Destroying resonance between Neptune and KBOs by stochastic planetesimal scatterings

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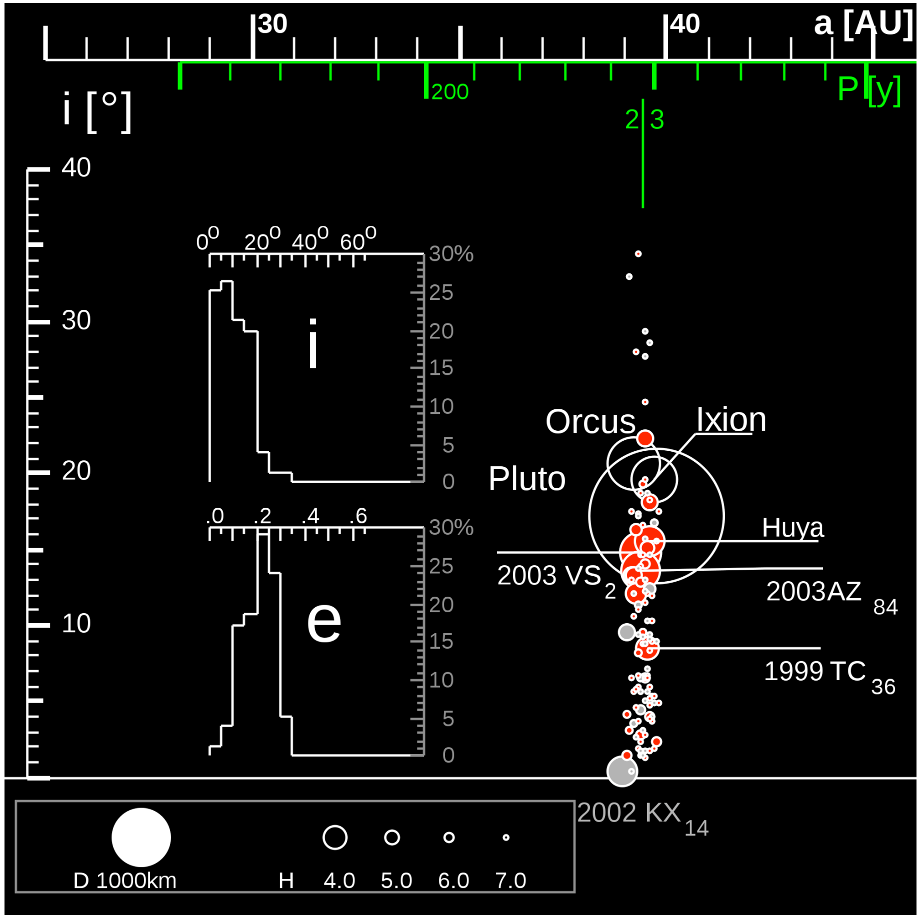
ISIMA 2011, KIAA-PKU

Resonant KBO

- The 3:2 MMR established by Neptune



Neptune migrated outward by several AU early in the history of the solar system and some KBOs was resonant captured by Neptune.

(Malhotra 1995)

wikipedia website

KBOs escape resonance

- The maximum libration amplitude as measured in semimajor axis

$$\Delta a_{\rm Nep,lib} = 2C_{lib} a_{\rm Nep} \left(\frac{M_{\rm Nep} e_{res}}{M_{\odot}} \right)^{1/2},$$ Murray & Dermott 1999

- When the change in semimajor axis of Neptune (or KBO) exceeds $\Delta a_{Nep,lib}/2 \approx 0.35$ AU (3:2 MMR), KBO escapes resonance
- The basic idea of our project is <u>to calculate the change semimajor axis of Neptune/KBO due to planetesimal scatterings and compare the result with $\Delta a_{Nep,lib}/2$ to address whether KBO can escape resonance</u>
- Our analysis is based on order-of-magnitude estimation

Estimate the change in semimajor axis

- The following equations are frequently used in this work to calculate the change in semimajor axis due to kicks

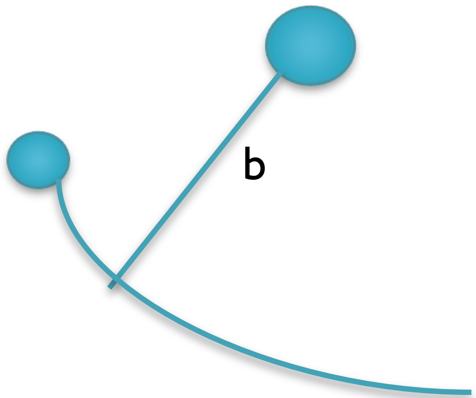
$$\Delta\left(-\frac{GM_\odot}{2a}\right) \sim \Delta\left(\frac{1}{2}v^2\right) + \Delta\left(-\frac{GM_\odot}{r}\right),$$

$$\frac{\Delta a}{a} \sim \frac{\Delta v}{v}$$

- The basic procedure:

1. estimate the change in momentum
2. calculate the change in velocity
3. estimate the change in semimajor axis

Gravitational encounter



$$\frac{\Sigma}{M_{\rm Plu}}\Omega_{\rm KBO} b_{min}^2 t \sim 1.$$

$$\Delta a_{\rm KBO} \sim \frac{\Delta v}{v} a_{\rm KBO} \sim \frac{M_{\rm Plu}}{M_\odot}\frac{a_{\rm KBO}^2}{be}.$$

$$N(b) \sim \frac{\Sigma}{M_{\rm Plu}}\Omega t b \Delta b.$$

$$\Delta a_{\rm KBO,rms}^{cum} = \left(\sum \Delta a(b)^2 N(b)\right)^{1/2}$$

Collisions

- Physical collisions

Neptune/KBO suffering a collision with a planetesimal change $\sim M_{per}u$ of its momentum. By using the specific energy equation, the change semimajor axis can be derived

