

# Origin of Hot Jupiters: Secular Chaos

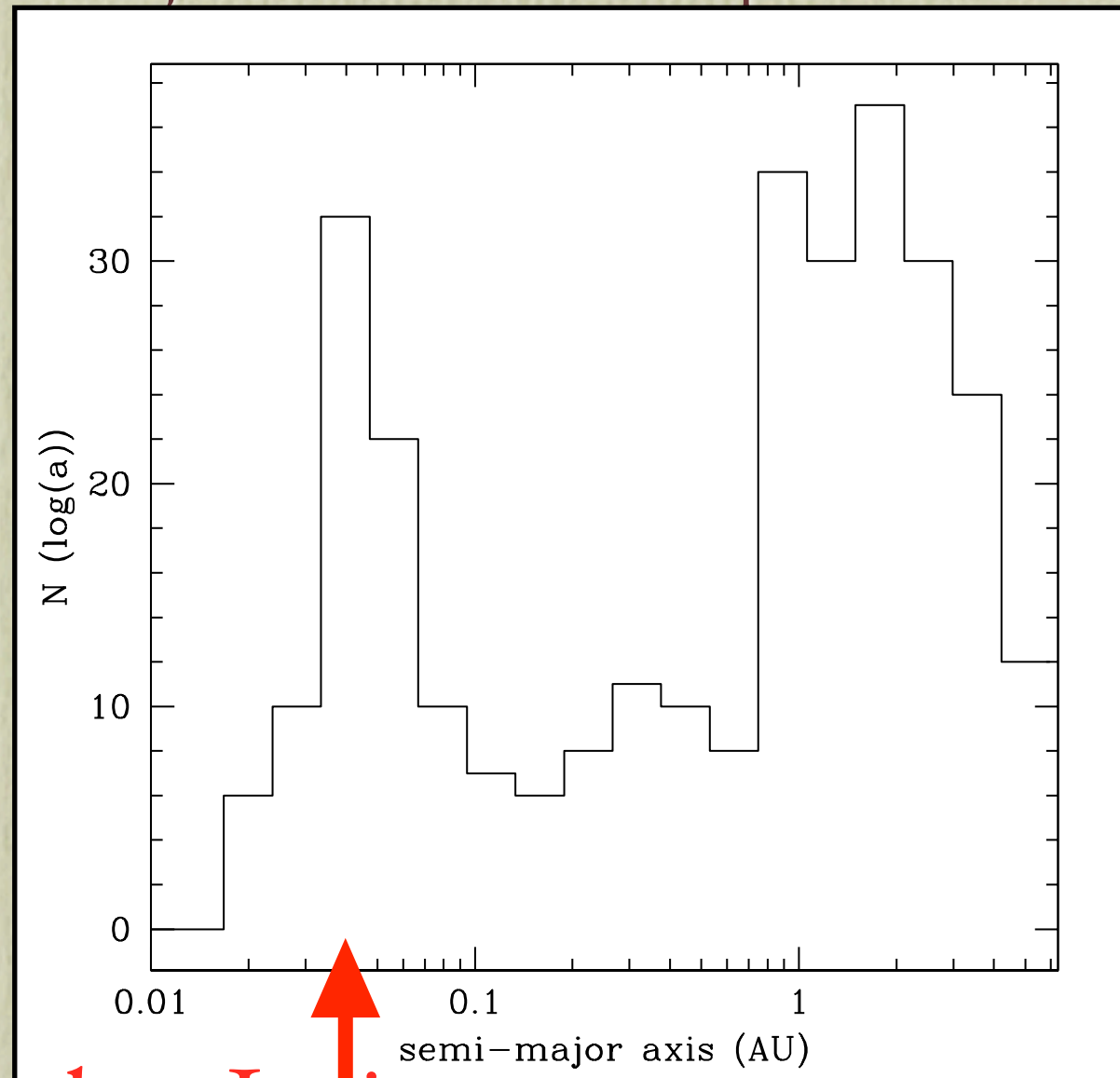
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# Hot Jupiters

Semi-major axis distribution of planets  $> 0.1 M_{\text{Jup}}$



hot Jupiters  
(~3 day orbits)

- ~ 1% of FGK stars have hot Jupiters
- How did Jupiters migrate from  $> 1$  AU to  $< 0.1$  AU?



# A New Migration Mechanism: Secular Chaos

(Wu & Lithwick 2011)

- Start with a few Jupiters beyond an AU, on widely-spaced, **mildly** eccentric & inclined orbits
- focus on **secular** (i.e. orbit-averaged) interactions.  
Okay if no close encounters or strong resonances.

