From Transitional Disks to the Solar system

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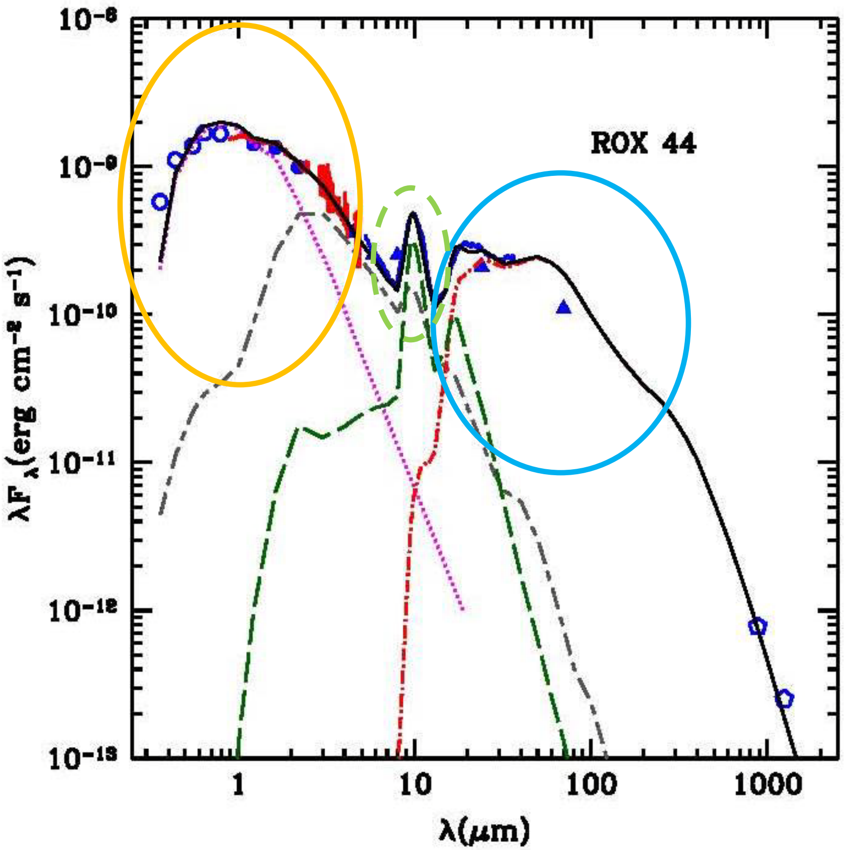
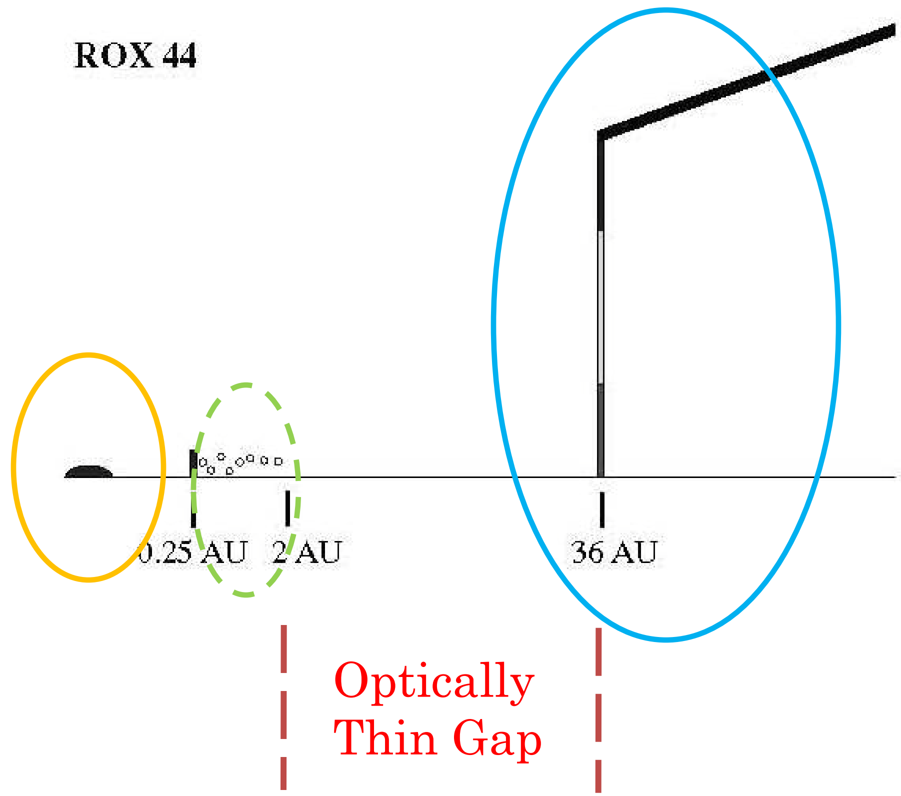
Observation of (pre-)Transitional disks --disk with a wide gap

