partial p}\right)$$

which translates, in terms of temperature, into the Schwarzchild criterion:

$$\left(\frac{\partial \ln T}{\partial \ln p}\right)_{ad} < \left(\frac{\partial \ln T}{\partial \ln p}\right)$$

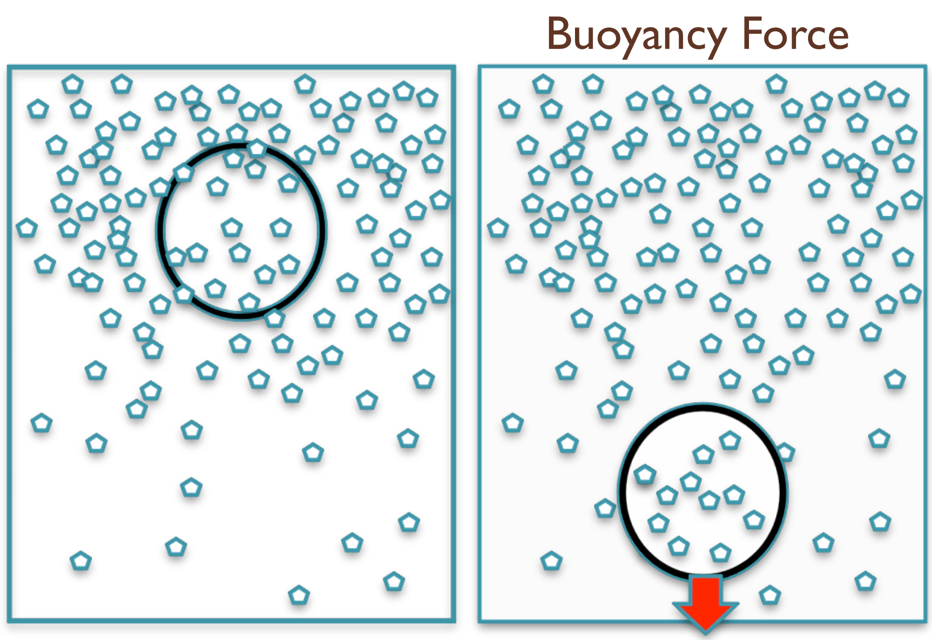
What we expect naively

- If density depends on composition only then we have compositional convection.

Instability criterion:

$$\rho = \beta S \Rightarrow$$

$$\rho_z > 0 \Leftrightarrow S_z > 0$$



What we expect naively

- Thermo-compositional convection is what happens when density depends both on temperature and composition.
- With

$$\rho = -\alpha T + \beta S \Rightarrow \rho_z = -\alpha T_z + \beta S_z$$

the new criterion for instability for overturning convection is

$$-\alpha T_z + \beta S_z > 0$$

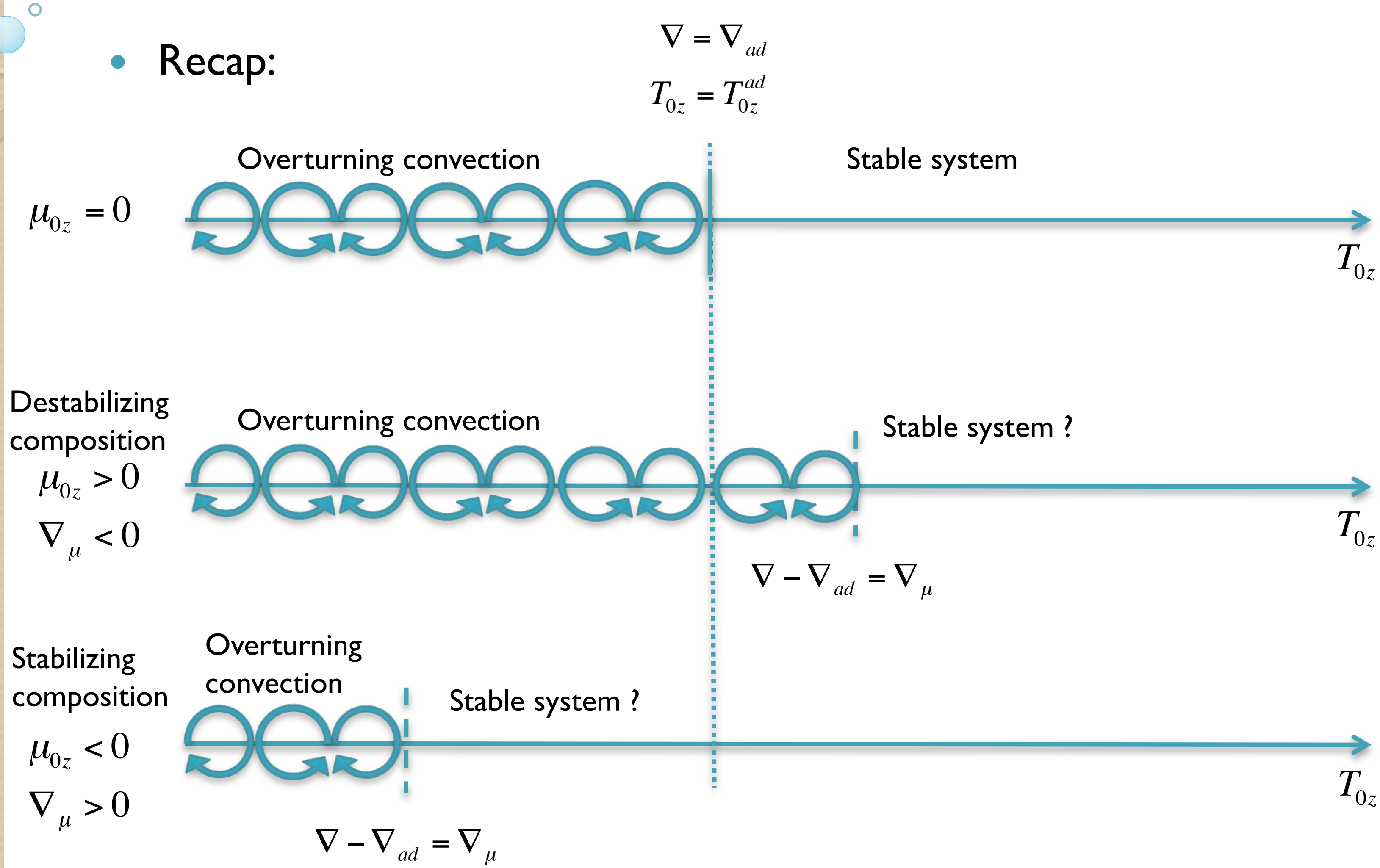
- For compressible fluids, the equivalent criterion for instability is the Ledoux criterion

$$\nabla - \nabla_{ad} + \nabla_{\mu} > 0 \Leftrightarrow$$

$$\left(\frac{\partial \ln T}{\partial \ln p}\right) - \left(\frac{\partial \ln T}{\partial \ln p}\right)_{ad} > \left(\frac{\partial \ln \mu}{\partial \ln p}\right)$$

What we expect naively

- Recap:



Thermo-compositional convection: the basics

- What we expect naively
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  - The fingering instability (thermohaline convection)
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Double-diffusive instabilities

- While this criterion "correctly" accounts for the onset of overturning thermo-compositional convection, this only begins to scratch the surface of the problem of mixing…

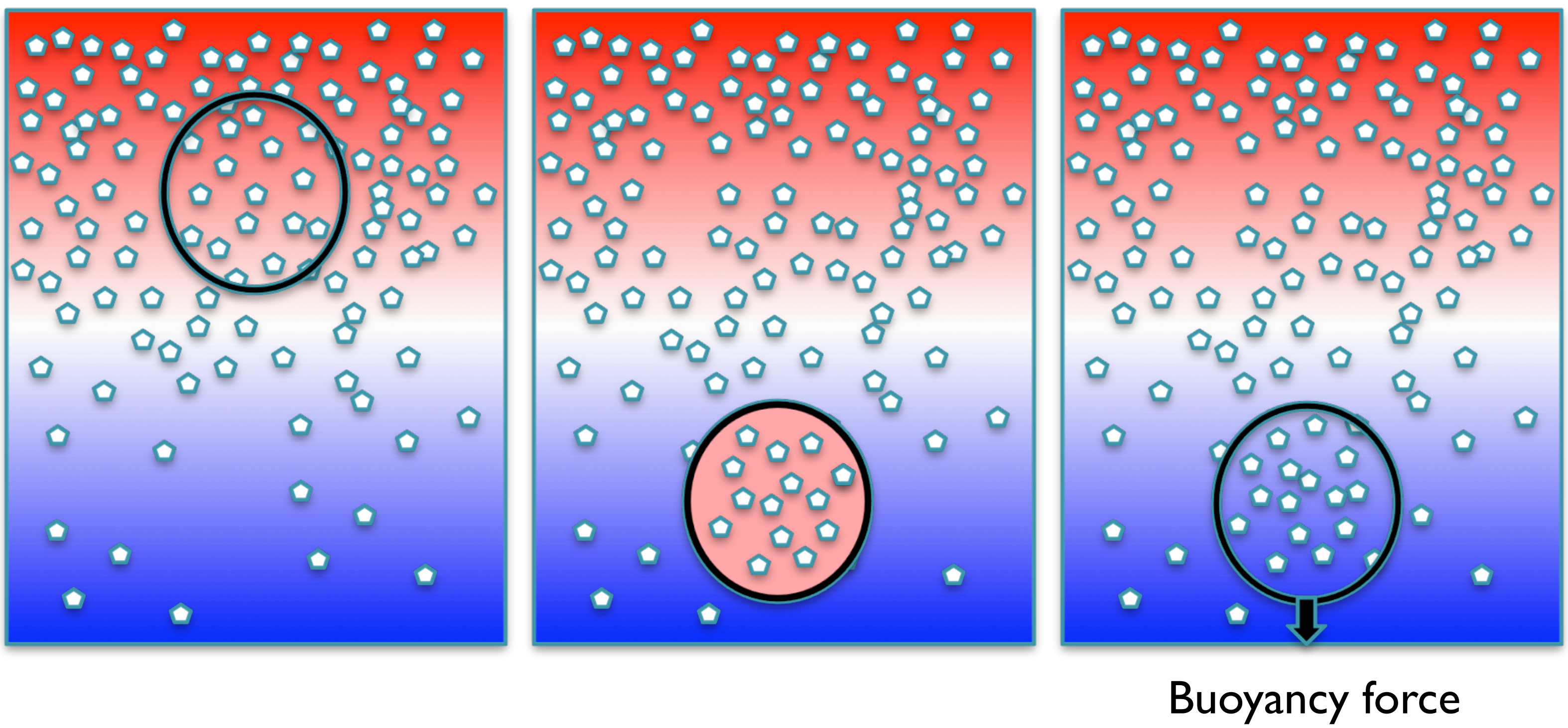
- Usually, the two components diffuse at different rates, with chemical species diffusion being much slower than heat.

- When this is the case, new linear instabilities can occur *even in the case of stably stratified density profiles.*

The fingering instability

- Example of the tropical ocean: the fingering instability.

$$\overline{T}_z, \overline{S}_z > 0 \text{ and } \overline{\rho}_z < 0$$



The fingering instability

- Fingering convection manifests itself in the form of long, thin, “fingers” of hot/salty, cold/fresh plumes of water.

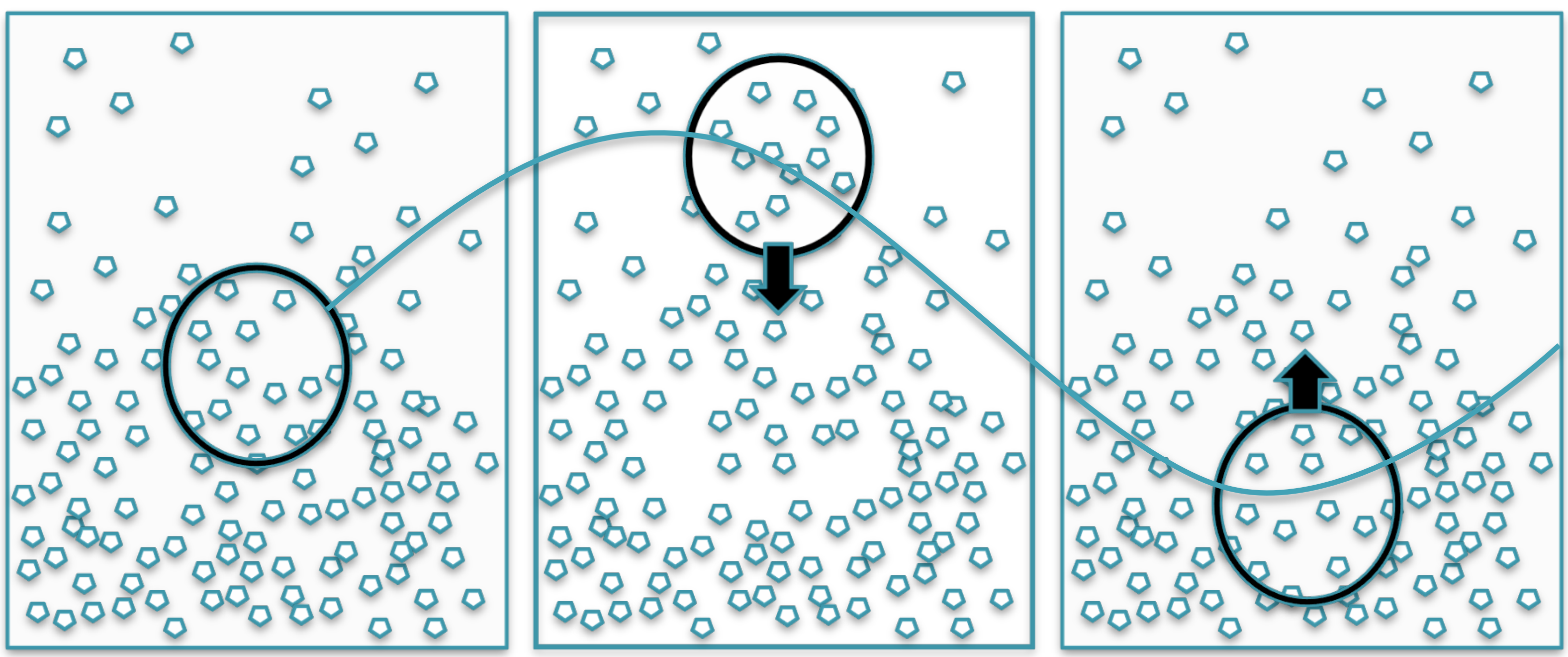
Hot & Salty dyed water

Cold & Fresh water

The oscillatory instability

- In compositionally stably stratified fluids

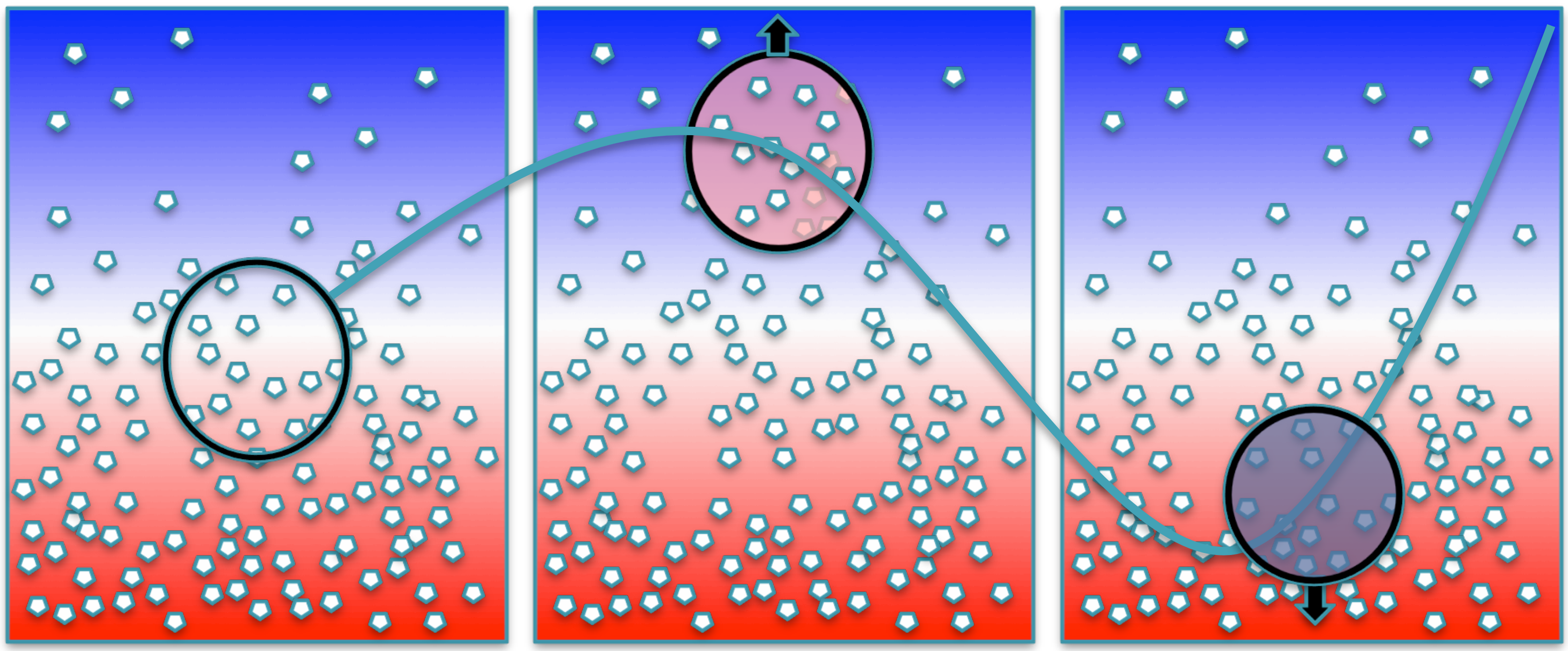
$$\overline{T}_z = 0,\ \overline{S}_z < 0 \text{ and } \overline{\rho}_z < 0$$



The oscillatory instability

- Now assume an unstable temperature gradient (cf. polar ocean): the oscillatory instability.

$$\overline{T}_z,\ \overline{S}_z < 0 \text{ and } \overline{\rho}_z < 0$$



Regimes of double-diffusive convection

- Summary: Double-diffusive instabilities occur when density depends on 2 components, which diffuse at different rates.