- Collision cross section

When $u > v_{esc}$, σ is exactly the physical surface area of Neptune

When $v_{esc} > u > v_H$, σ is enhanced by gravitational focusing with a factor of $\sim (v_{esc}/u)^2$

When $u < v_H$, σ is enhanced by a factor of $\sim(v_{esc}/v_H)^2$

Goldreich et al. 2004

Models of planetesimal disk

- 10 Pluto mass objects; existing 5 Gyrs
  based on the current observation (Brown 2008)

- Lots of Comets (10 km sized objects); disk surface density Σ =0.2 g/cm^2; existing 1 Gyrs (Kenyon & Luu 1999)

- Nice model : 100 km sized objects; disk surface density Σ =0.2 g/cm^2; existing 1 Gyrs (Gomes et al. 2005)

- Planet X : an outer planet beyond Pluto (Lykawka & Mukai 2008) a ~100 AU; M ~ 0.1M_{earth}; e ~ 0.4; t ~ 5 Gyrs

10 Pluto mass objects ; 5 Gyrs

- Kicks on Neptune

	$u < v_H$	**$v_H < u < v_{esc}$**	**$u > v_{esc}$**	**collision**
Single Δa (AU)	0.00013	0.0035e	N/A	N/A
No. of encounters	10^5	$0.6/e^4$		
Cumulative Δa (AU)	0.04	0.003/e		

10 Pluto mass objects ; 5 Gyrs

- Kicks on KBO (u ~ 1 km/s)

	$u < v_H$	$v_H < u < v_{esc}$	$u > v_{esc}$	collision
Single Δa (AU)	N/A	N/A	0.03	N/A
No. of encounters				
Cumulative Δa (AU)			0.08	

Lost of comets ; 1 Gyrs

- Kicks on Neptune

	$u < v_H$	**$v_H < u < v_{esc}$**	**$u > v_{esc}$**	**collision**
Single Δa (AU)			N/A	3×10^{-9} e
No. of encounters				$5 \times 10^{7} / e^2$
Cumulative Δa (AU)	tiny	0.0001/e		2×10^{-5}

Lost of comets ; 1 Gyrs

- Kicks on KBO

	$u < v_H$	**$v_H < u < v_{esc}$**	**$u > v_{esc}$**	**collision**
Single Δa (AU)	N/A	N/A	10^{-6}	0.008
No. of encounters				100
Cumulative Δa (AU)			0.003	0.08

Nice model

- Kicks on Neptune

	$u < v_H$	**$v_H < u < v_{esc}$**	**$u > v_{esc}$**	**collision**
Single Δa (AU)			N/A	$3x10^{-6}$ e
No. of encounters				$5x10^{4}$ /e^2
Cumulative Δa (AU)	tiny	0.003/e		$6x10^{-4}$

Nice model

- Kicks on KBO

	$u < v_H$	**$v_H < u < v_{esc}$**	**$u > v_{esc}$**	**collision**
Single Δa (AU)	N/A	N/A	6×10^{-4}	N/A
No. of encounters				
Cumulative Δa (AU)			0.06	

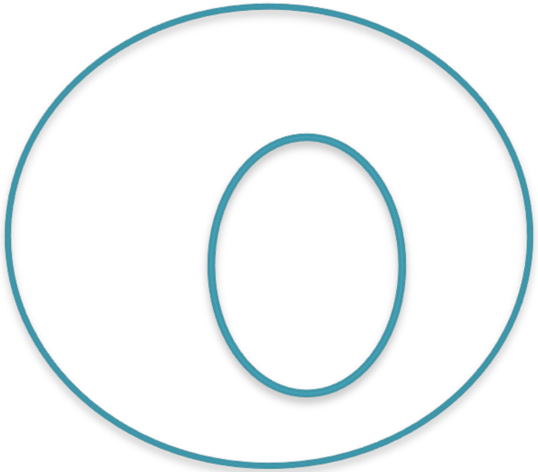
Planet X

- The minimum impact parameter

$$b_{min} \sim (1 - e_X)a_X - (1 + e_{\rm KBO})a_{\rm KBO} \sim 10 \text{ AU}.$$

- The number of encounters

$$N \sim \frac{t\Omega_X}{2\pi} \sim 5 \times 10^6.$$



By using impulse approximation, the change semimajor axis ~ 0.0001 AU

The cumulative change ~ 0.2 AU

Case No.	Encounter Type	Δa^a (AU)	Remark

Conclusion

II	Single Kick on Neptune	<0.0035	No destroying
"10 Plutos,	Cumulative Small Kicks on Neptune	0.04	No destroying
5 Gyrs"	Single Kick on KBO	0.03	No destroying
	Cumulative Small Kicks on KBO	0.08	No destroying
III	Single Kick on Neptune	too small	No destroying
"Lots of Comets,	Cumulative Small Kicks on Neptune	$0.0001/e$	No destroying
1Gyrs"	Single Kick on KBO	0.008	No destroying
	Cumulative Small Kicks on KBO	0.08	No destroying
IV	Single Kick on Neptune	too small	No destroying
"Nice model"	Cumulative Small Kicks on Neptune	$0.003/e$	No destroying
	Single Kick on KBO	6×10^{-4}	No destroying
	Cumulative Small Kicks on KBO	0.06	No destroying
V	Single Kick on Neptune	10^{-4}	No destroying
"Planet X"	Cumulative Small Kicks on Neptune	0.2	No destroying
	Single Kick on KBO	10^{-4}	No destroying
	Cumulative Small Kicks on KBO	0.2	No destroying

[a]The change semimajor axis reported in this table is the maximum value among the result in different encounter regimes.