SED from observations



Model



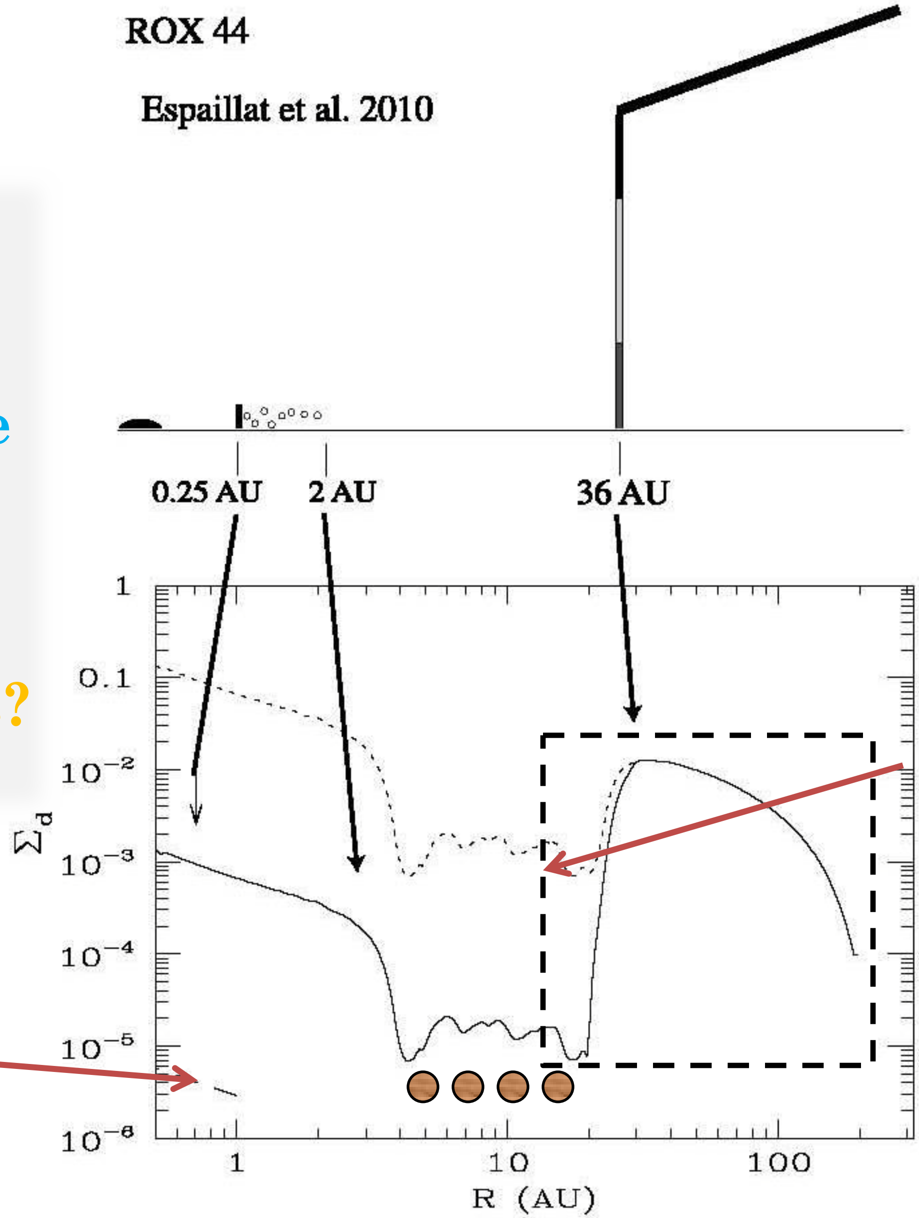
Espaillat et al 2010

Gap Opening by Planet(s) KARAOKE

Ideas:

How to deplete the dust fraction in the gap?

How to form multiple gas giants?



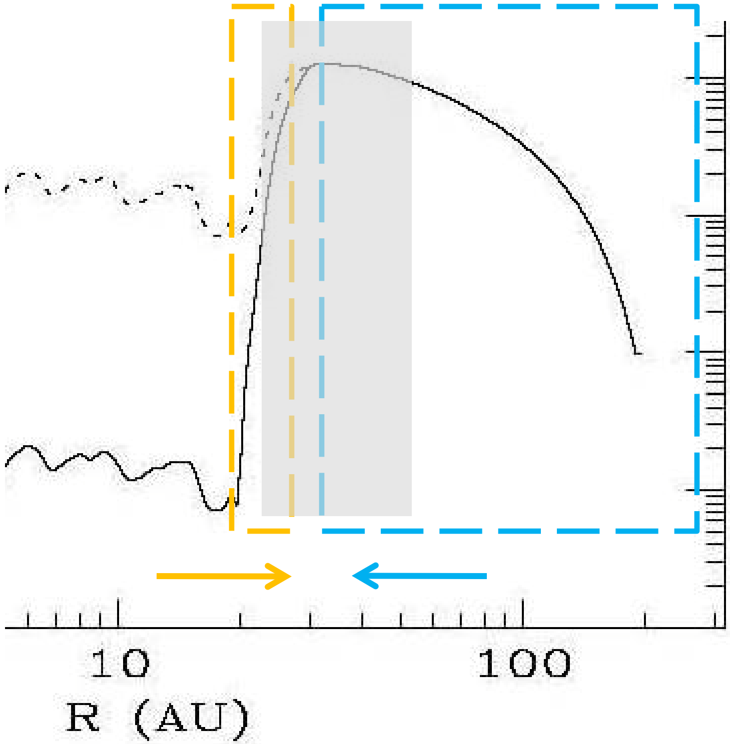
Vp

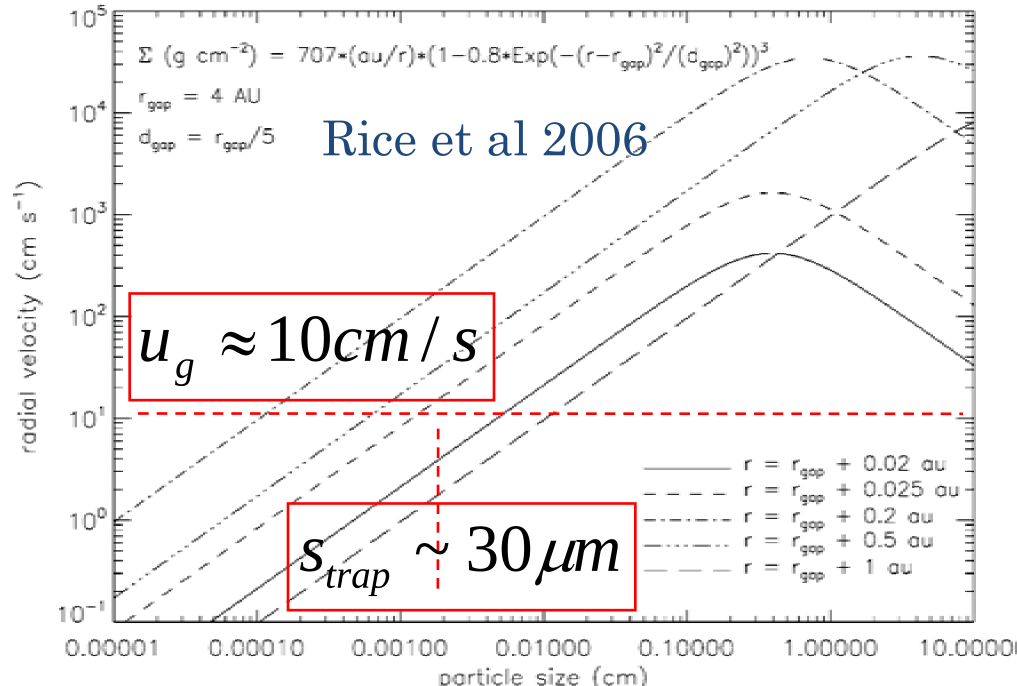
Dust depletion By 10^2 ~ 10^5

4 Jupiters In resonance

Zhu et al 2011

‘Filtration’ effect at the rim of the gap





$$u_p(s) = \frac{u_g}{4\pi^2 St^2 + 1} - 2\eta v_k \frac{2\pi St}{4\pi^2 St^2 + 1}$$

Positive(Negative) pressure gradient

->Particles feeling Tail(head) wind & gaining loosing) angular momentum

->Particles drifting outward (inward)