  - **Fingering regime ("thermohaline")**: temperature/entropy is stably stratified, composition unstable

    Direct instability, long tall finger-like plumes.

  - **Oscillatory regime (semiconvection)**: temperature/entropy is unstably stratified, composition stable

    Oscillatory instability, overstable gravity waves.

Many other types of DD-instabilities exist:

- Angular momentum vs entropy: the GSF instability
- Magnetic buoyancy vs entropy: magnetic DD
- Etc..

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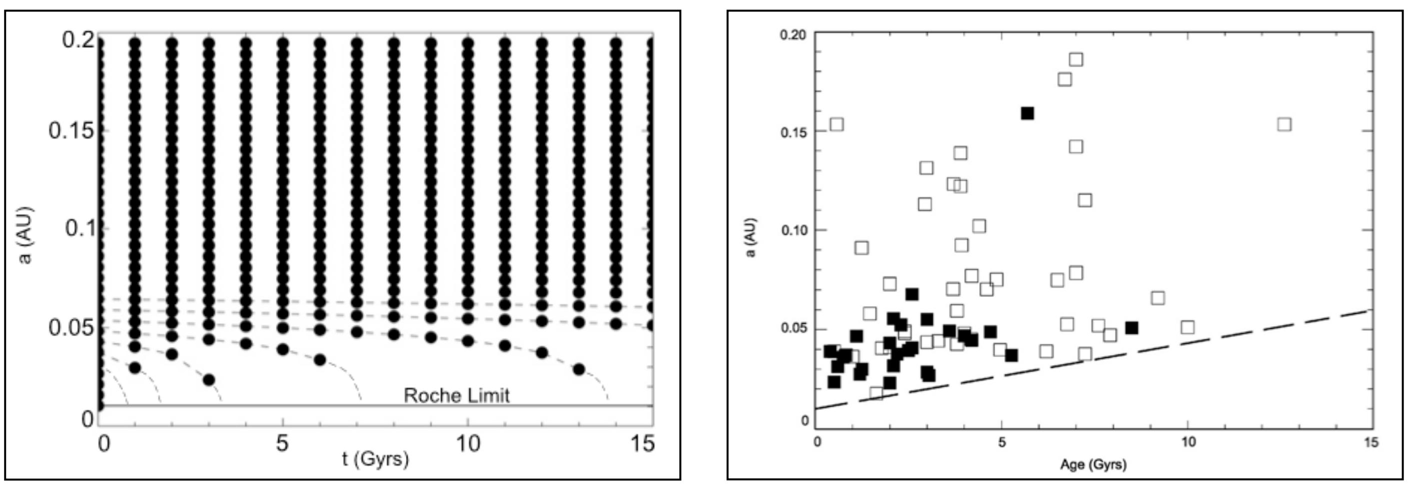
Fingering convection in astrophysics

- Example in the astrophysical context: the "Missing Mohicans" problem.
  - Interactions between the planet and the disk (cf. migration), or the planet and another planet (secular chaos) thought to lead to very close orbits (3-day period planets)
  - Tidal interactions with the central stars then causes further orbital decay on Gyr timescale.

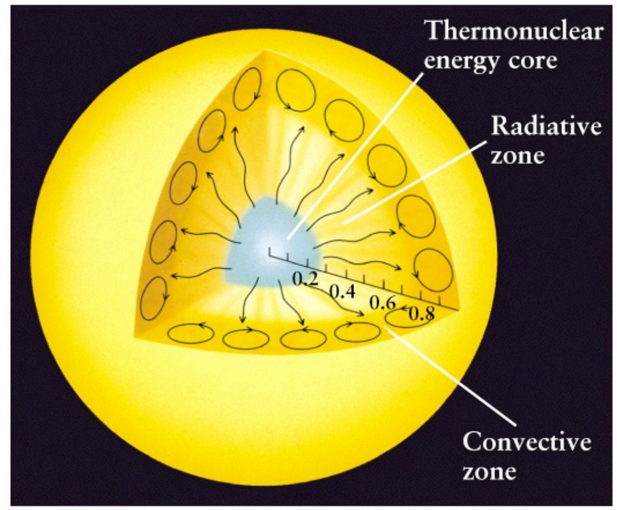
Jackson et al 2009



Fingering convection in astrophysics

- Example in the astrophysical context: the "Missing Mohicans" problem.

  - Solar-type stars have outer convective zones.
  - If infalling planetary material is mixed within the outer convection zone only, we may expect to see (Gonzalez, 1997):



  - Higher metallicity (dispersion) in planet-host stars
  - Even higher metallicity (dispersion) in planet-host stars with shallower convection zone.

Fingering convection in astrophysics

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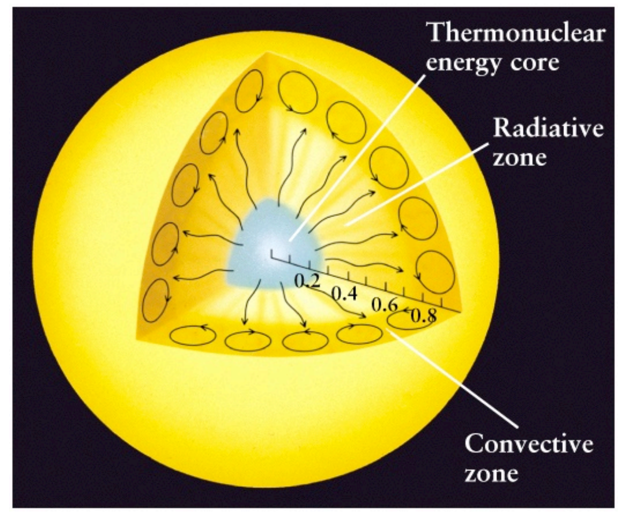
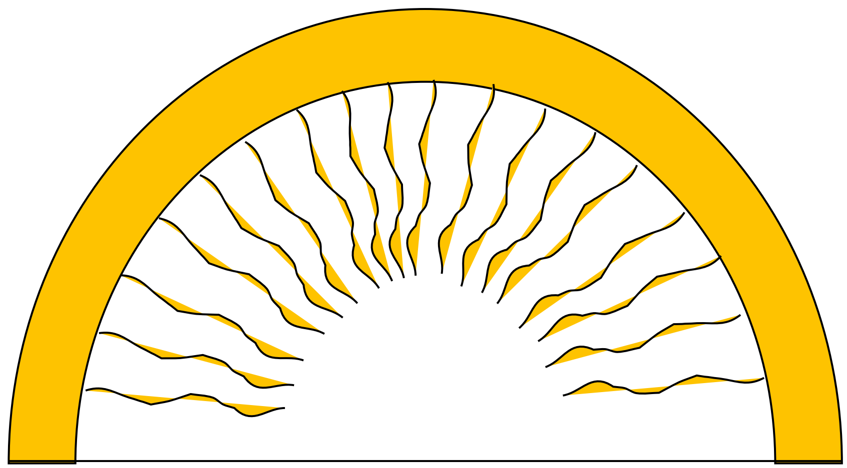


Not observed!

Fingering convection in astrophysics

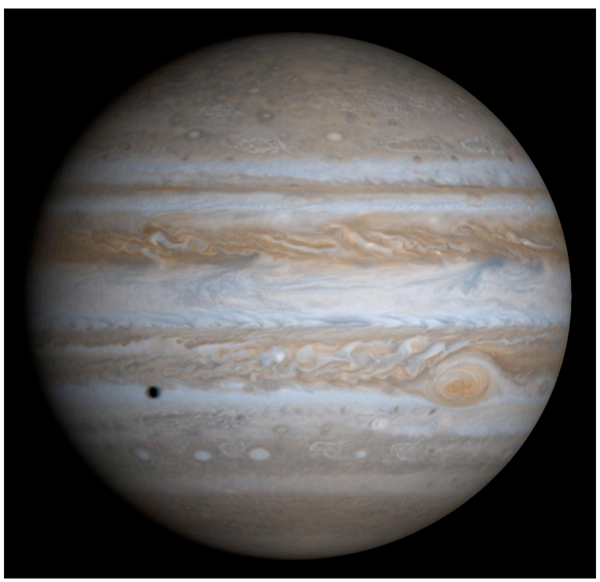
- Example in the astrophysical context: the "Missing Mohicans" problem.
  - Possible resolution of the problem: dilution of the additional metallicity by fingering convection (Vauclair 2004)



  - But until recently, we had no reliable estimate of the mixing timescale by fingering convection in stellar environments.

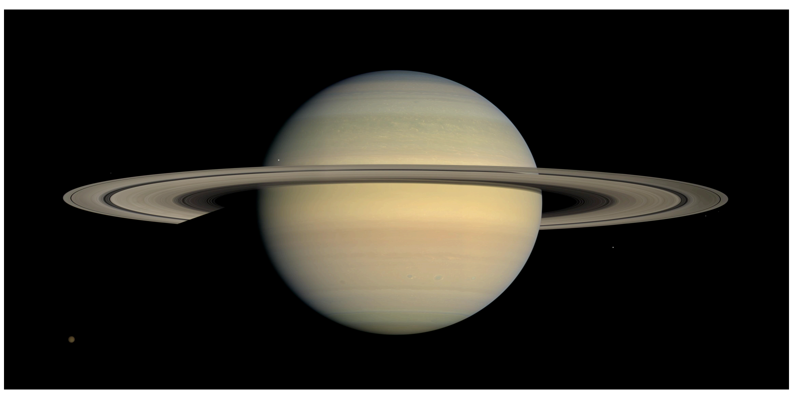
Oscillatory convection in astrophysics

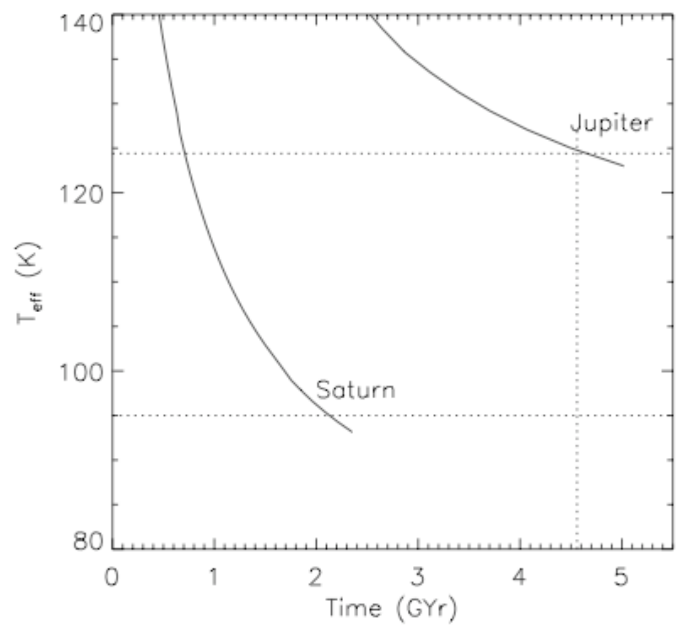
- Example in the astrophysical context: The effect of compositional gradients in giant planets
  - Standard planetary structure/evolution theory fails to fit observations for all 4 giant planets in our solar system:
    - Jupiter's present core is probably too small to account for envelope accretion on reasonable timescale



Oscillatory convection in astrophysics

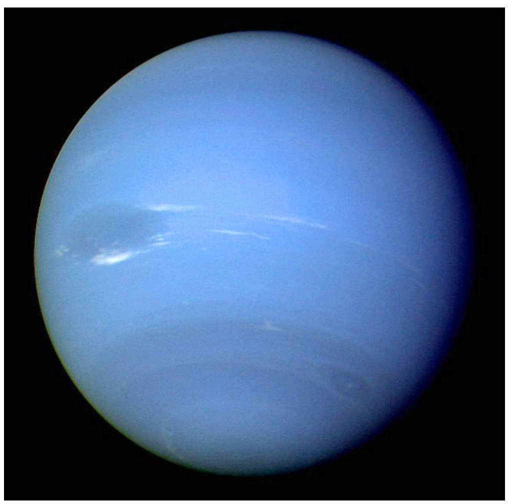
- Example in the astrophysical context: The effect of compositional gradients in giant planets
  - Standard planetary structure/evolution theory fails to fit observations for all 4 giant planets in our solar system:
    - Saturn's current luminosity is much higher than predicted by cooling models

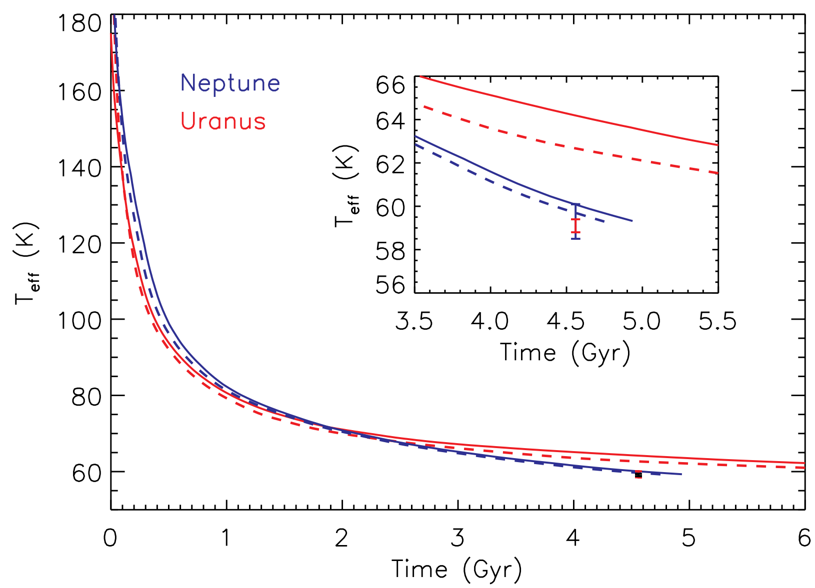




Oscillatory convection in astrophysics

- Example in the astrophysical context: The effect of compositional gradients in giant planets
  - Standard planetary structure/evolution theory fails to fit observations for all 4 giant planets in our solar system:
    - Uranus' intrinsic heat flux is consistent with zero, contrary to models, and Neptune's intrinsic heat flux also too small





Oscillatory convection in astrophysics

- Example in the astrophysical context: The effect of compositional gradients in giant planets
  - Core erosion by convective motions could explain:
    - Jupiter's present core size
    - The existence of compositional gradients in Uranus/Neptune which could stifle convective efficiency
  - Helium "rain-out" in Saturn could also create He compositional gradients which would then inhibit convection and prolong cooling.