->Piling up at local pressure maximum

$\Omega < \Omega_K$

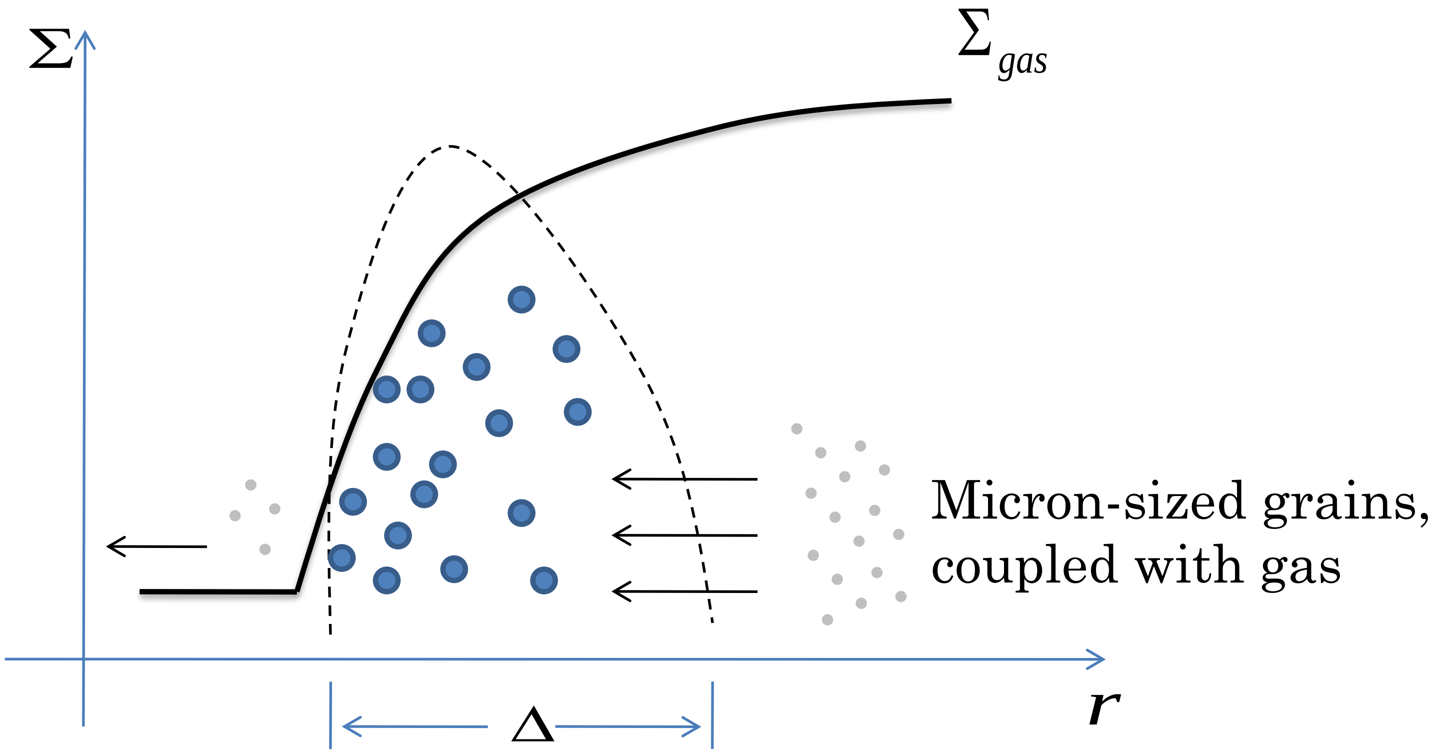
∇P

$\frac{GM_*}{r^2}$

$\Omega > \Omega_K$

∇P

Two groups of particles with only coagulation

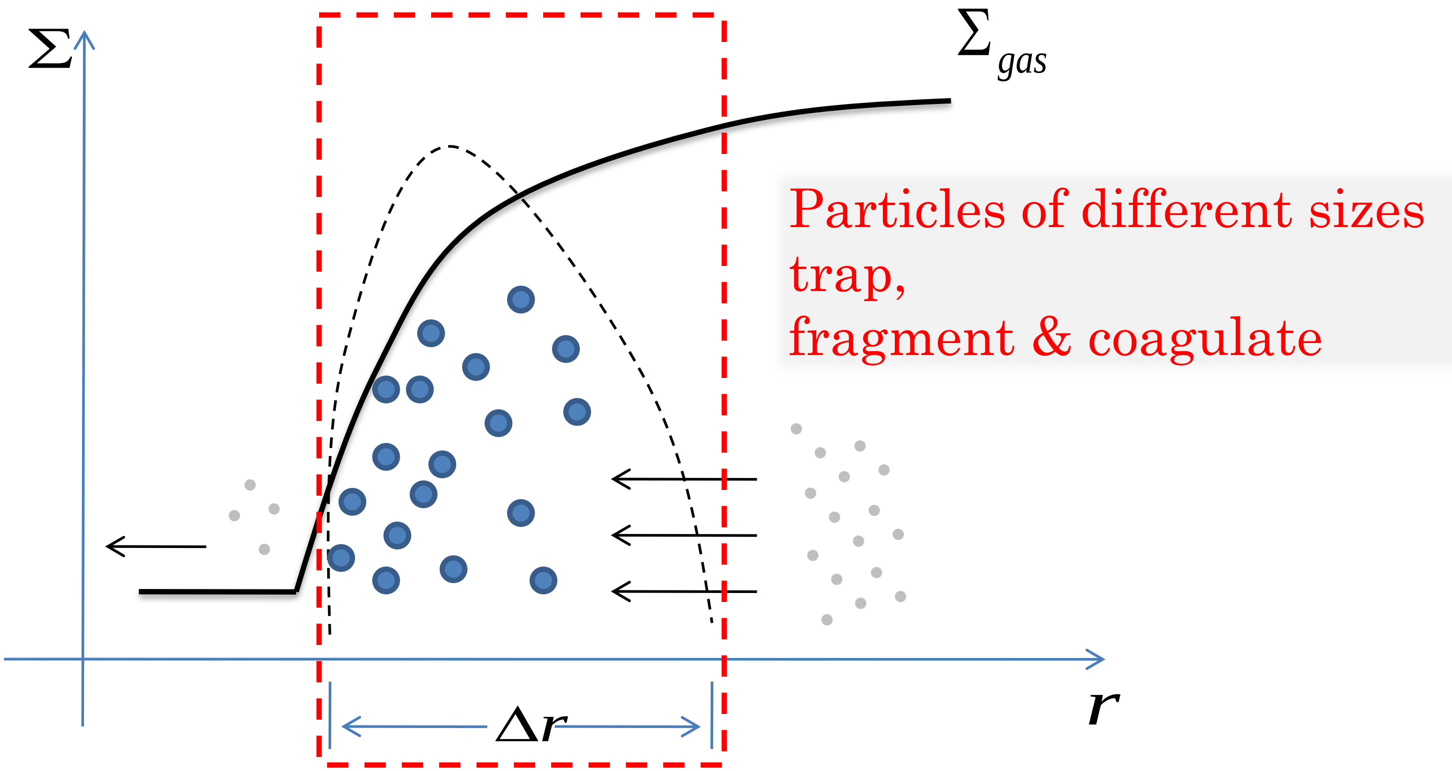


Fraction of small particles get trapped $\pi s_{trap}^2 n_{trap} \Delta \approx 1$

Total mass needed to be trapped to get a significant amount of depletion

$$\frac{4\pi}{3}\rho_s s_{trap}^3 n_{trap} h_{trap} 2\pi r = 0.05 M_\oplus \left(\frac{\rho_s}{1g/cm^3}\right)\left(\frac{s_{trap}}{30\mu m}\right)\left(\frac{h_{trap}}{1AU}\right)\left(\frac{r}{50AU}\right)$$