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Mathematical model

- Governing equations (Boussinesq approximation):

$$\frac{\partial u}{\partial t} + u \cdot \nabla u = -\frac{\nabla p}{\rho_0} - \frac{\rho}{\rho_0} g \hat{e}_z + \nu \nabla^2 u$$

$$\frac{\partial T}{\partial t} + u \cdot \nabla T = \kappa_T \nabla^2 T$$

$$\frac{\partial S}{\partial t} + u \cdot \nabla S = \kappa_S \nabla^2 S$$

$$\nabla \cdot u = 0$$

$$\frac{\rho}{\rho_0} = -\alpha T + \beta S$$

Mathematical model

Goal: to study double-diffusive instability "in the field".

- Double-diffusive convection scale much, much smaller than system scale.
- Model considered here:
  - Assume **background** temperature and concentration profiles are linear (constant gradients T_{0z}, S_{0z})
  - Assume that all **perturbations** are triply-periodic in domain (L_x,L_y,L_z):

$$q(x,y,z,t) = q(x+L_x,y,z,t)$$
$$= q(x,y+L_y,z,t) = q(x,y,z+L_z,t)$$

  - As a result,

$$T(x,y,z,t) = L_z T_{0z} + T(x,y,z+L_z,t)$$

Mathematical model

- Governing non-dimensional equations (+ fingering case and – oscillatory case):

$$\frac{1}{\Pr}\left(\frac{\partial u}{\partial t} + u\cdot\nabla u\right) = -\nabla p + (T - S)\mathbf{e_z} + \nabla^2 u$$

$$\frac{\partial T}{\partial t} + u\cdot\nabla T \pm w = \nabla^2 T$$

$$\frac{\partial S}{\partial t} + u\cdot\nabla S \pm \frac{w}{R_0} = \tau\nabla^2 S$$

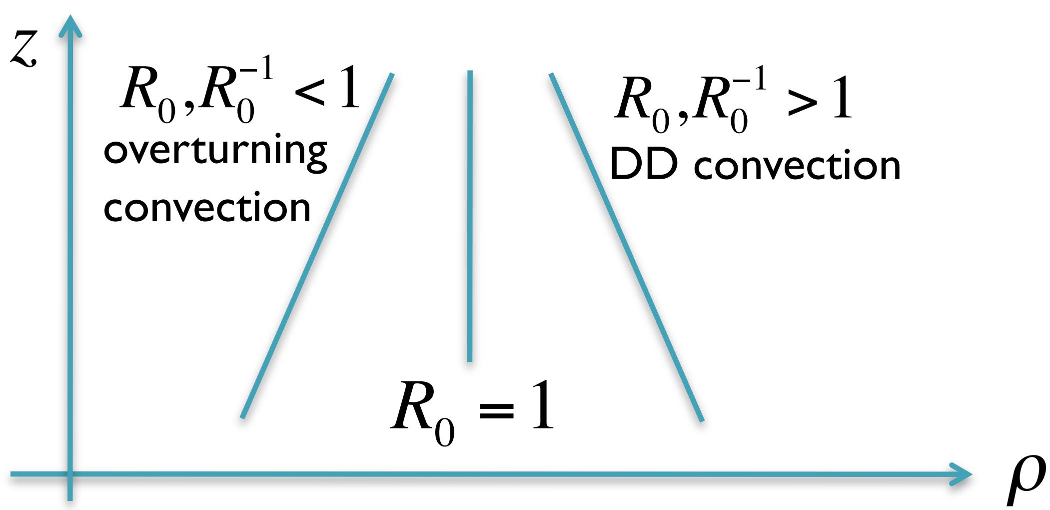
$$\nabla\cdot u = 0$$

$$\boxed{\Pr = \frac{\nu}{\kappa_T}, \quad \tau = \frac{\kappa_S}{\kappa_T} \qquad R_0 = \frac{\alpha T_{0z}}{\beta S_{0z}}}$$

$$[l] = d = \left(\frac{\kappa_T \nu}{\alpha g |T_{0z}|}\right)^{1/4}, \quad [t] = \frac{d^2}{\kappa_T}, \quad [T] = d|T_{0z}|, \quad [S] = \frac{\alpha}{\beta} d |T_{0z}|$$

Mathematical model

- The dynamics of double-diffusive instabilities depends principally on the non-dimensional **density ratio:**
- Fingering: $$R_0 = \frac{\alpha T_{0z}}{\beta S_{0z}} = \frac{\text{Stabilizing temperature stratification}}{\text{Destabilizing salinity stratification}}$$
- Oscillatory: $$R_0^{-1} = \frac{\beta S_{0z}}{\alpha T_{0z}} = \frac{\text{Stabilizing salinity stratification}}{\text{Destabilizing temperature stratification}}$$



Mathematical model

- Linear stability analysis:
  - Normal mode solution

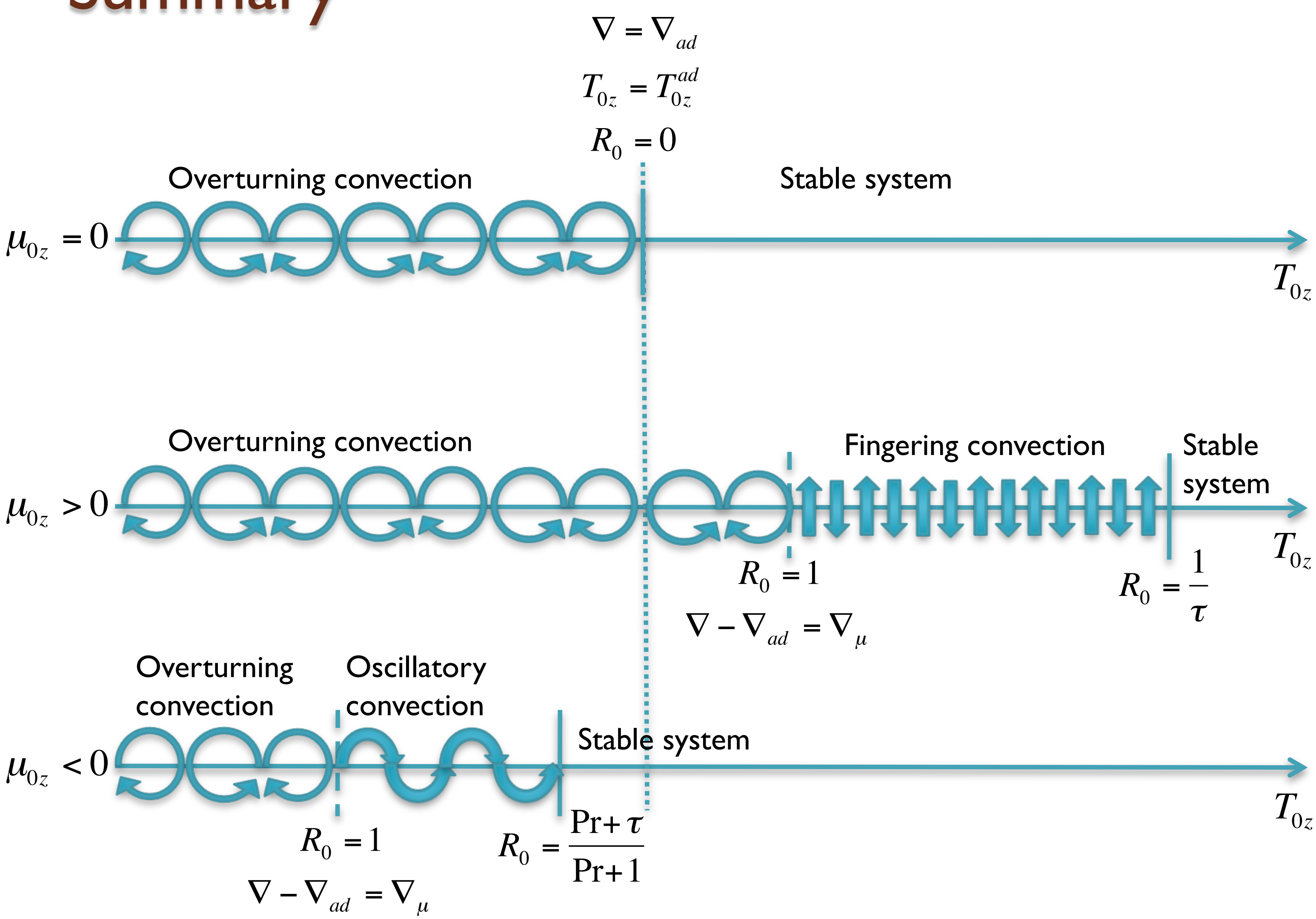
$$q = \hat{q} e^{ik_x x + ik_y y + ik_z z + st}$$

  - Growth rate satisfies a cubic equation in s which depends on **k**, R_ρ, Pr and τ,
  - Fastest growing modes are "elevator modes" ($k_z = 0$)
  - Instability only occurs for

$$1 < R_0 < \frac{1}{\tau} \quad \text{in fingering case}$$

$$1 < R_0^{-1} < \frac{\Pr + 1}{\Pr + \tau} \quad \text{in oscillatory case}$$

Summary



$\nabla = \nabla_{ad}$
$T_{0z} = T_{0z}^{ad}$
$R_0 = 0$
Overturning convection
Stable system
$\mu_{0z} = 0$
T_{0z}
Overturning convection
Fingering convection
Stable system
$\mu_{0z} > 0$
T_{0z}
$R_0 = 1$
$\nabla - \nabla_{ad} = \nabla_\mu$
$R_0 = \frac{1}{\tau}$
Overturning convection
Oscillatory convection
Stable system
$\mu_{0z} < 0$
T_{0z}
$R_0 = 1$
$\nabla - \nabla_{ad} = \nabla_\mu$
$R_0 = \frac{\mathrm{Pr}+\tau}{\mathrm{Pr}+1}$

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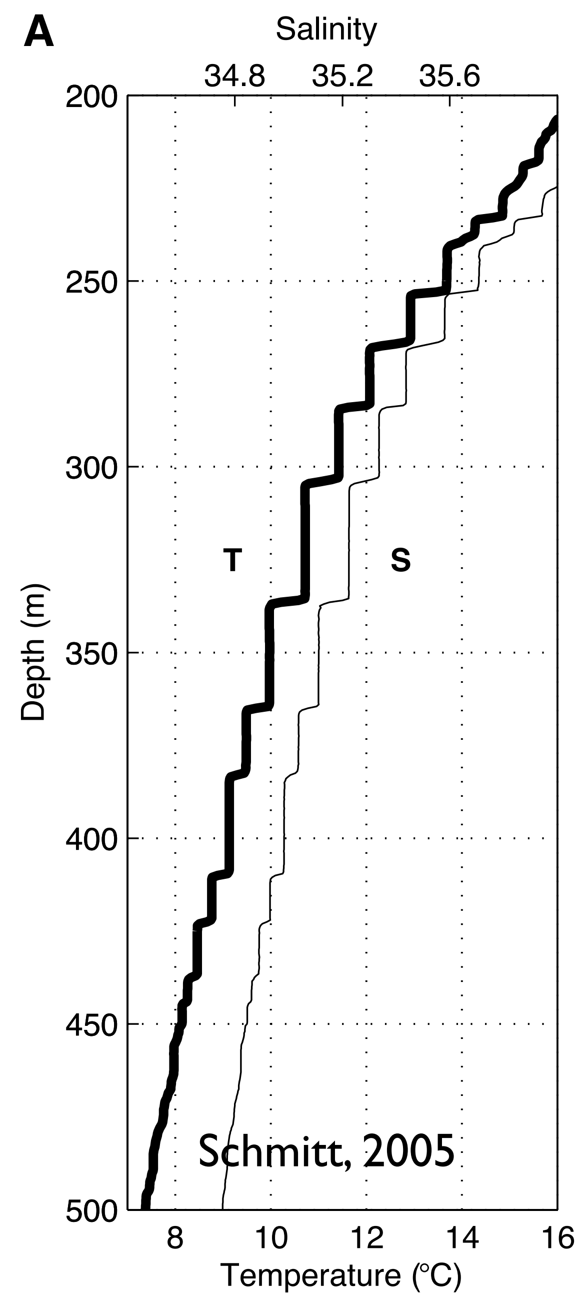
Outstanding questions

- What are the transport properties of double-diffusive convection in various regimes, as function of the background parameters & fluid properties?
- Can we explain and predict the large-scale dynamics observed to be associated with double-diffusive convection, and in particular large-scale gravity waves, and the formation of **thermocompositional staircases**?
- Can we understand how transport is modified in the presence of large-scale structures?
- Does this resolve the discrepancies between theories and observations?

Thermohaline staircases

- Thermohaline staircases are often observed in ocean thermocline with active fingering convection
  - Layers are typically 10m – 100m deep
  - Can have large horizontal extent (hundreds of kilometers)
  - Individual layers persist for months or more
  - **Transport through staircase much larger than through standard fingering convection**
- Similar staircases are observed in the polar ocean, i.e. in oscillatory convection, **with the same increased transport properties.**



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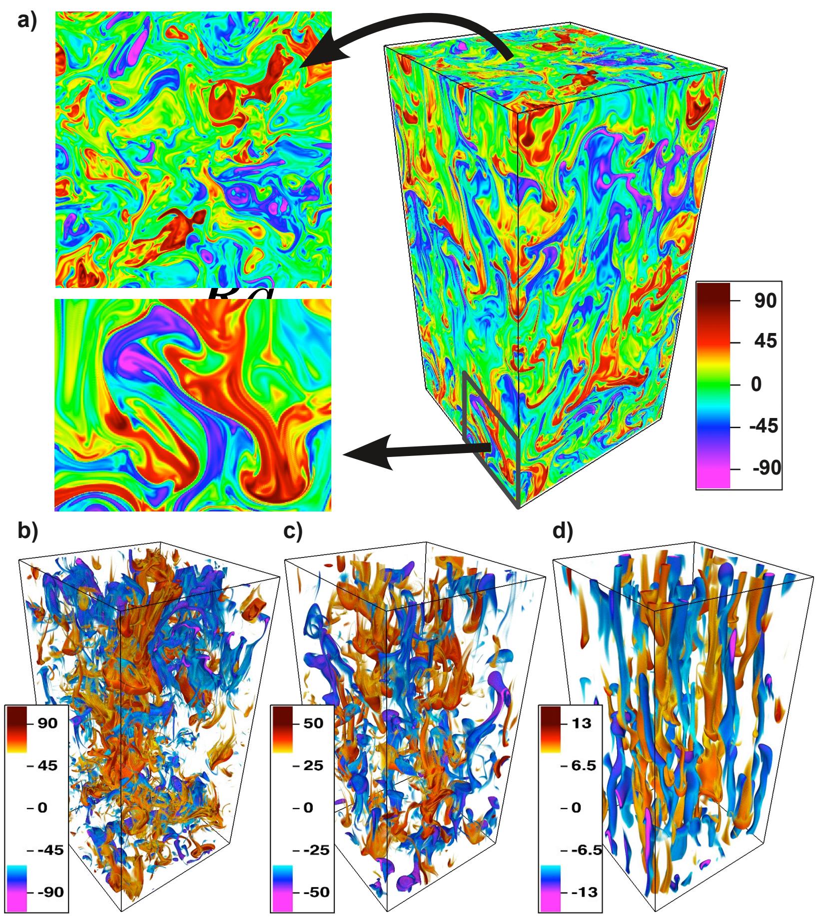
Outstanding questions

- Transport by fingering convection
  - Transport by small-scale motion
  - The formation of large-scale structures
  - Staircases in stellar interiors?
  - Consequences for models of stellar pollution by planetary infall.

- Transport by oscillatory convection
  - Transport by small-scale motion
  - The formation of staircases
  - Transport through a staircase
  - Staircase equilibration?

Numerical experiments

- Stephan Stellmach developed high-performance 3D code to study double-diffusive convection
- Code is pseudo-spectral, triply periodic, fully resolved on all scales.