'One-box' Model

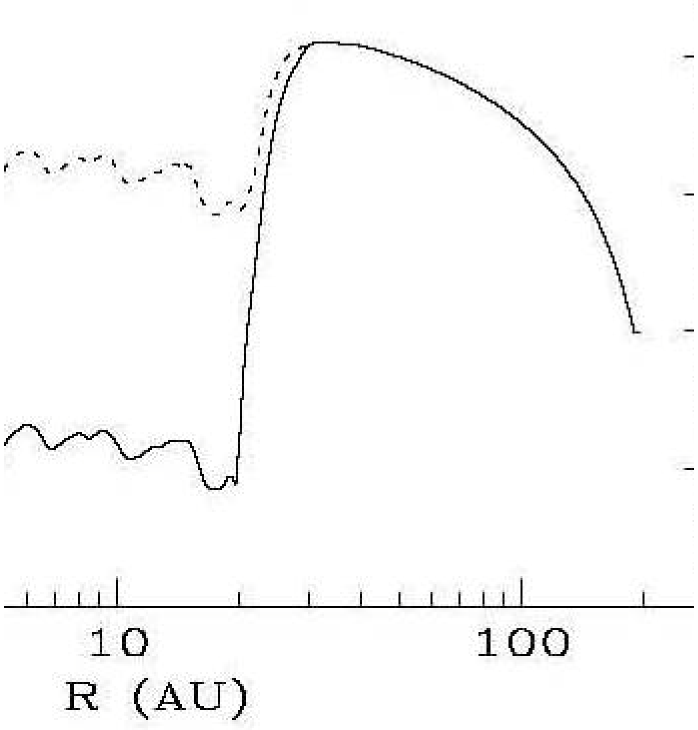


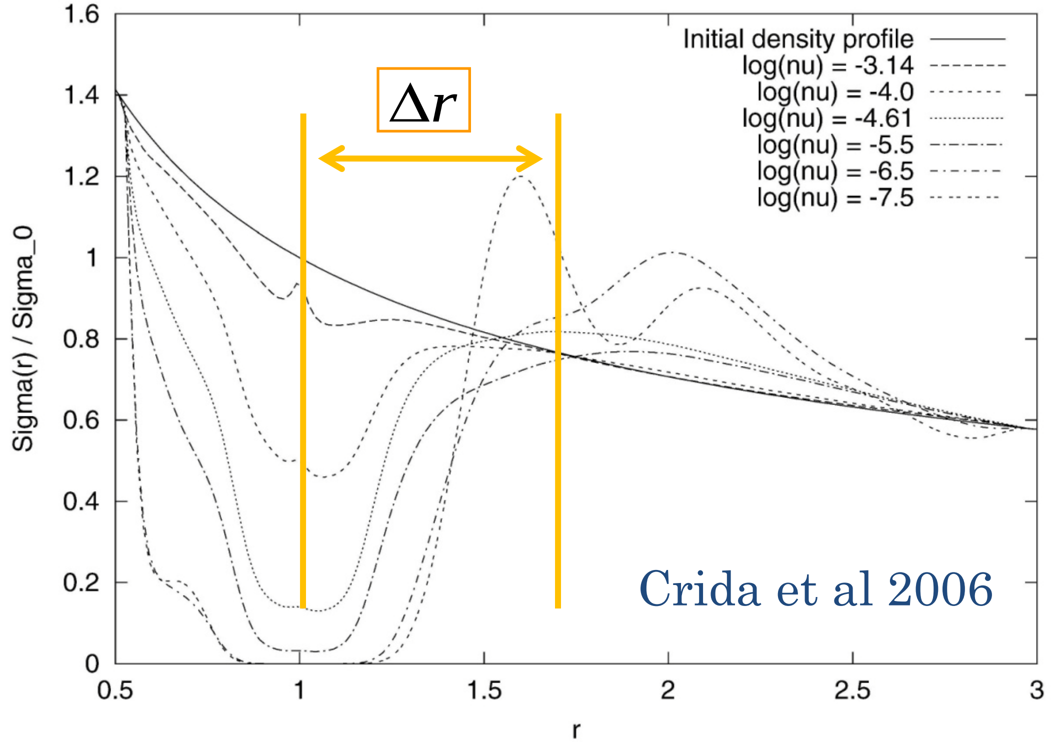
Mass Conservation: $\dot{M}_p^{box} = \dot{M}_p^{in} - \dot{M}_p^{out}$

$\dot{M}_p^{in} = z^{in}\dot{M}_{acc}$ Infinite reservoir of small grains

$\dot{M}_p^{out} = 2\pi r u_g \sum_p^{box}(s < s_{trap})$ Depends on gas background profile

'One-box' Model
--gas background





$$\nabla \cdot (\rho_g u_g) = \nabla \cdot (D_t \nabla \rho)$$

advection-diffusion balance

$$\frac{\rho_g u_g}{\Delta r} = \frac{\rho_g D_t}{\Delta r^2}$$

The distance from planet to pressure maximum

$$\Delta r = \frac{D_t}{u_g} = \frac{\nu_t}{u_g} = \frac{2}{3} r$$

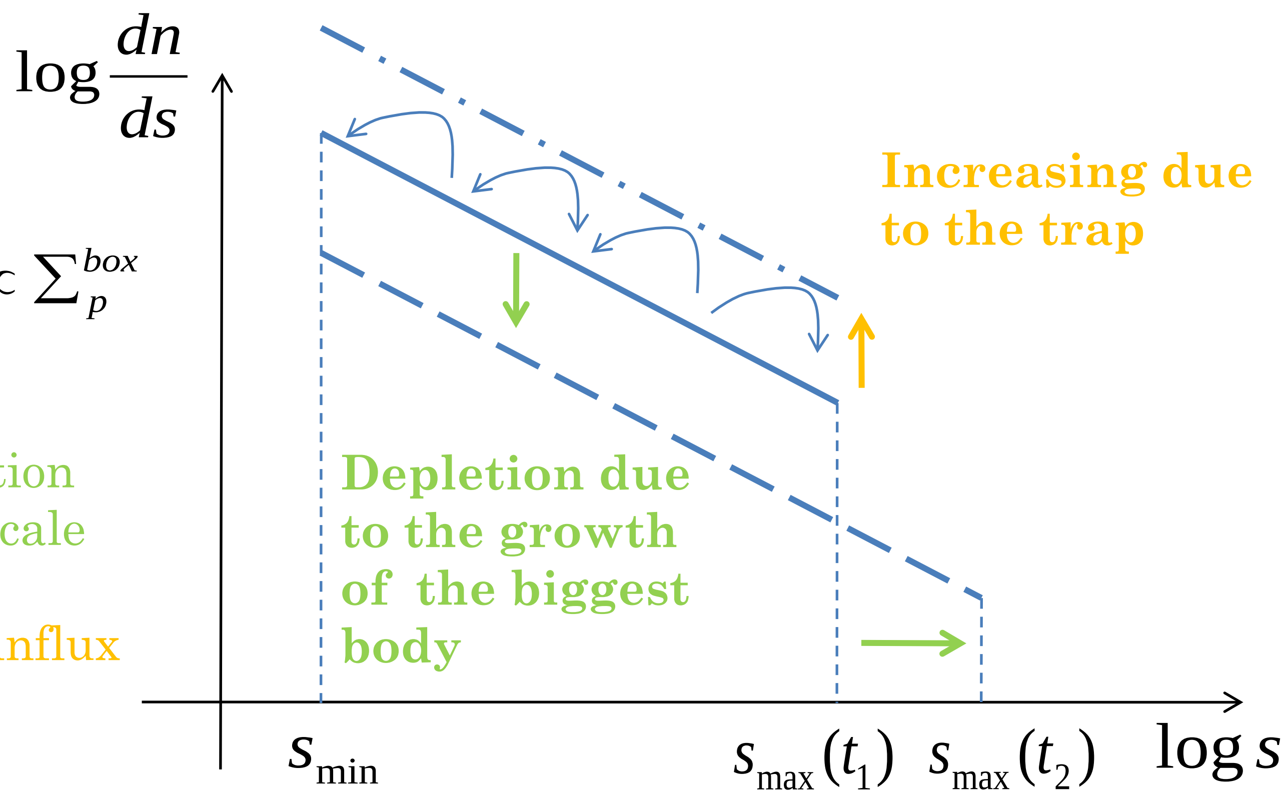
Minimum size of the trapped particles

$$s_{trap} \approx 40 \mu m$$

‘One-box’ Model
--coagulation and fragmentation

$$\frac{dn}{ds} \propto \sum_p^{box}$$

Depletion time-scale
Vs
Mass influx



Coagulation & Fragmentation Balance

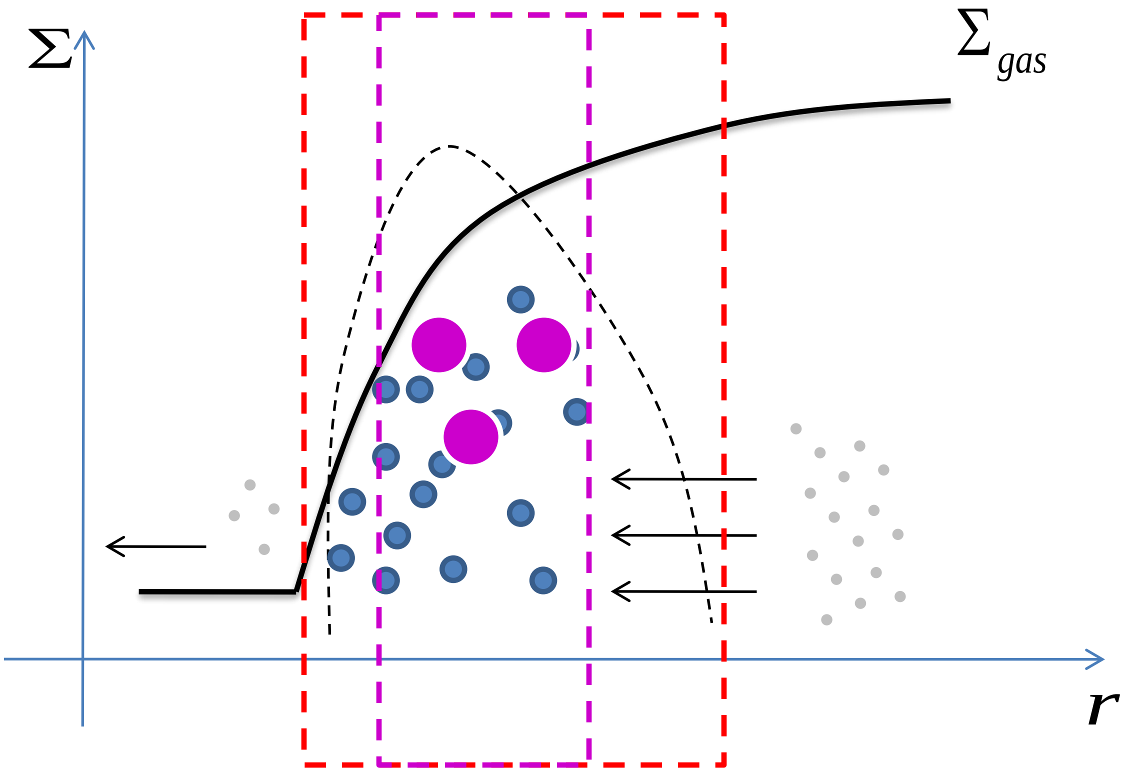
$$\frac{dn}{ds} \propto s^{-3.5}$$

$$\tau_{collision} << \tau_{dynamic}$$

$$\frac{dm_{\max}}{dt} = \int_{s_{\min}}^{s_{\max}} \frac{dn}{ds'}(s')m(s')\Delta v(s,s')A(s_{\max},s')\varepsilon ds'$$

Bierstiel et al 2011, Mathis et al 1977 etc. ISM for KBOs and ABOs.

‘One-box’ Model
--concentration of big bodies



Local surface density of particles

$$\Sigma_p^{box} = \frac{M_p^{box}}{2\pi r \Delta r_p} \qquad \Delta r_p = \frac{D_t}{u_p} = \frac{\nu_t / Sc_{eff}}{\overline{u}_p}$$

Equations

Evolution equations:

$$\frac{dM_p^{box}}{dt} = \dot{M}_p^{in} - \dot{M}_p^{out}$$

$$\frac{ds_{\max}^{box}}{dt} = f(\Sigma_p^{box}, s_{\max}^{box}, disk\ \ parameters)$$

$$\Sigma_p^{box} = \frac{M_p^{box}}{2\pi r \Delta r_p}$$

$$\Delta r_p = g(s_{\max}^{box}, disk\ \ parameters)$$

Diagnostic equation:

$$z_{dust2gas} = \frac{\Sigma_{small}}{\Sigma_{gas}^{box}} = \frac{\Sigma_p^{box}}{\Sigma_{gas}^{box}} \sqrt{\frac{s_{trap}}{s_{\max}^{box}}} \frac{\Delta r(s_{\max}^{box})}{\Delta r}$$

Spread of small particles

Initial conditions

- Viscous disk (MMSN) after 1Myr
  - $M_* = M_{solar}$
  - $z^{in} = 0.01$
  - $s_{\max}^{in} = 1mm$
  - $\alpha = 10^{-3}$
  - $M_{d0} = 0.02 M_*$
  - $\varepsilon = 0.1$
  - $r_{planet} = 50AU$
  - $\dot{M}_{acc} \approx 10^{-8} M_{solar} / yr$
  - $\Sigma_g^{box} \approx 1 g / cm^2$