$\mathrm{Pr} = 7$

$\tau = 0.01$

Numerical experiments

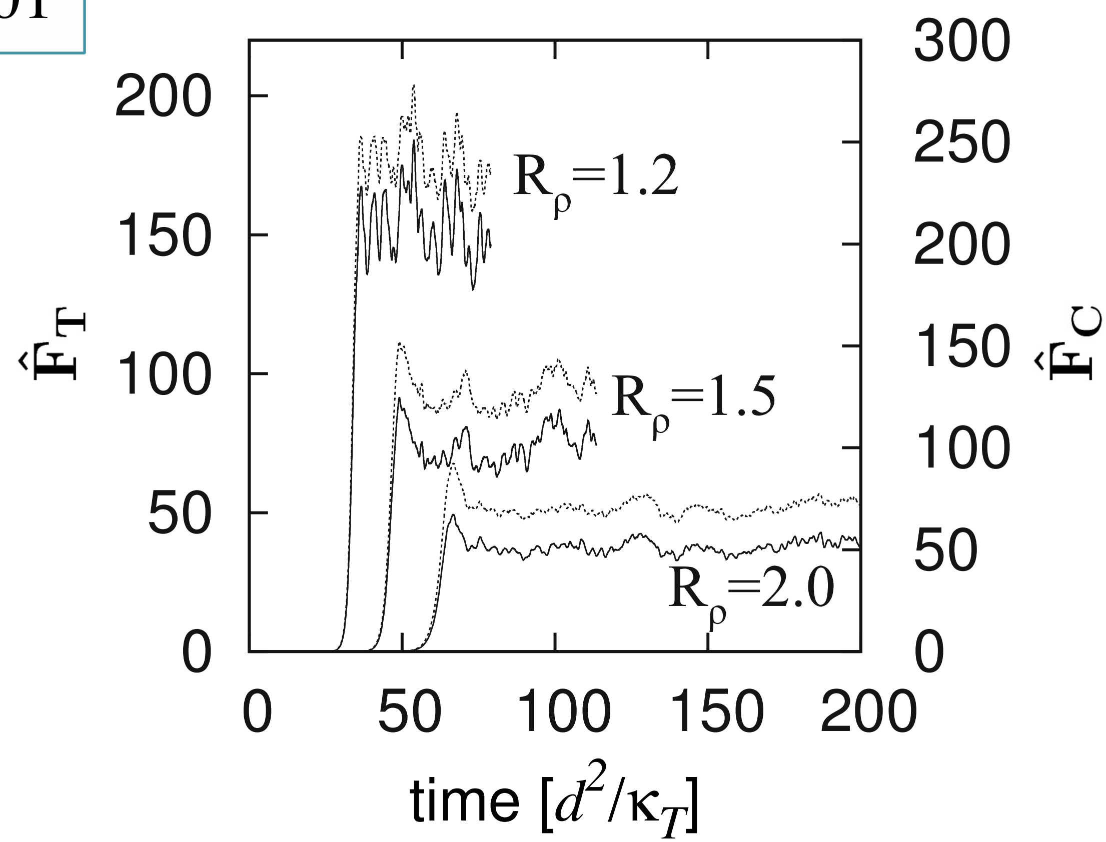
- We define non-dimensional fluxes:
  - The Nusselt number $Nu = \dfrac{\text{Total heat flux}}{\text{Diffused heat flux}}$
  - The total flux ratio $\gamma_{tot} = \dfrac{\text{Total buoy. flux from heat}}{\text{Total buoy. flux from salt}}$

- These quantities are functions of Pr, τ and R_0 and can be measured from "small box" simulations.
  - Box size chosen to contain about 5 x 5 x 10 unstable modes
  - Basic instability grows rapidly, and saturate into homogeneous, statistically steady state.
  - Transport properties are measured.

Numerical experiments

$\mathrm{Pr} = 7$

$\tau = 0.01$



From Traxler et al. 2011

Numerical experiments

- In astrophysical systems, typical parameters Pr and τ are <<1 because thermal diffusion increased by photon transport while other diffusion coefficients are not.

- Planetary interiors: $\mathrm{Pr},\ \tau \approx 10^{-3}$
- Stellar interiors: $\mathrm{Pr},\ \tau \approx 10^{-6}$

- The stellar parameter regime is not achievable numerically – scale separation too large. Planetary regime maybe approachable.

Mixing by fingering convection in astrophysics

- We ran a series of numerical experiments for gradually decreasing values of Pr, and τ between 0.1 and 0.03.

Set	1	2	3	4	5	6
Pr	1/3	1/3	1/10	1/10	1/10	1/30
τ	1/3	1/10	1/3	1/10	1/30	1/10

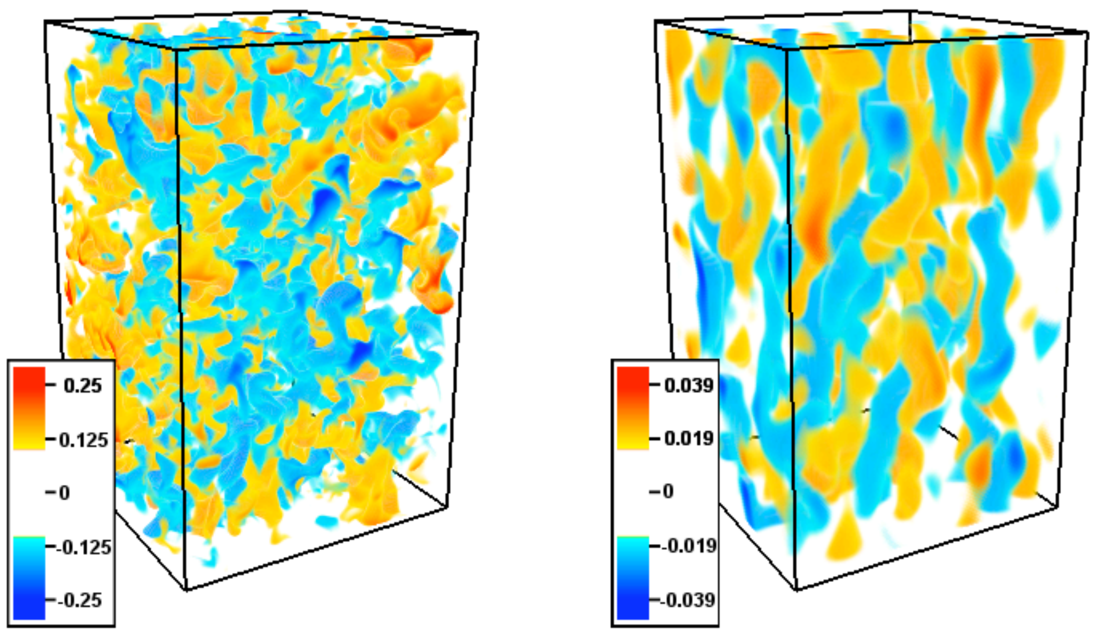
- In each case, density ratio is varied across whole instability range

$$1 < R_0 < \tau^{-1}$$

Mixing by fingering convection in astrophysics

- Fingering convection is more turbulent as R_0 approaches one (overturning convection limit), less turbulent as R_0 approaches $1/\tau$

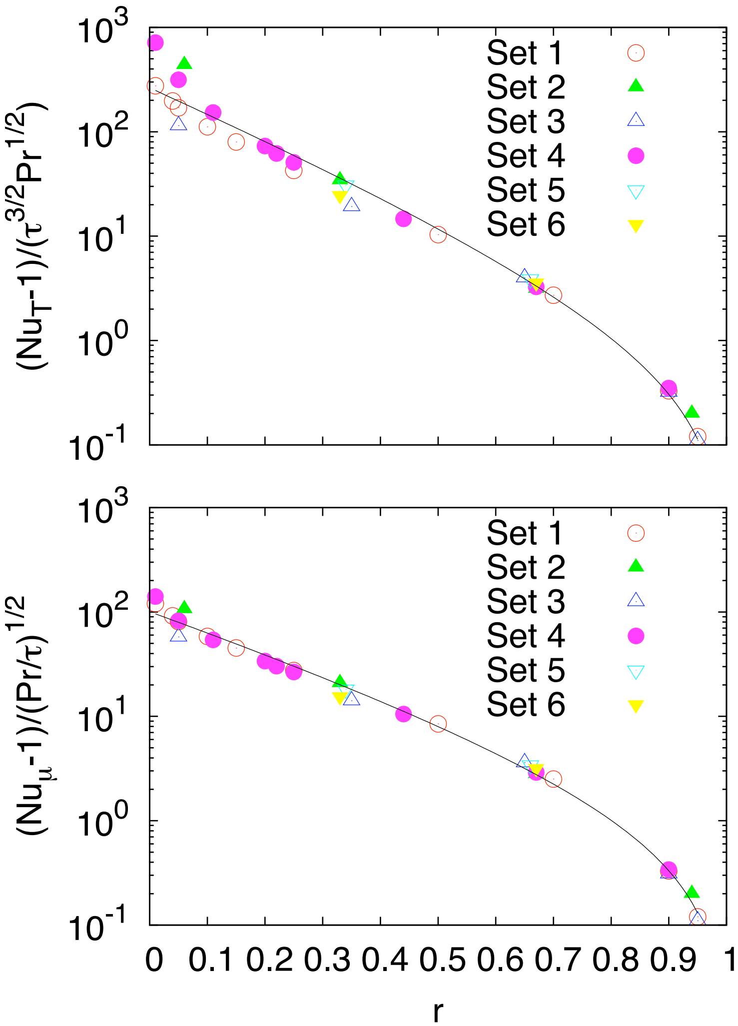
$$\mathrm{Pr} = 0.1$$
$$\tau = 0.1$$



$R_0 = 1.45$ $\qquad$ $R_0 = 9.1$

Traxler et al. 2011

Mixing by fingering convection in astrophysics



Result: Turbulent heat and compositional transport follow *universal asymptotic law*:

If $\mathrm{Pr} << 1, \ \tau << 1, \ \mathrm{Pr} \sim \tau$

$$\mathrm{Nu_T} - 1 = \mathrm{Pr}^{1/2}\,\tau^{3/2} F(r)$$

$$\mathrm{Nu}_\mu - 1 = \sqrt{\frac{\mathrm{Pr}}{\tau}}\, G(r)$$

$$\text{where } r = \frac{R_0 - 1}{R_c(\mathrm{Pr},\tau) - 1}$$

Traxler et al. 2011

Mixing by fingering convection in astrophysics

Physical implications:

Turbulent diffusivity = (Nu-1) (microscopic diffusivity).

- $\mathrm{Nu}_{\mathrm{T}} - 1 \;=\; \mathrm{Pr}^{1/2}\,\tau^{3/2} F(r)$: turbulent heat transport by fingering convection typically negligible in stellar or planetary interiors.

- $\mathrm{Nu}_{\mu} - 1 \;=\; \sqrt{\mathrm{Pr}/\tau}\, G(r)$: turbulent compositional transport by fingering convection up to a few orders of magnitude larger than diffusion in stellar/planetary interiors.

Outstanding questions

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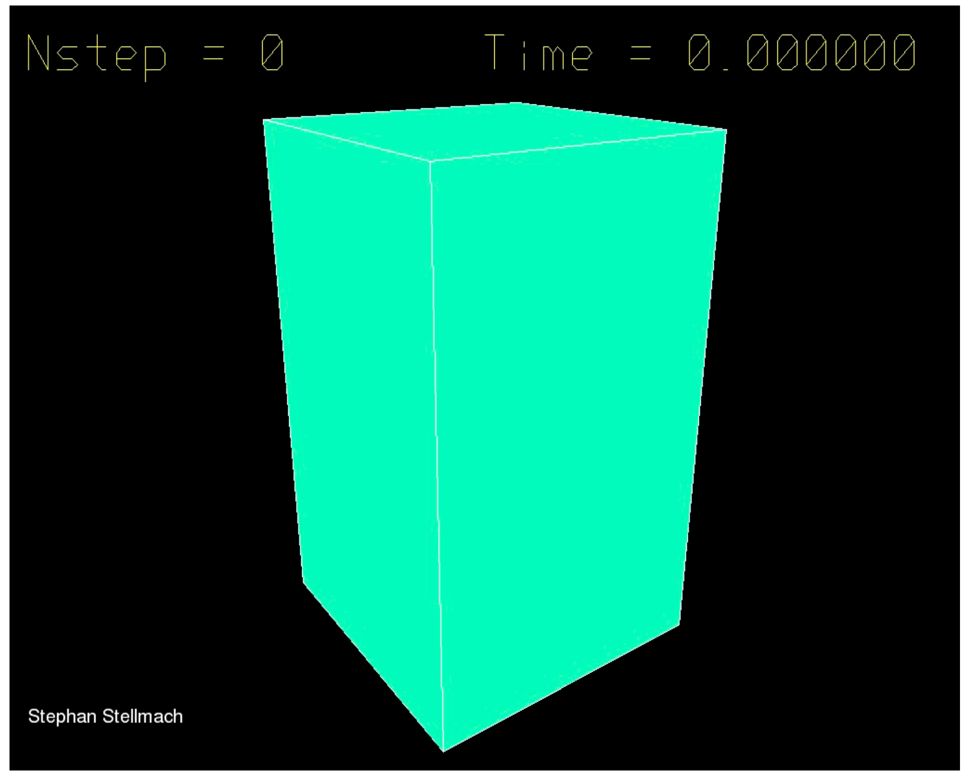
  - Transport by small-scale motion
  - The formation of large-scale structures
  - Staircases in stellar interiors?
  - Consequences for models of stellar pollution by planetary infall.
- Transport by oscillatory convection
  - Transport by small-scale motion
  - The formation of staircases
  - Transport through a staircase
  - Staircase equilibration?

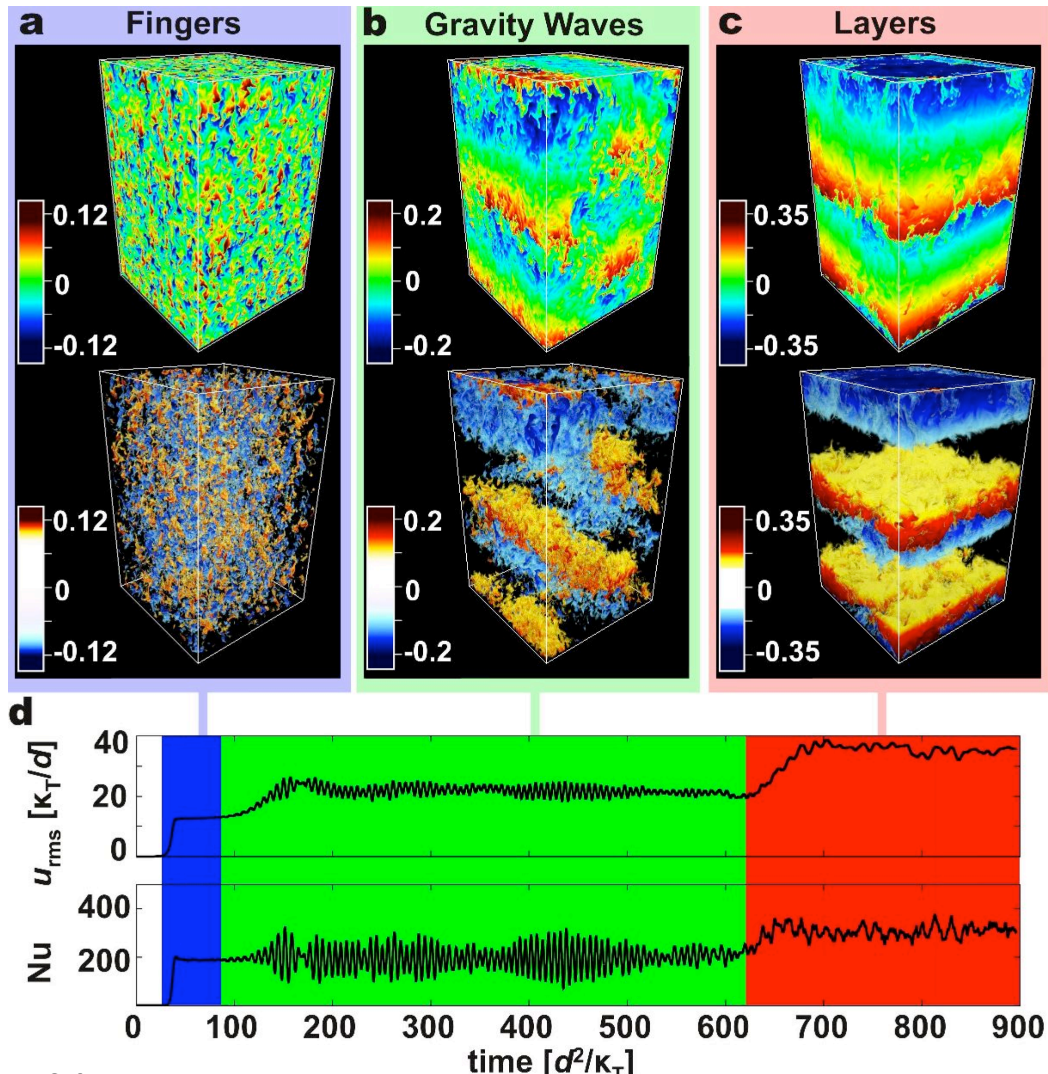
Large-scale dynamics in fingering convection

- All previous runs were done in "small" domain sizes (5x510 FGW).
- Let's look at a larger domain run
  - 25 x 25 x 40 FGW domain size,
  - Pr = 7, τ = 1/3 (more like water-fluid)
  - R_0 = 1.1 (close to convective overturning instability)



Stellmach et al. 2011

Large-scale dynamics in fingering convection



Stellmach et al. 2011

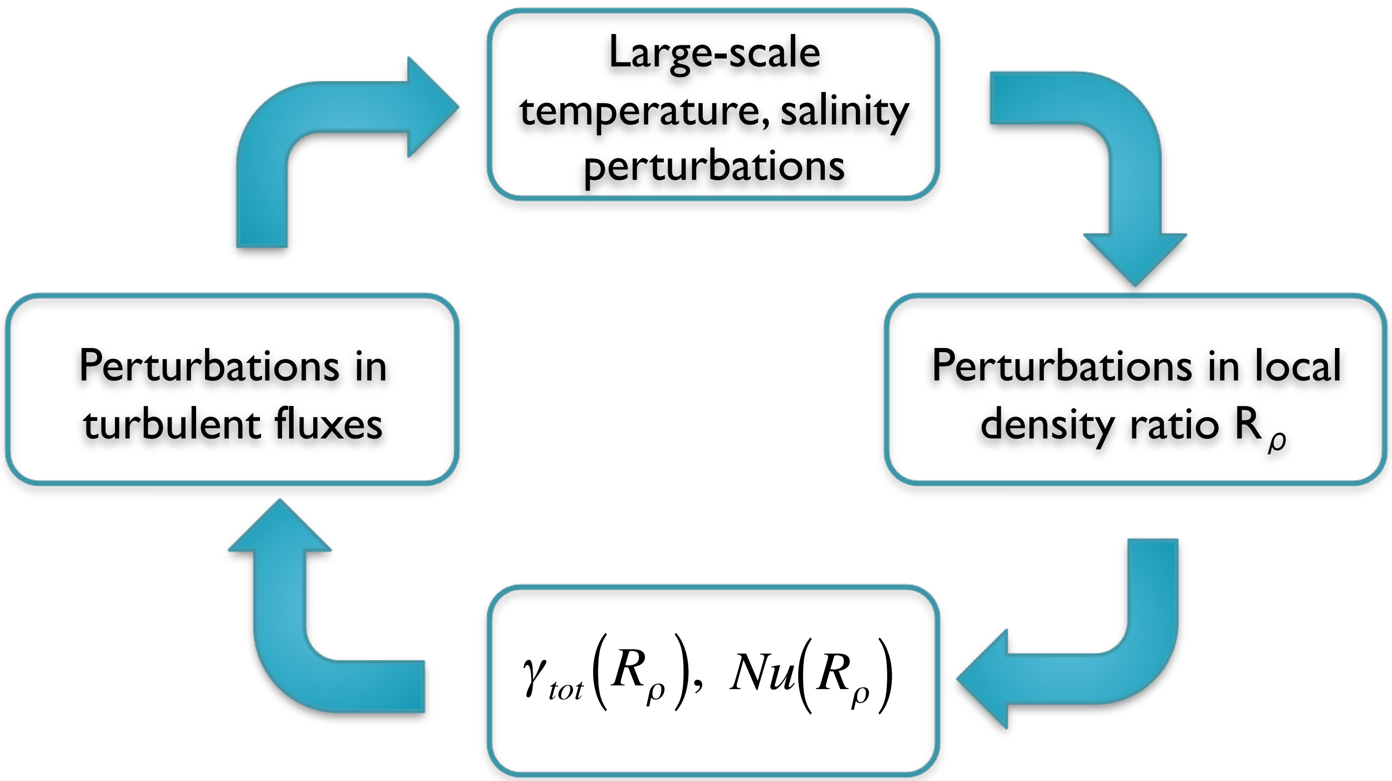
Large-scale dynamics in fingering convection

- Emergence of large-scale dynamics (gravity waves and layers) can be understood using "mean-field" theory
  - Long tradition of this approach for fingering convection: Stern & Turner, 1969; Walsh & Ruddick, 1995; Stern et al. 2001; Radko 2003. …

- Mean-field theory
  - Note that emerging structure scale >> finger scale
  - Spatially average governing equations over small scales
  - Use empirically motivated closure to model turbulent transport by the small-scales
  - Study the resulting evolution of the large-scale fields

Large-scale dynamics in fingering convection

- Spontaneous formation of large-scale structures induced by positive feedback between large-scale temperature/salinity perturbation and induced fluxes.

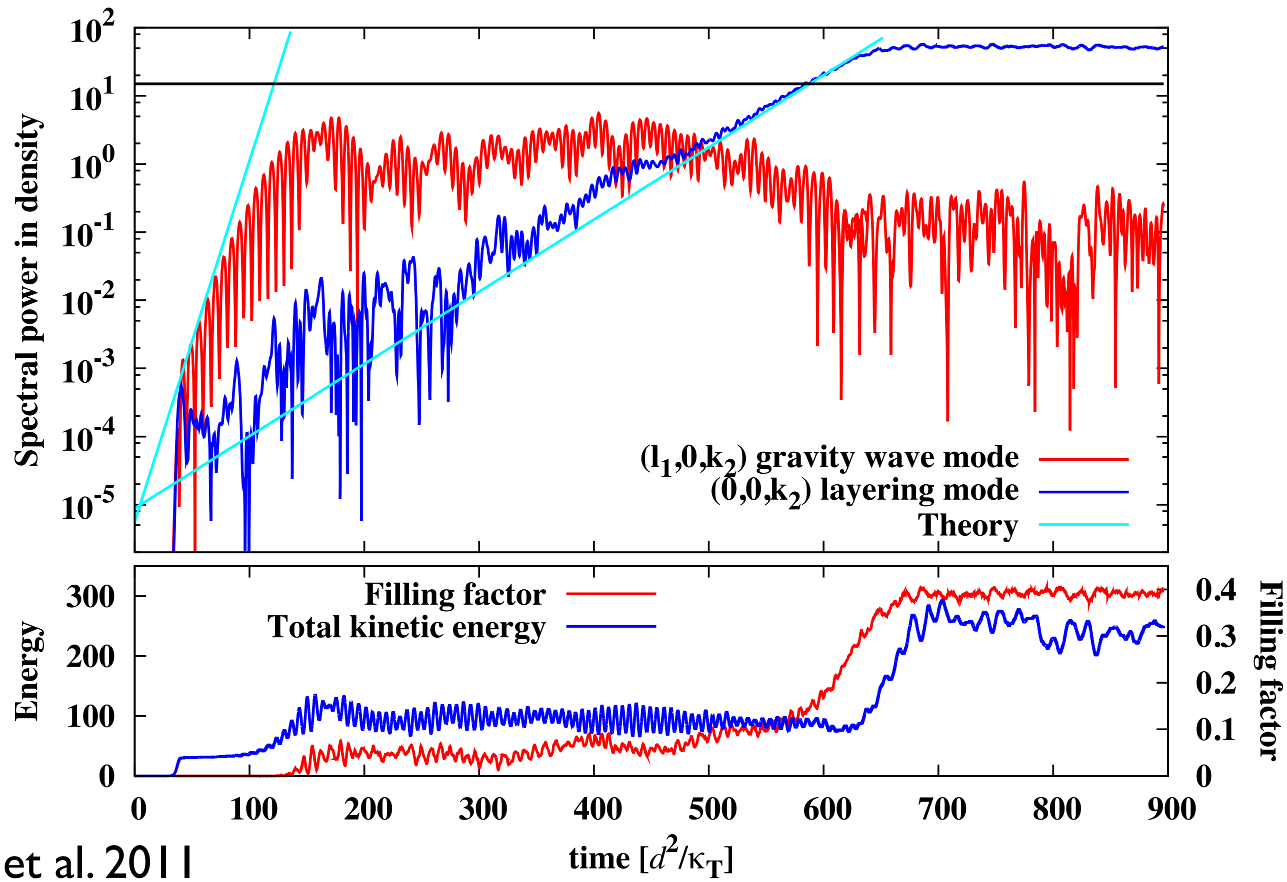


Large-scale dynamics in fingering convection

- Theory predicts:
  - Large-scale gravity wave excitation if $Nu(R_\rho)$ is large enough
  - Large-scale layering modes if $\gamma_{tot}\ (R_\rho)$ is a decreasing function
  - Mode growth rates depend on $Nu(R_\rho)$ and $\gamma_{tot}\ (R_\rho)$
- Layering mode overturns into a staircase when amplitude is large enough

Large-scale dynamics in fingering convection

- Comparison with large-domain numerical simulations reveals very good agreement with theory …



Stellmach et al. 2011

Large-scale dynamics in fingering convection

- **In short:**
  - We can now predict, for any parameter regime, whether large-scale gravity waves and/or layers will form, and at what rate they grow
  - What is the initial spacing of the staircase
  - The only requirement is to measure small-scale transport laws from small-domain simulations…

- **Note that:**
  - Mean-field theory can equally well be applied to the oscillatory regime. Predicts layer formation also when γ^{-1} is a decreasing function of R^{-1}

Outstanding questions

- Transport by fingering convection

    Transport by small-scale motion

    The formation of large-scale structures

    Staircases in stellar interiors?

    Consequences for models of stellar pollution by planetary infall.

- Transport by oscillatory convection

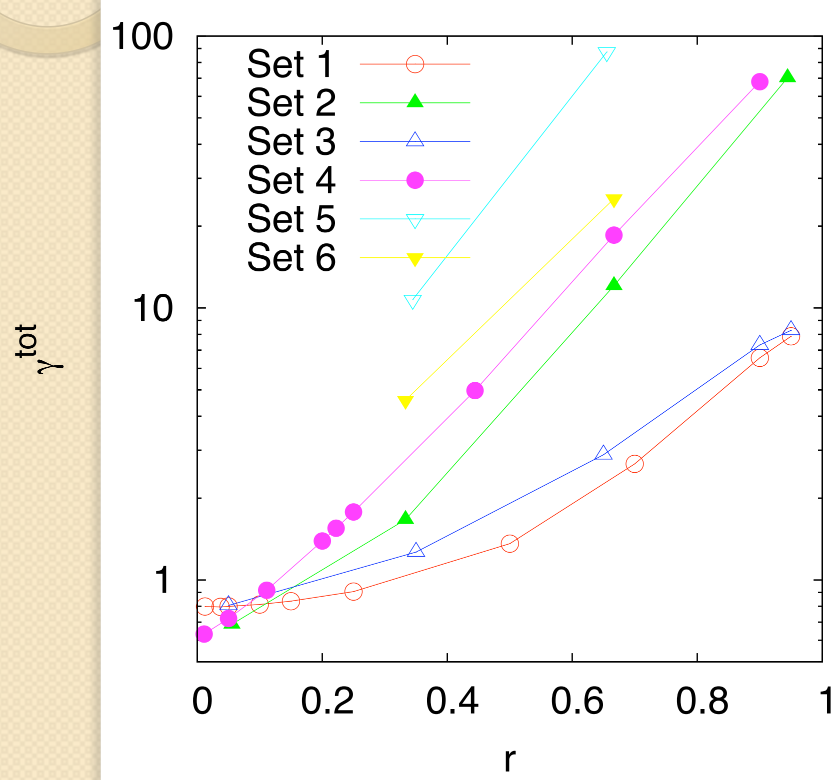
    Transport by small-scale motion

    The formation of staircases

    Transport through a staircase

    Staircase equilibration?

Large-scale dynamics in fingering convection in the astrophysical regime



- For low Pr, we find that:
  - Nu is very small
  - γ_{tot} "never" decreases with R_ρ
- Implications for low Pr fingering systems:
  - **No spontaneous staircase formation through γ – instability mechanism**
  - **No large-scale gravity waves**
  - **Transport properties dominated by small-scale dynamics**

Traxler et al. 2011

Mixing by fingering convection in astrophysics

Physical implications:

Turbulent diffusivity = (Nu-1) (microscopic diffusivity).

- $\mathrm{Nu}_{\mathrm{T}} - 1 = \mathrm{Pr}^{1/2}\, \tau^{3/2} F(r)$: turbulent heat transport by fingering convection typically negligible in stellar or planetary interiors.

- $\mathrm{Nu}_{\mu} - 1 = \sqrt{\mathrm{Pr}/\tau}\, G(r)$: turbulent compositional transport by fingering convection up to a few orders of magnitude larger than diffusion in stellar/planetary interiors.

- **These laws can be used "as is" in stellar evolution models (please do).**

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- Transport by oscillatory convection
  - Transport by small-scale motion
  - The formation of staircases
  - Transport through a staircase
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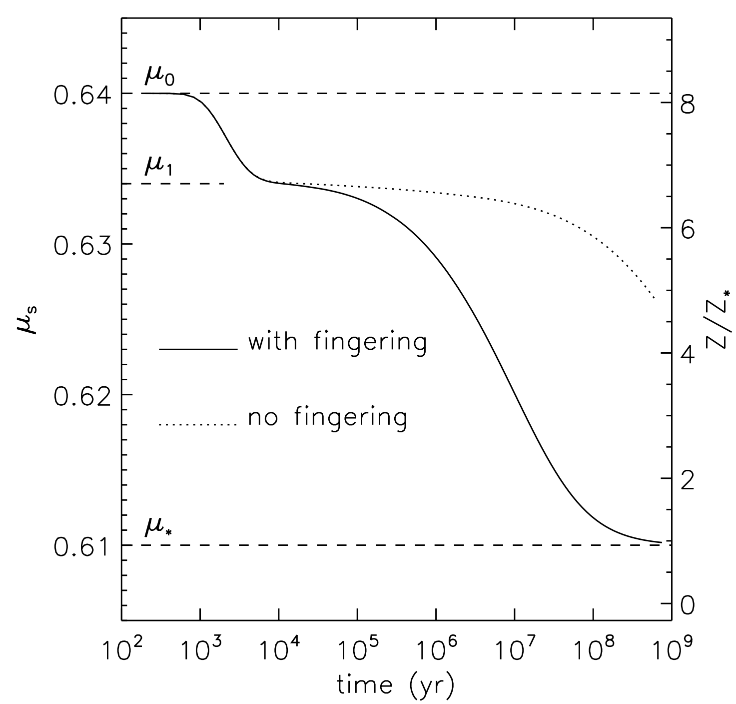
Planetary infall

- To study the effect of planetary infall on the surface metallicity of a star, one must study the evolution of the metallicity profile within.
  - Effect of dynamical mixing
  - Effect of fingering convection



Garaud 2011

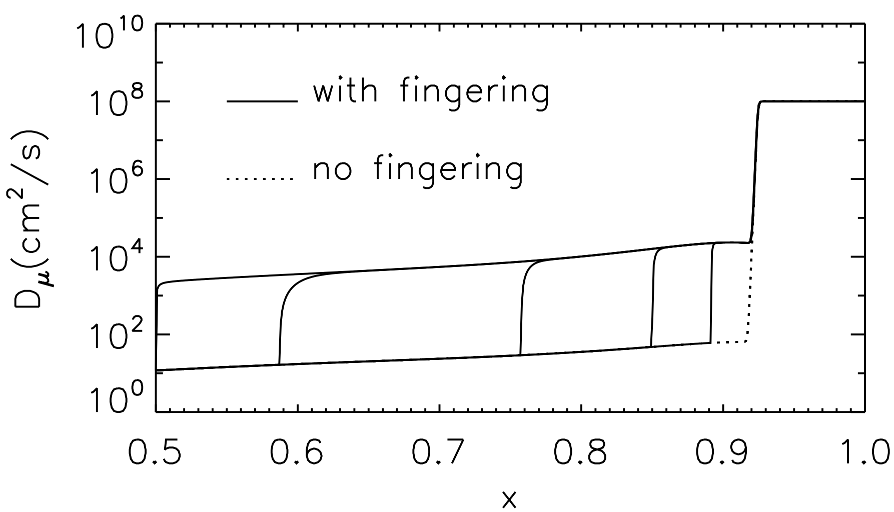
- Fingering convection strongly enhances dilution of metals into stellar interior.