Parameter space

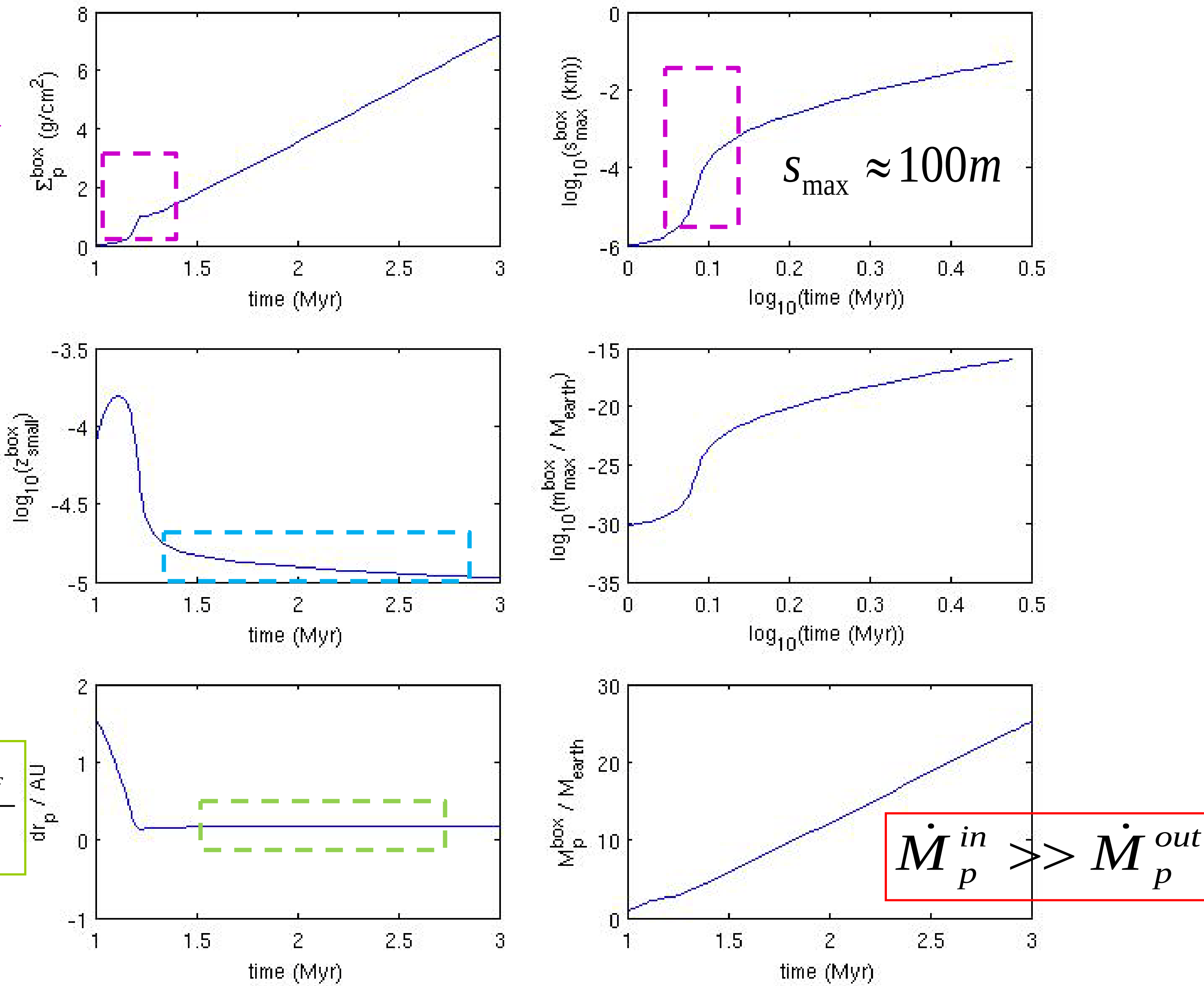
Observational constrains

Dust depletion at 50AU after 3Myr

Concentration due to growth

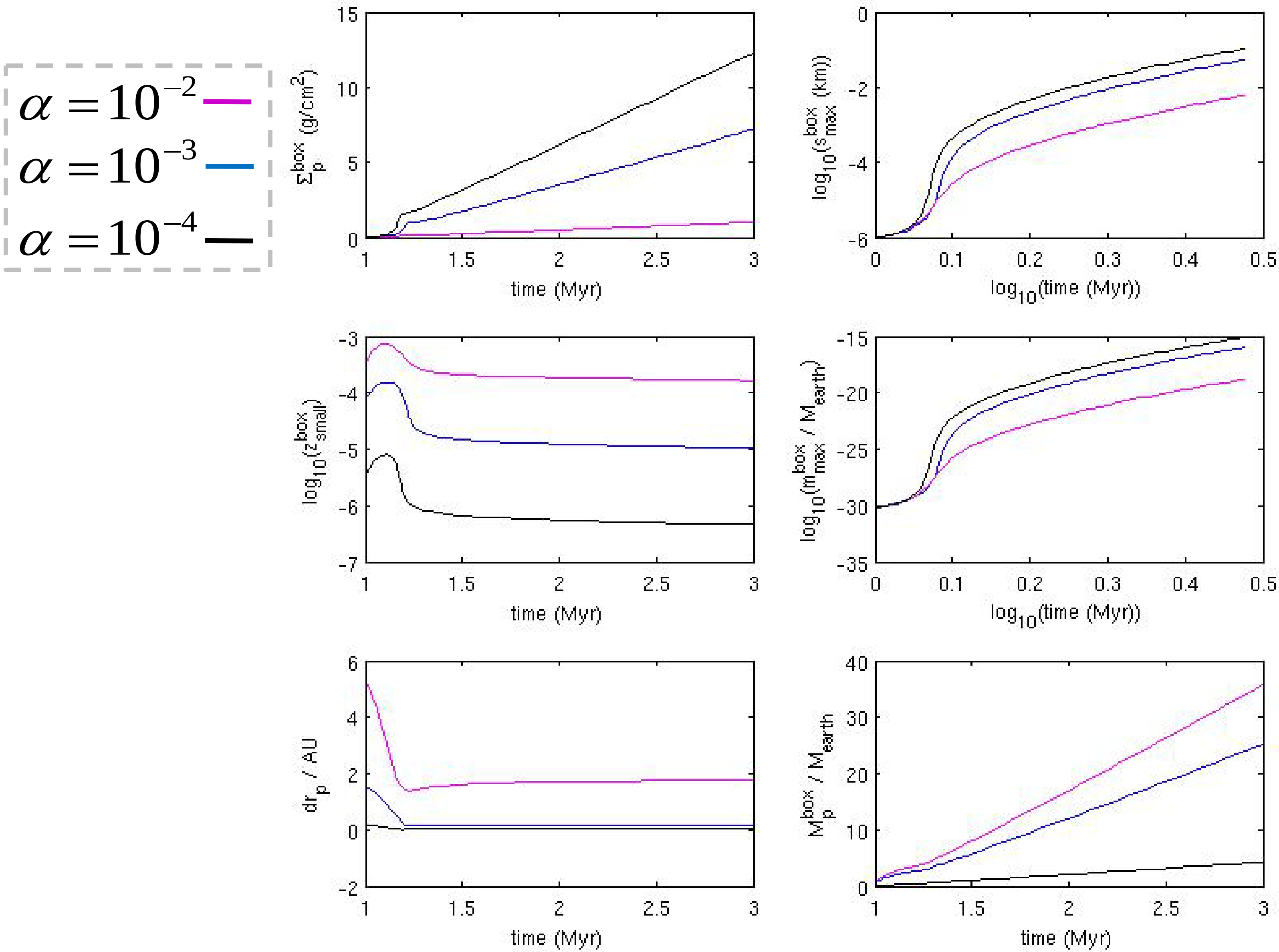
Depletion by a factor of 10

$$\Delta r_p = \frac{\nu_t / Sc_{eff}}{\overline{u}_p}$$

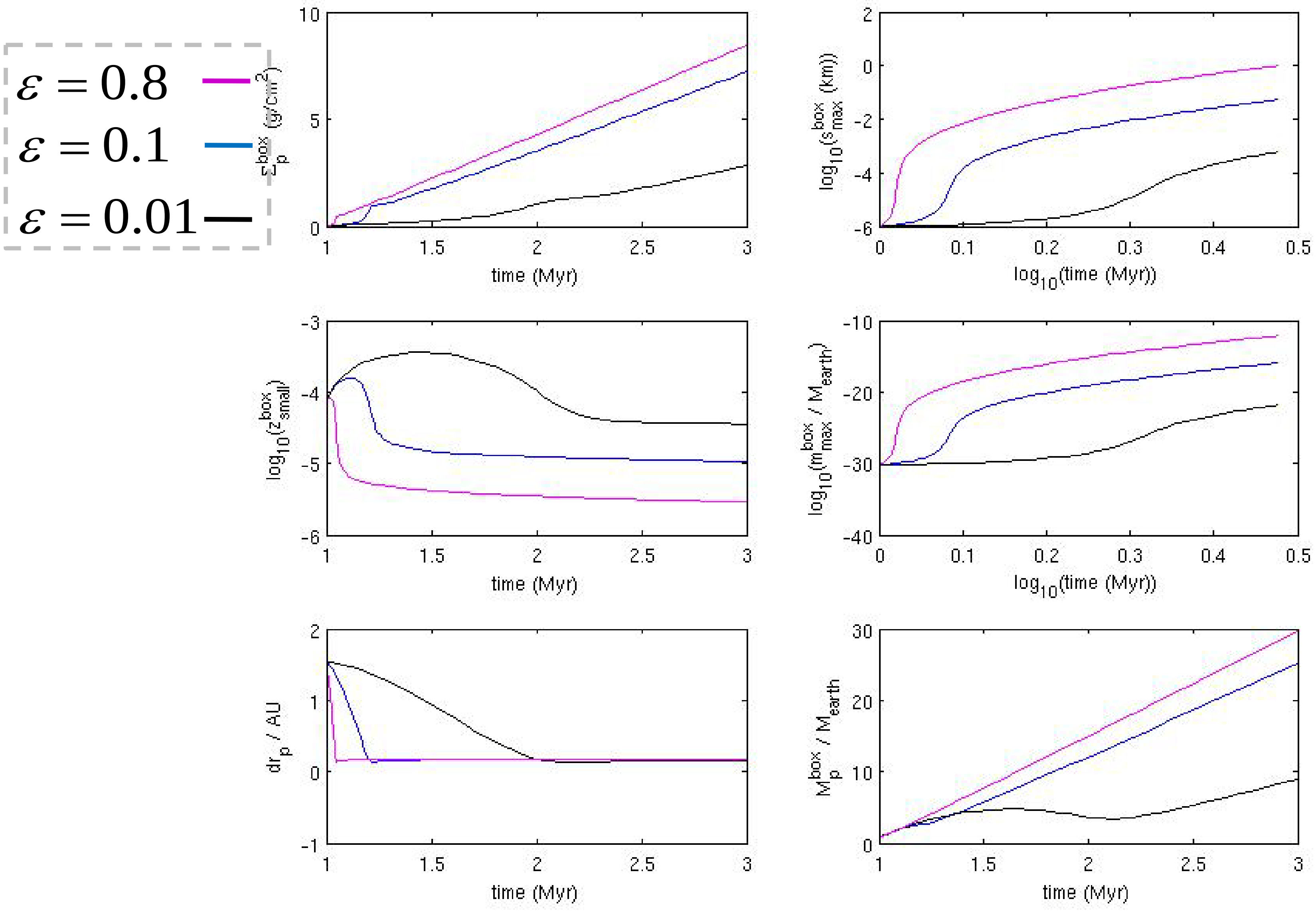


$$\dot{M}_p^{in} >> \dot{M}_p^{out}$$

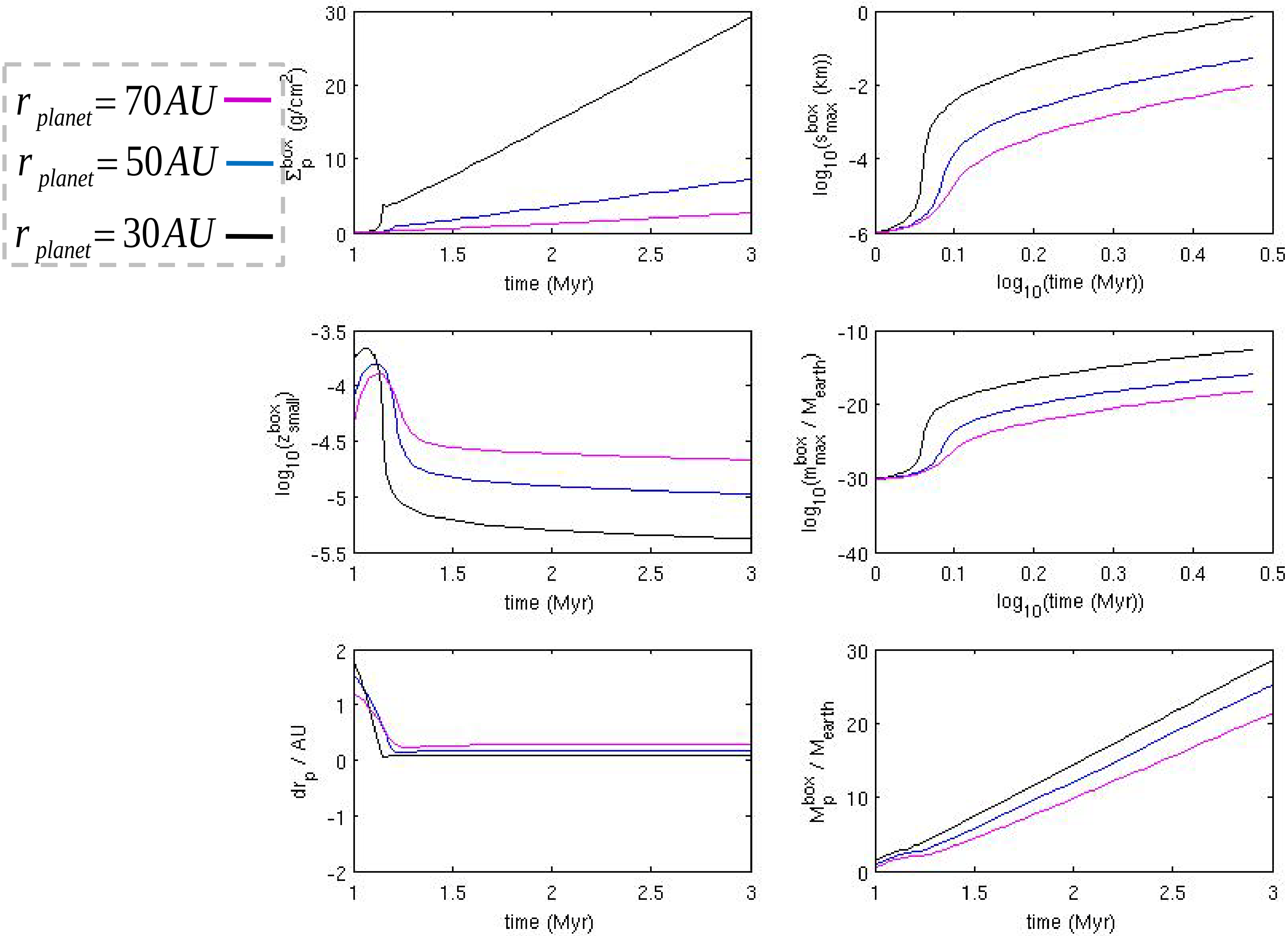
Parameter space: turbulent parameter



Parameter space: sticking efficiency



Parameter space: planet position



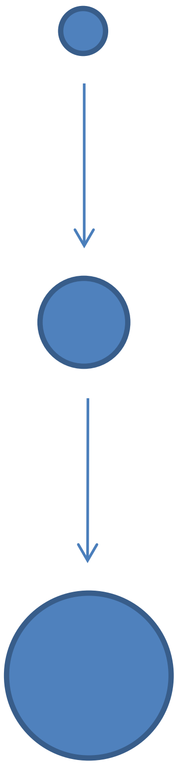
Planet formation in the 'box'?

- Only forming asteroids at ~50AU
- How about at 5AU?