- After about 100Myr, all evidence for planetary infall has disappeared.

Observed planet-metallicity trend must be primordial!

Planetary infall

- To study the effect of planetary infall on the surface metallicity of a star, one must study the evolution of the metallicity profile within.
  - Effect of dynamical mixing
  - Effect of fingering convection (Vauclair 2004)



- In addition, the fingering region extends all the way to the Li-burning region

Could explain observed (but controversial) claim of higher Li-depletion in planet-bearing stars.

Could explain dispersion in solar-type stars

Garaud 2011

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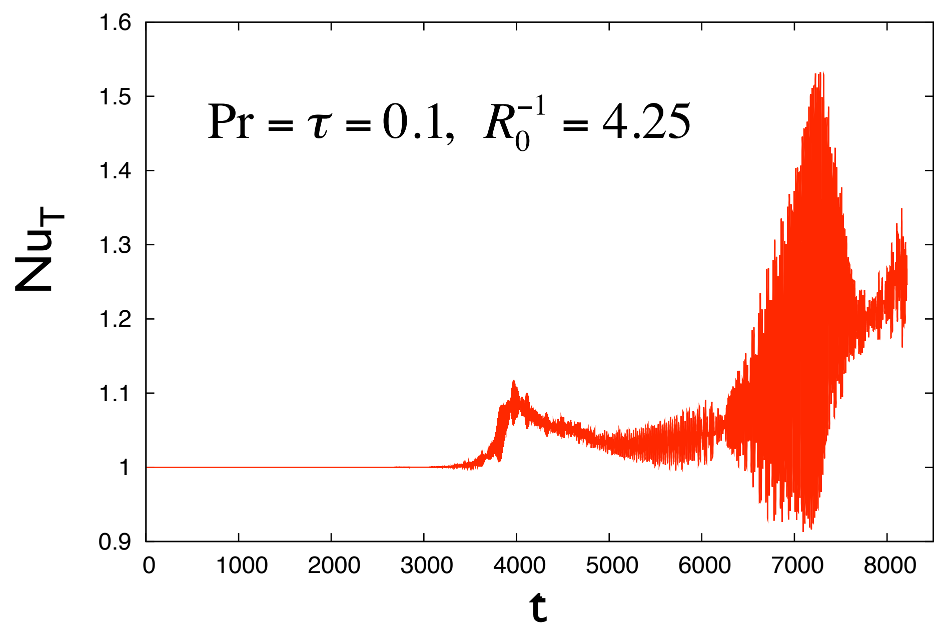
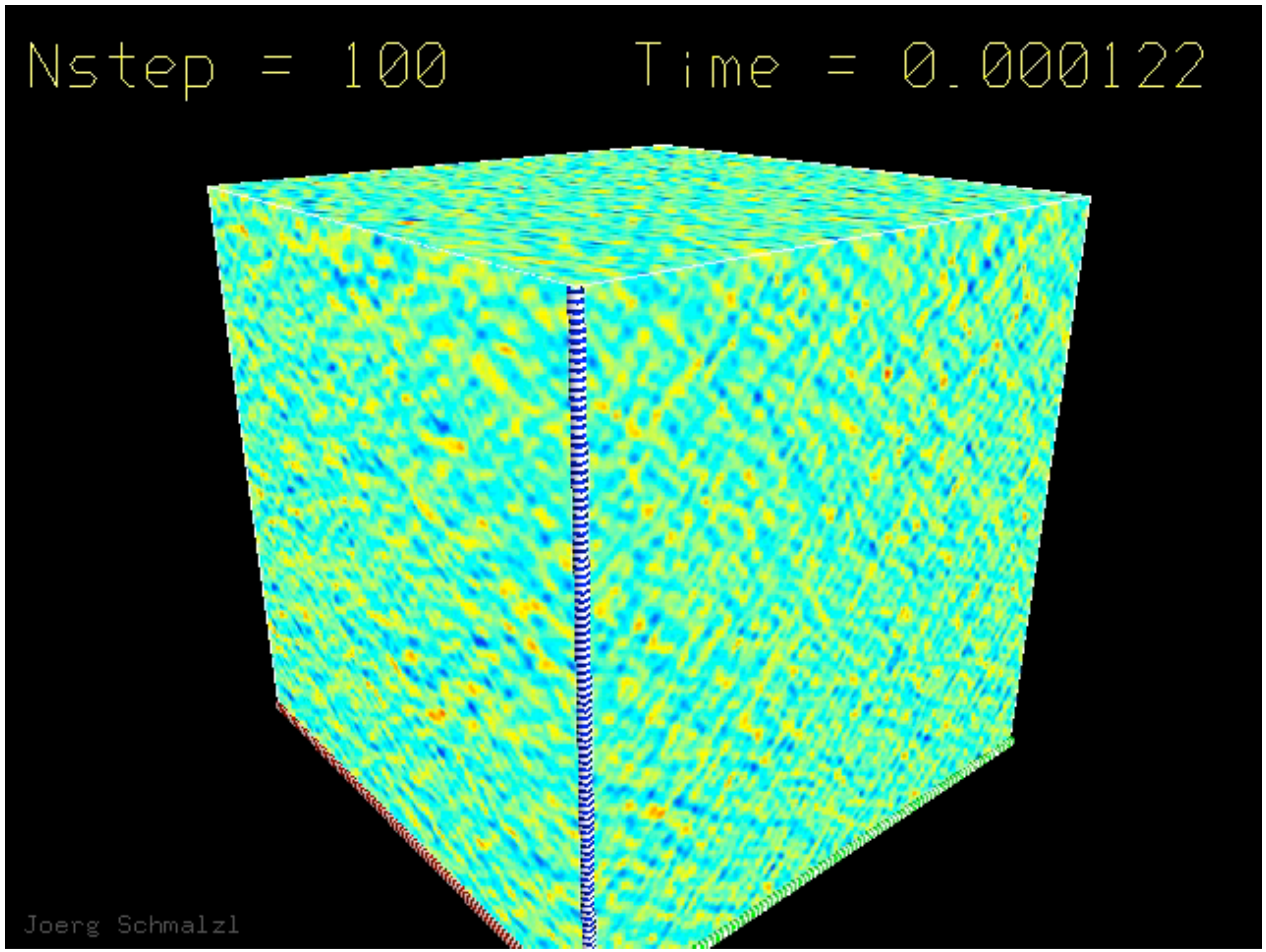
Mixing by oscillatory convection in astrophysics

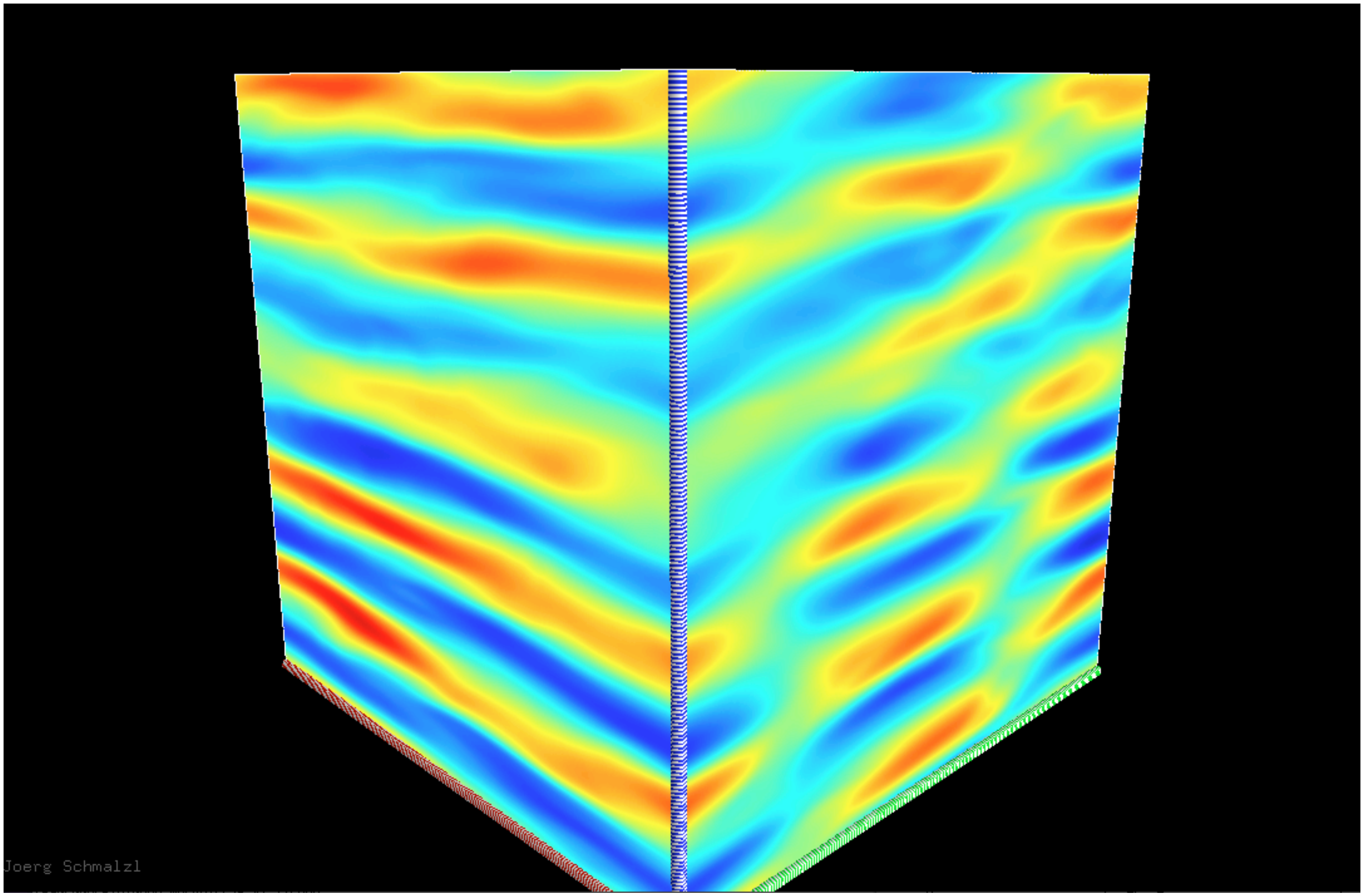
Preliminary results:

- Similar setup as in the fingering case
- *Caveat*: in this case the Nusselt numbers sometimes do not appear to give statistically steady transport values:



Mixing by oscillatory convection in astrophysics

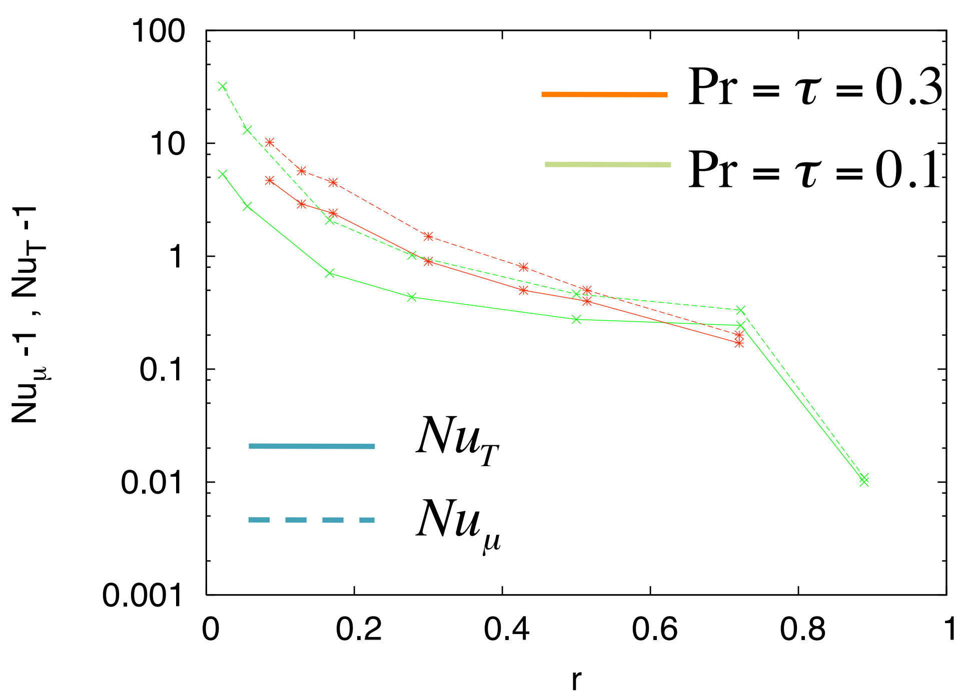



Joerg Schmalzl

Mixing by oscillatory convection in astrophysics

Preliminary results: turbulent mixing by small-scale motion does not appear to follow same asymptotic law as fingering convection.

More runs needed to determine what law it actually is.



$$r = \frac{R_0^{-1} - 1}{R_c^{-1}(\mathrm{Pr},\tau) - 1}$$

Outstanding questions

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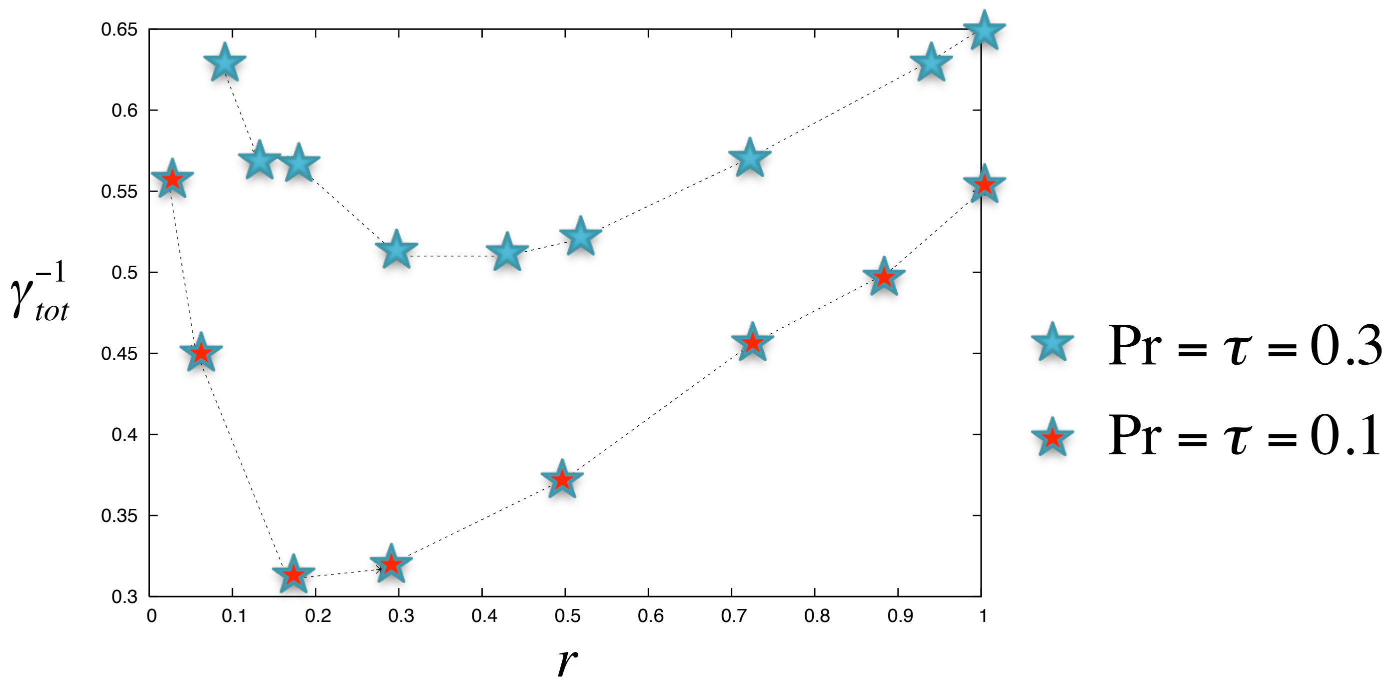
Large-scale dynamics in oscillatory convection in the astrophysical regime

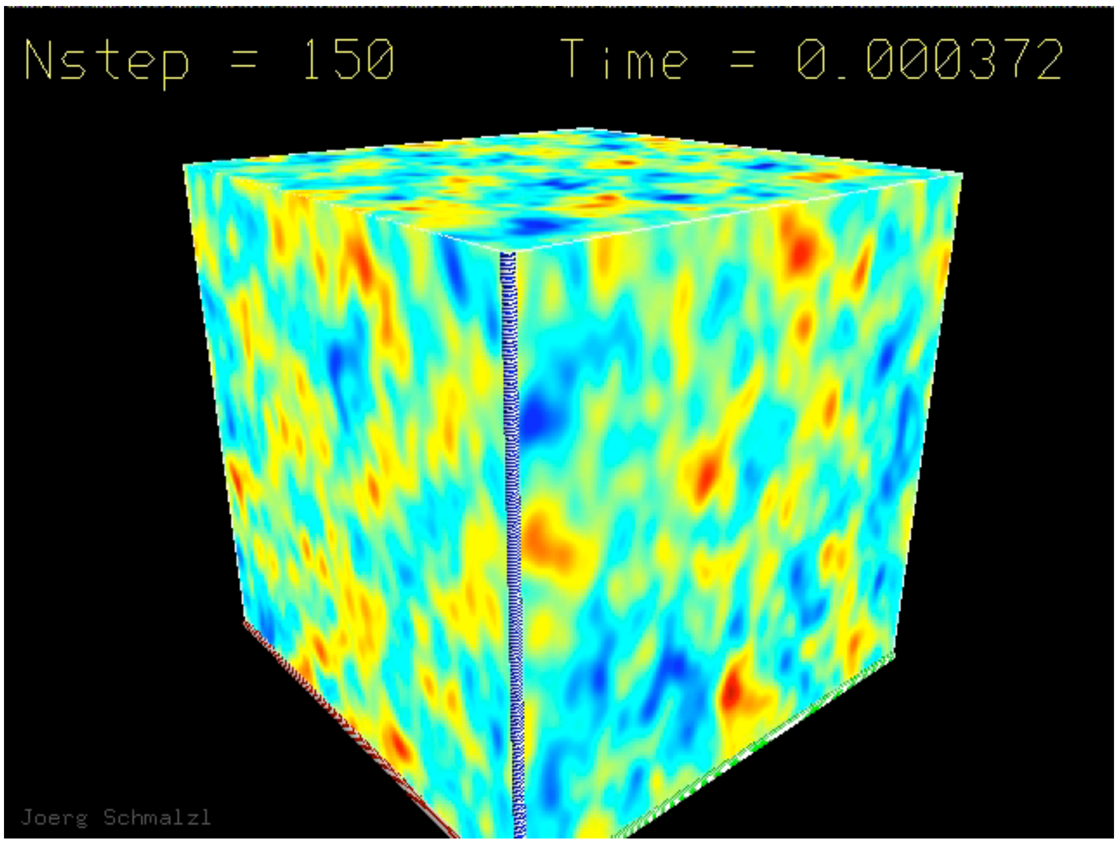
Results:

- In this case, for low Pr, γ_{tot} does decrease for low R_ρ so **staircase formation is expected in weakly stratified systems**



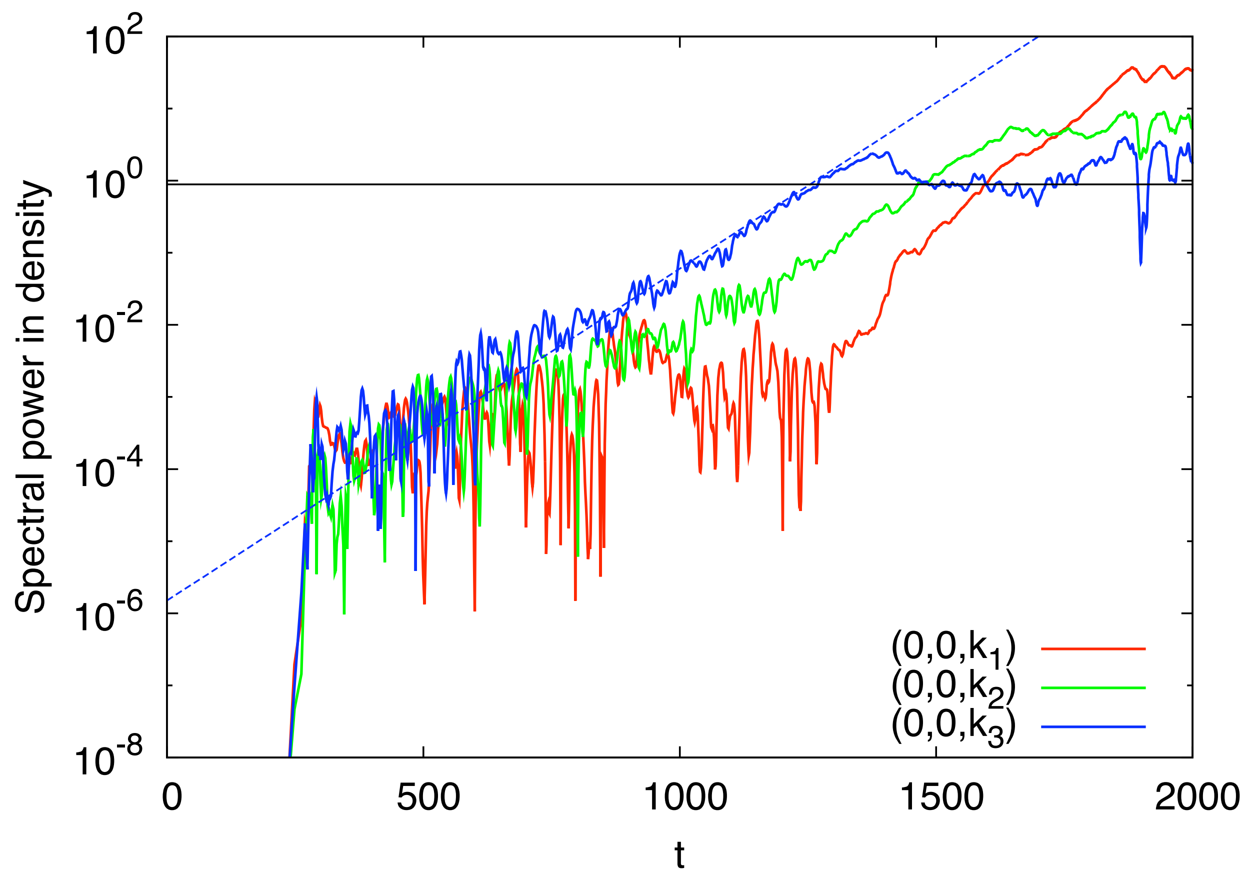
Large-scale dynamics in oscillatory convection in the astrophysical regime

- Simulation for $\mathrm{Pr} = \tau = 0.3, R_0^{-1} = 1.2$



Large-scale dynamics in oscillatory convection in the astrophysical regime

- Layer formation in this regime is also explained by Radko's γ-instability:



Outstanding questions

- Transport by fingering convection
  - Transport by small-scale motion
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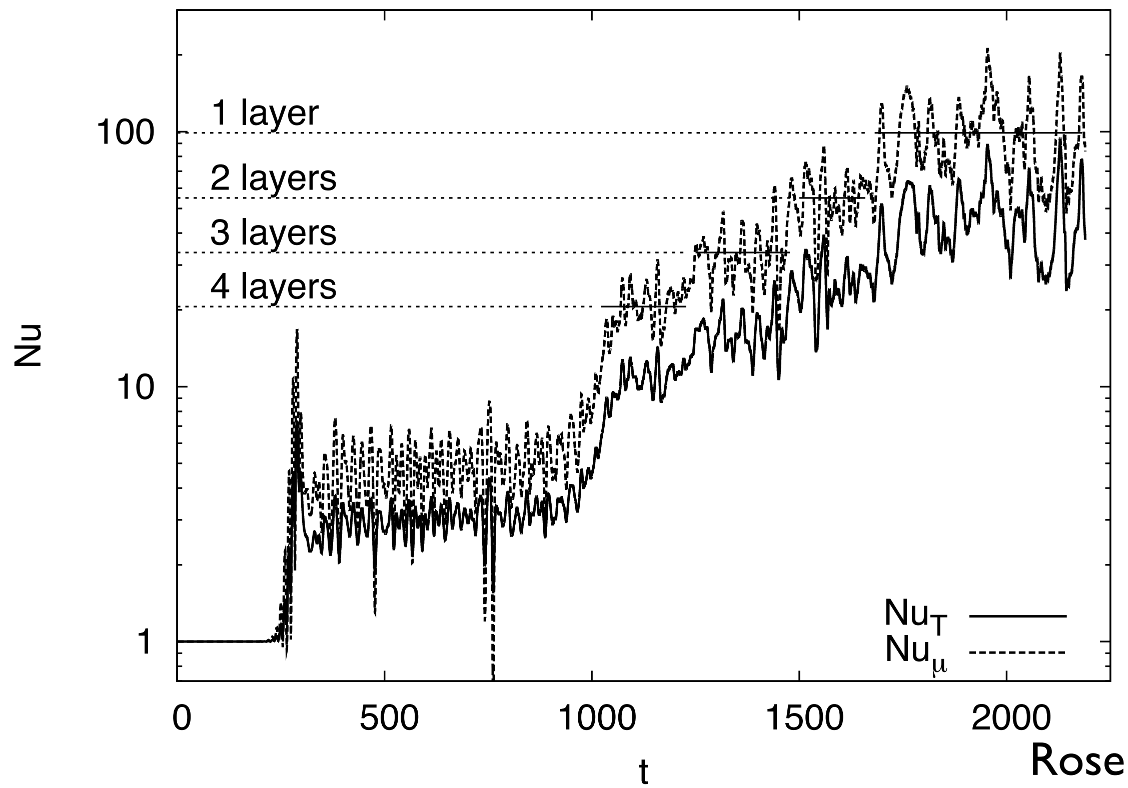
Staircase transport in oscillatory convection in the astrophysical regime

Results:

- Staircase formation and each subsequent merger increases turbulent transport.



Rosenblum et al. 2011.

Staircase transport in oscillatory convection in the astrophysical regime

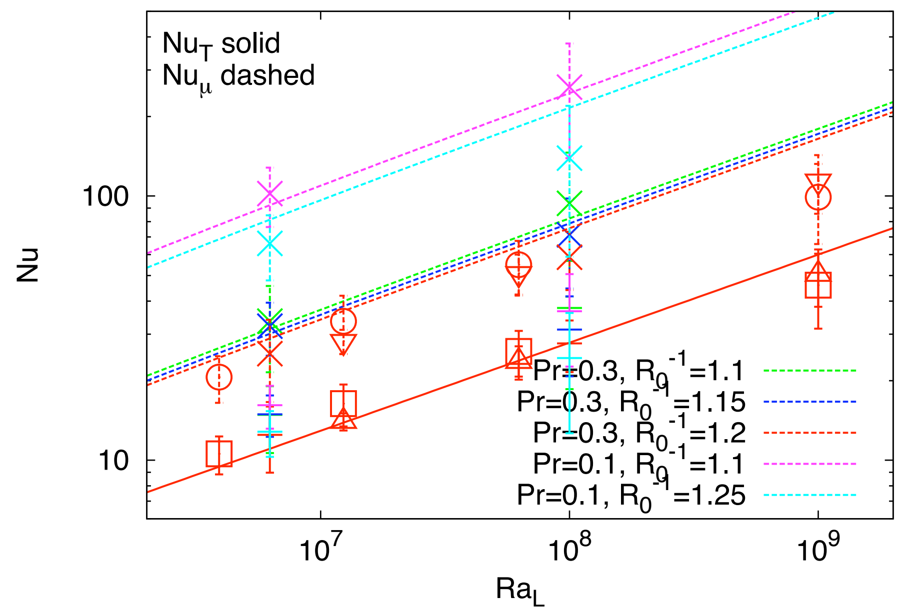
Results:

- Transport properties in the layered convection case seems to be "well" explained with Rayleigh-Benard scaling laws based on layer height h

$$Ra_L = \frac{\alpha g |T_{0z}| h^4}{\nu \kappa_T}$$

$$Nu_T = 0.06 Ra_L^{1/3}$$

$$Nu_\mu = 1 + \frac{R_0}{\tau}(Nu_T - 1)$$

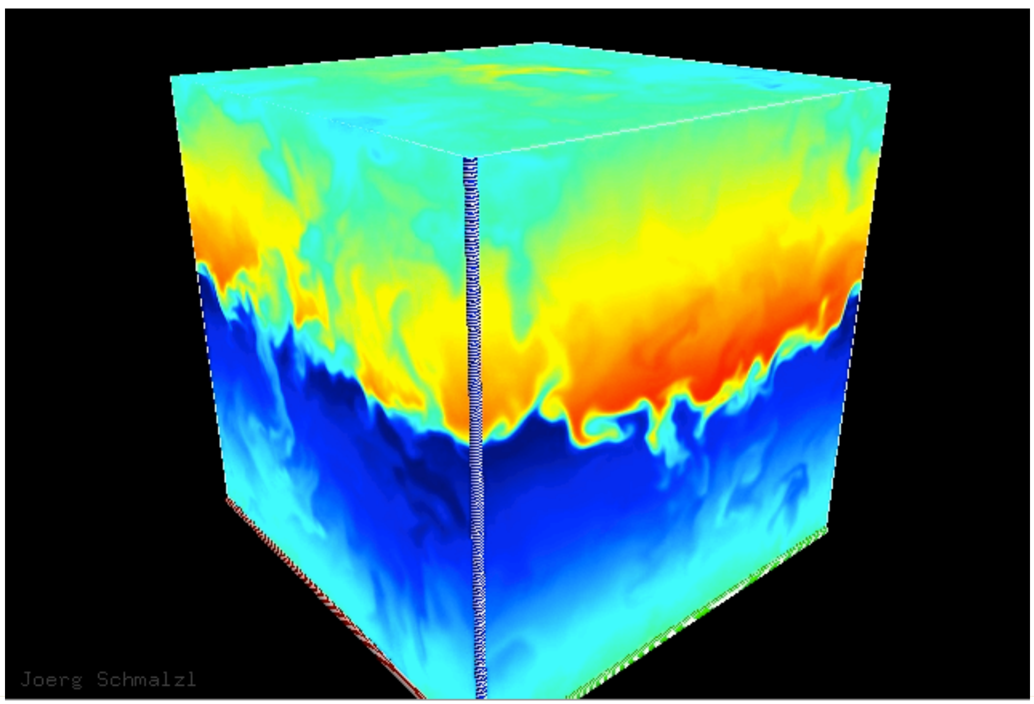


Rosenblum et al 2011, + new results in prep.

Staircase transport in oscillatory convection in the astrophysical regime

However, what determines the ultimate layer height?

- In simulations, final layer is always as tall as box



Outstanding questions

- Transport by fingering convection
  - Transport by small-scale motion
  - The formation of large-scale structures
  - Staircases in stellar interiors?
  - Consequences for models of stellar pollution by planetary infall.
- Transport by oscillatory convection
  - Transport by small-scale motion
  - The formation of staircases
  - Transport through a staircase
  - Staircase equilibration?
  - Consequences for planetary models.

Staircase equilibration

Actually, no idea…

(well, that's not entirely true, but …)

Outstanding questions

- Transport by fingering convection
    - Transport by small-scale motion
    - The formation of large-scale structures
    - Staircases in stellar interiors?
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    - Consequences for planetary models.

Staircase equilibration

To be determined by Tingtao this summer!