- Problem of the time-scale for
  Saturn's core formation in situ in MMSN

Jupiter's formation
->Trapping particles in the outer disk
->Triggering Saturn formation

Gravitational Stirring

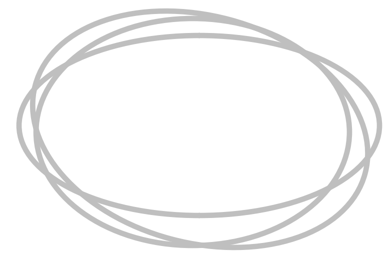


<10km:

-velocity dispersion: *Turbulence V.S. Gas drag*
-cross section: *Geometrical cross section*
-dispersion: *Advection V.S. Diffusion*

10km-100km:

-velocity dispersion:
Gravitational stirring V.S. Gas drag
-cross section: *Gravitational focusing*
-dispersion: *Eccentricity & inclination*



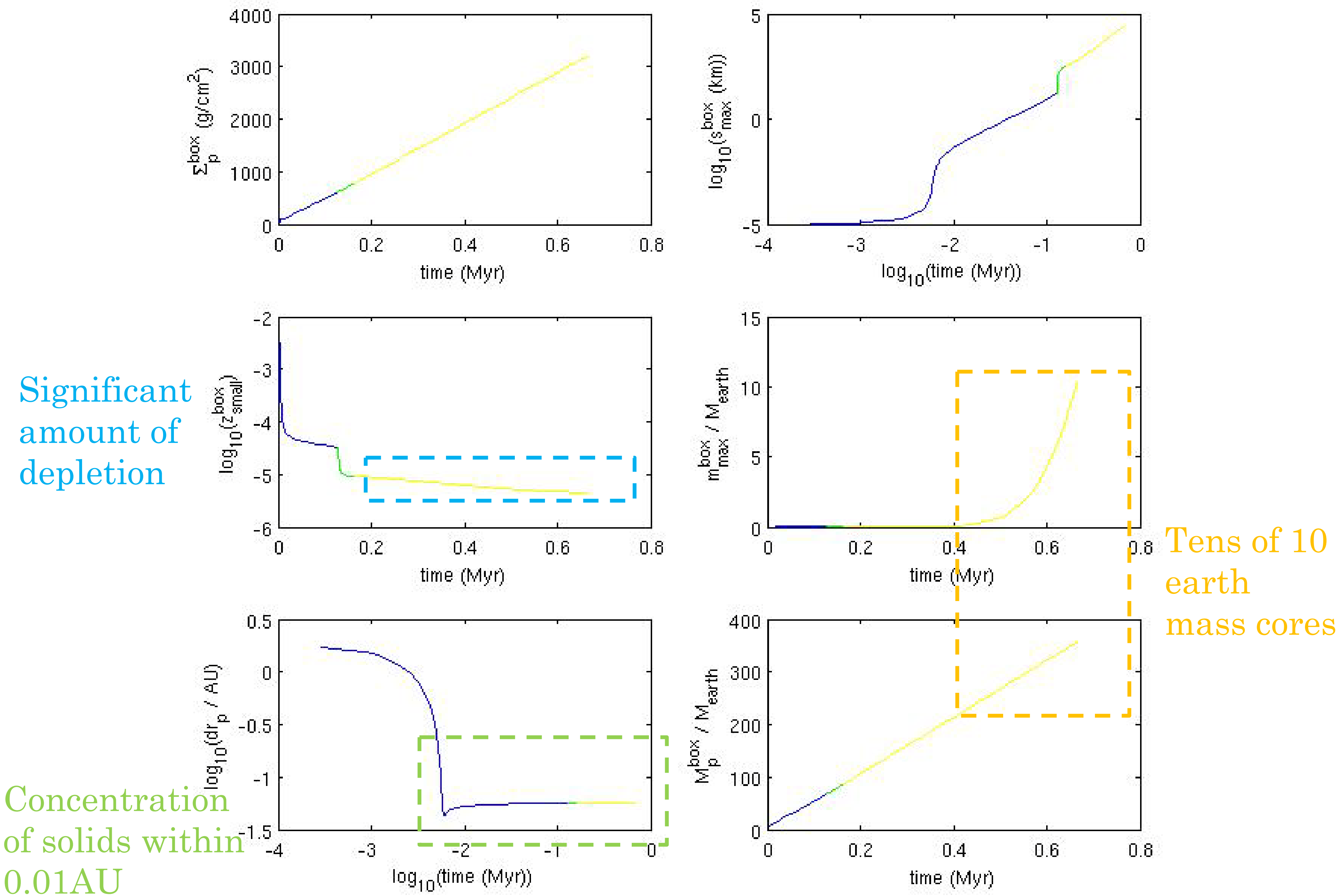
>100km:

-velocity dispersion:
Gravitational stirring V.S. Tidal damping
-cross section: *Gravitational focusing*
-dispersion: *Eccentricity & inclination*

$$\frac{dm_{\max}}{dt} = \int_{s_{\min}}^{s_{\max}} \frac{dn}{ds'}(s')m(s')\Delta v(s,s')A(s_{\max},s')\varepsilon ds'$$

$$\Delta r_p = \left\langle e^2 \right\rangle^{1/2} r$$

Planet's core formation at 5AU after ~1Myr



Conclusion & Future Work

- **Conclusion**
  - We don't necessarily need multiple gas giants to explain the optically thin gap in (pre-)transitional disks.
  - We can form the core of Saturn in a short time-scale
- **Future Work**
  - More realistic model of coagulation & fragmentation
  - Solving radial distribution
  - Forming a series of planets
  - …