Conclusions/prospects

Conclusions/prospects

- We are well on the way to having a comprehensive theory of double-diffusive convection in astrophysics, in both fingering and oscillatory regimes

- **Fingering regime:**
  - Asymptotic transport laws have been determined
  - Layer formation and gravity wave formation unlikely
  - **Implications for astrophysics (examples):**
    - Fingering convection unlikely to be sufficient to explain peculiar abundances of AGB stars (Denissenkov, 2011, Traxler et al. 2011)
    - Fingering convection plays an important role in metallicity dilution in planet-host stars after impact by a planet, and possible role on Li depletion (Vauclair, 2004; Garaud, 2011)

Conclusions/prospects

- We are well on the way to having a comprehensive theory of double-diffusive convection in astrophysics, in both fingering and oscillatory regimes

- **Oscillatory regime**
  - Asymptotic transport laws remains TBD
  - Gravity wave excitation unlikely
  - Layer formation likely
  - Potential transport law through staircase established, but equilibrium layer height remains TBD.
  - **Implications for astrophysics (examples):**
    - Possible role of layered convection in explaining the diversity of gas giant planet heat fluxes / radii. (Chabrier & Baraffe, 2007)

Traxler, Stellmach, Garaud, Radko & Brummell 2011 (JFM)

The dynamics of fingering convection I: Small-scale fluxes and large-scale instabilities

Stellmach, Traxler, Garaud, Brummell & Radko 2011 (JFM)

The dynamics of fingering convection II: The formation of thermohaline staircases

Traxler, Garaud & Stellmach, 2011 (ApJL)

Turbulent transport by fingering convection in astrophysics

Rosenblum, Garaud, Traxler & Stellmach 2011 (ApJ)

Layer formation and evolution in semi-convection

Extras

Large-scale dynamics in fingering convection

- Averaged equations:

$$\frac{1}{\Pr}\left(\frac{\partial \overline{u}}{\partial t} + \nabla \cdot R\right) = -\nabla \overline{p} + \left(\overline{T} - \overline{S}\right)\mathbf{e}_z + \nabla^2 \overline{u}$$

$$\frac{\partial \overline{T}}{\partial t} + \nabla \cdot F_T + \overline{w} = \nabla^2 \overline{T}$$

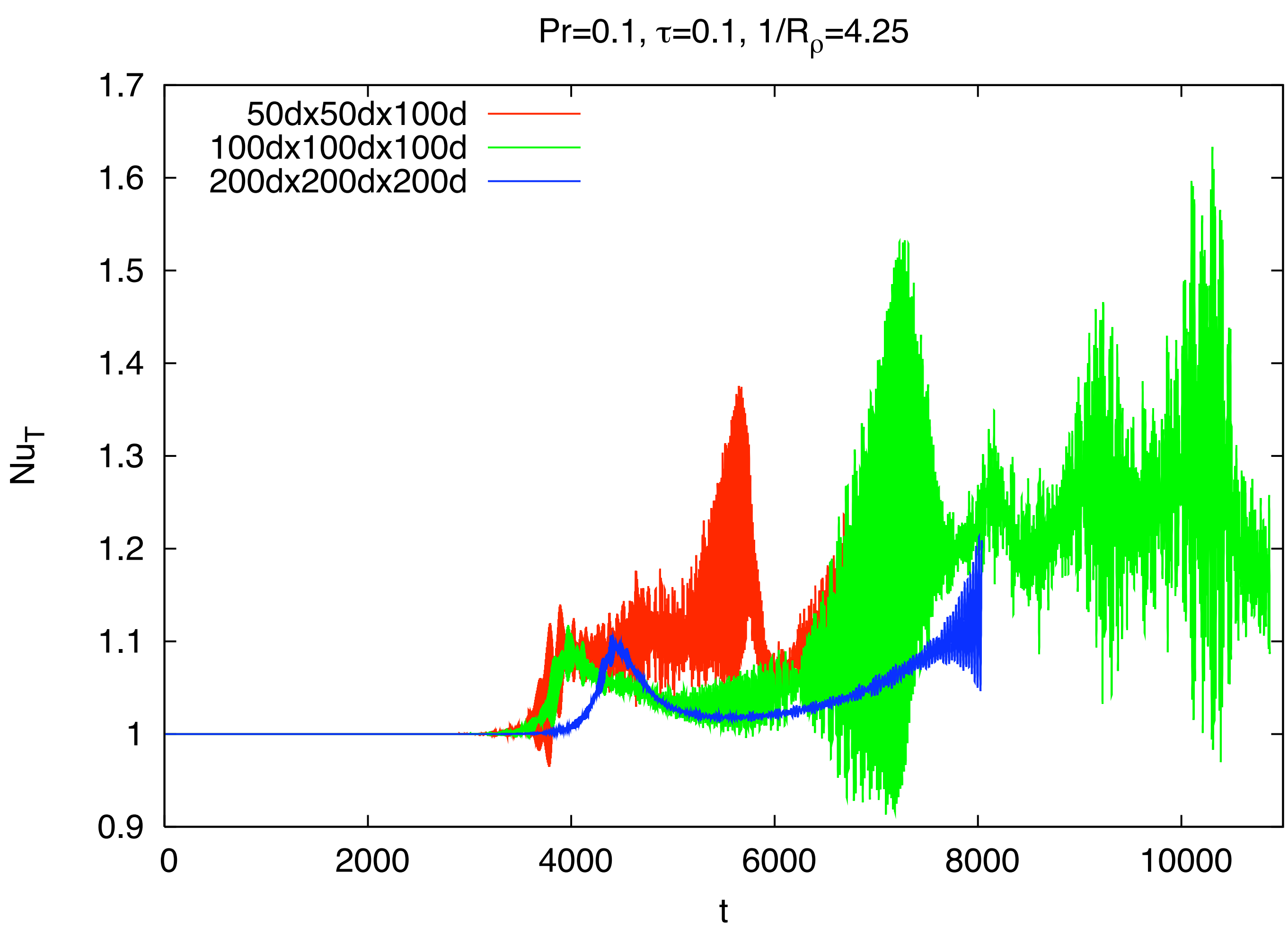
$$\frac{\partial \overline{S}}{\partial t} + \nabla \cdot F_S + \frac{\overline{w}}{R_0} = \tau \nabla^2 \overline{S}$$

$$\nabla \cdot \overline{u} = 0$$

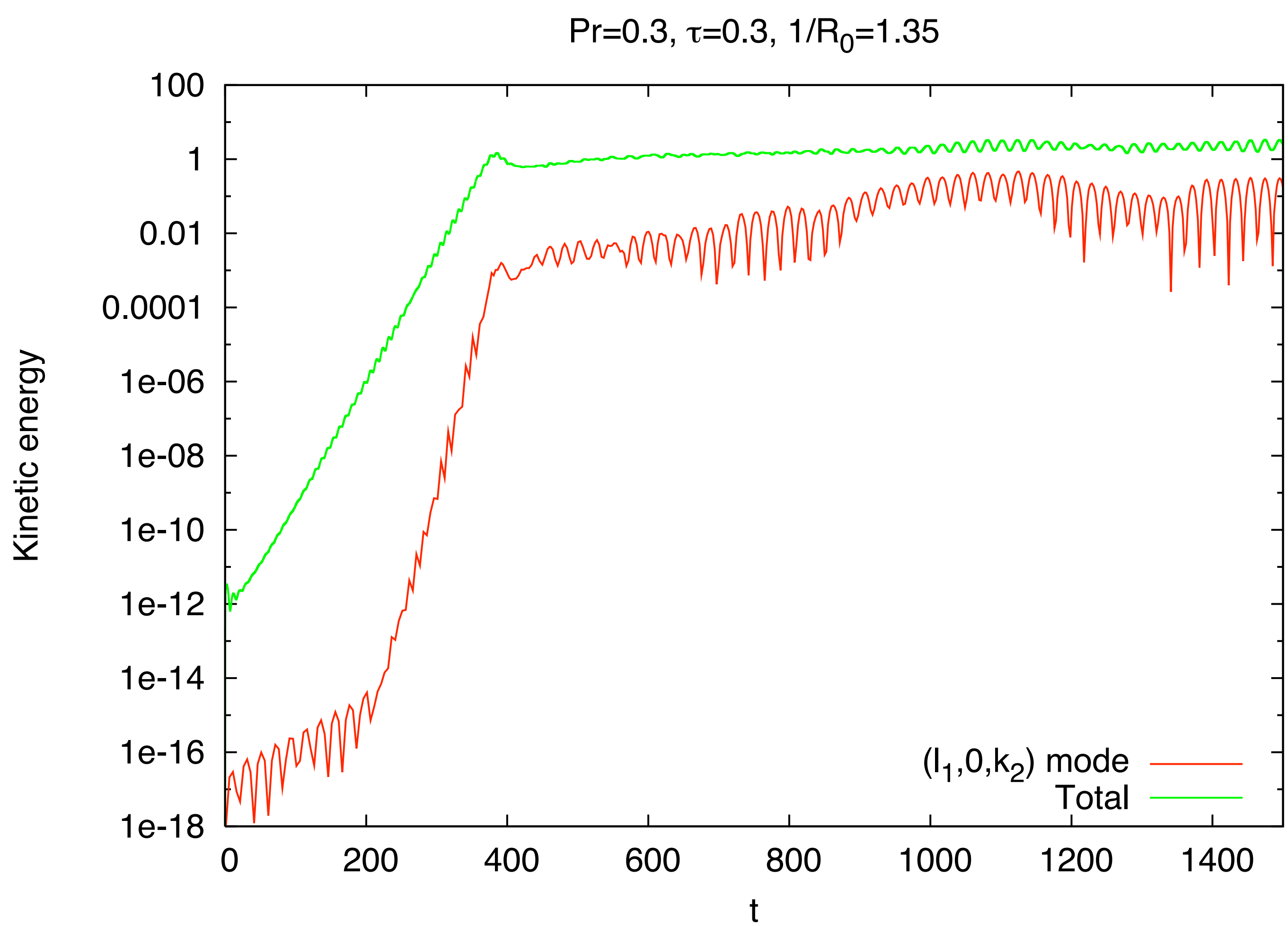
- Non-dimensional fluxes:
  - Measured with Nu and γ_{tot}
  - Empirically known from small box simulations!

- Flux model:
  - Neglect Reynolds stresses
  - Salinity, heat fluxes only have vertical component
  - Non-dimensional fluxes only depend on local

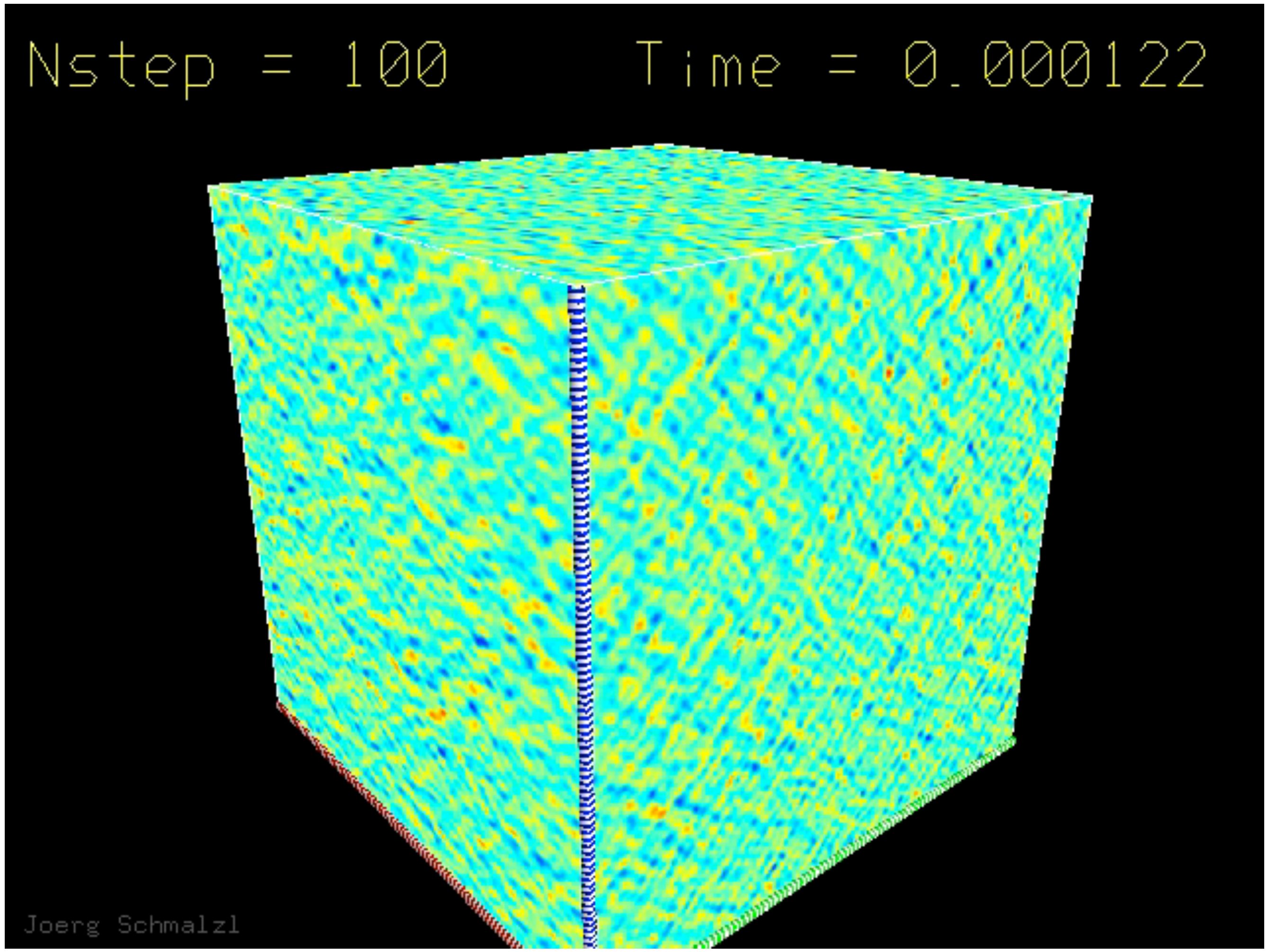
$$R_\rho = \frac{T_{0z} + \overline{T}_z}{S_{0z} + \overline{S}_z}$$



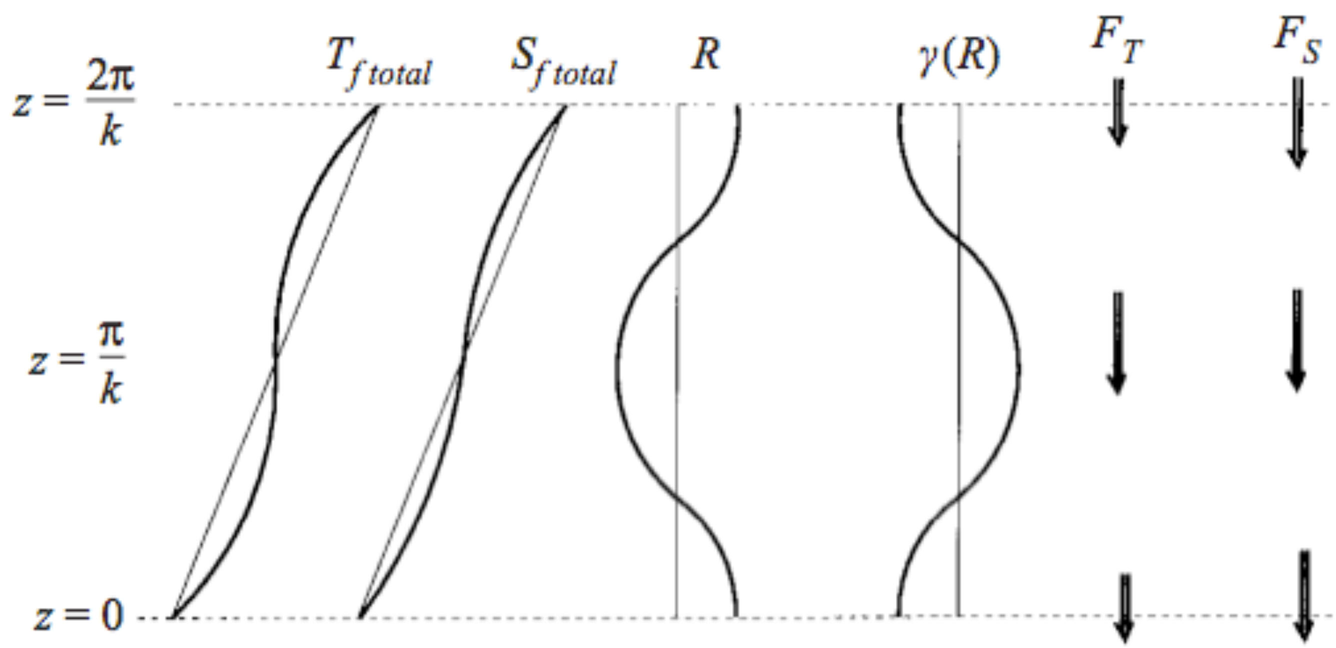
Pr=0.1, τ=0.1, 1/Rρ=4.25
50dx50dx100d
100dx100dx100d
200dx200dx200d
NuT
t
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1
0.9
0
2000
4000
6000
8000
10000



Pr=0.3, τ=0.3, 1/R0=1.35
Kinetic energy
100
1
0.01
0.0001
1e-06
1e-08
1e-10
1e-12
1e-14
1e-16
1e-18
0
200
400
600
800
1000
1200
1400
t
(l1,0,k2) mode
Total


Nstep = 100
Time = 0.000122
Joerg Schmalzl

Radko's interpretation



Radko's theory for layer equilibration in fingering convection