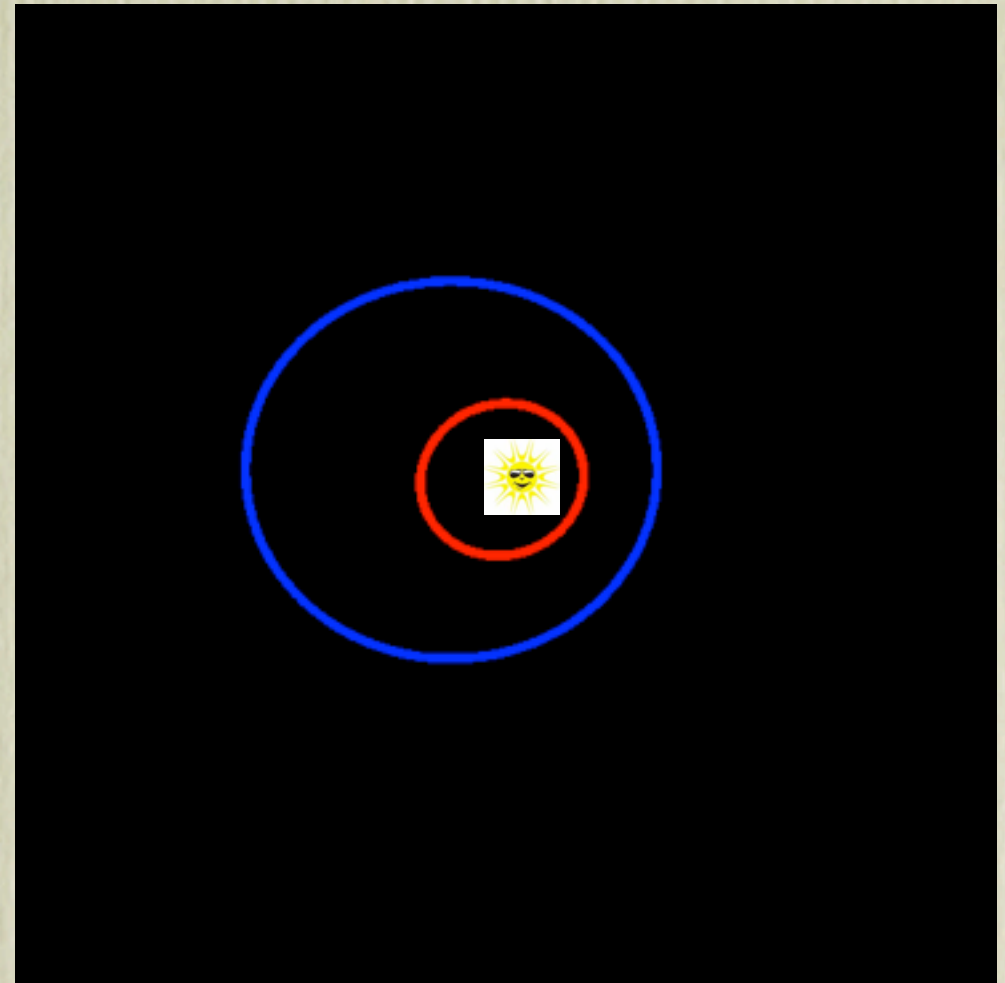
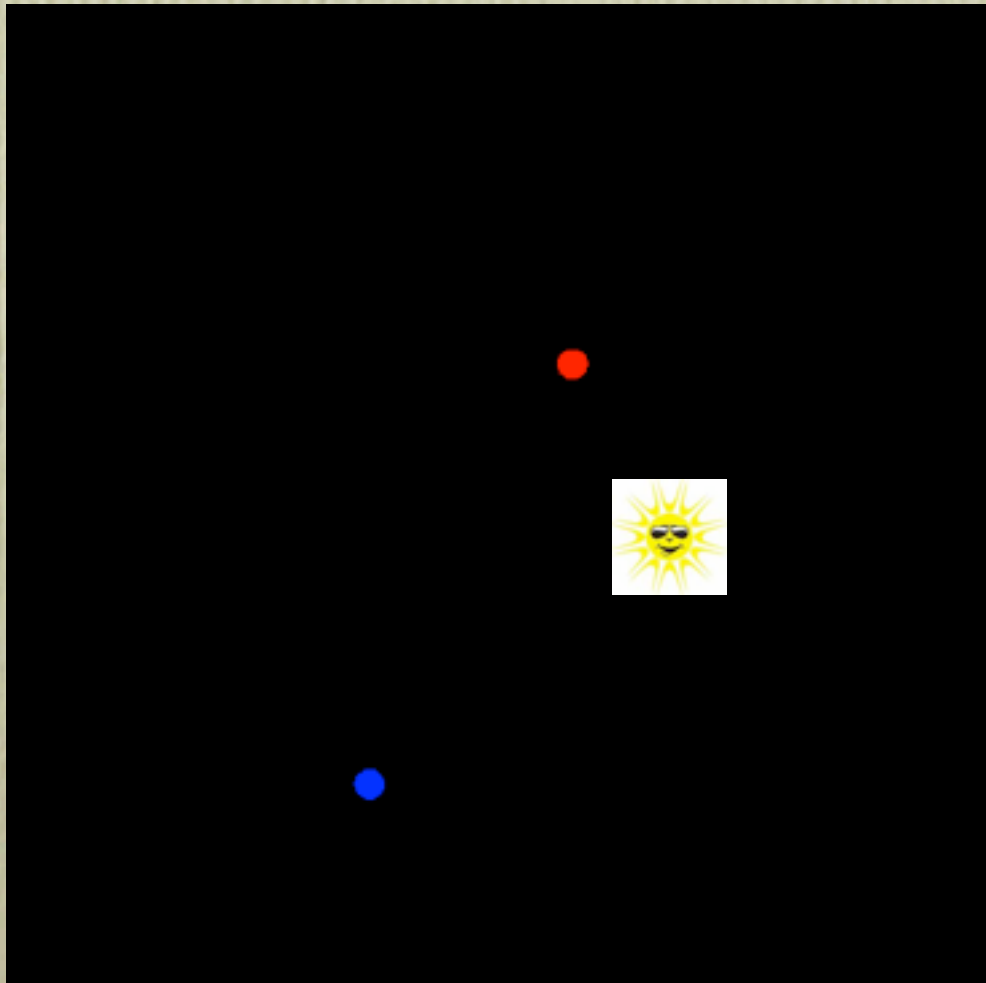




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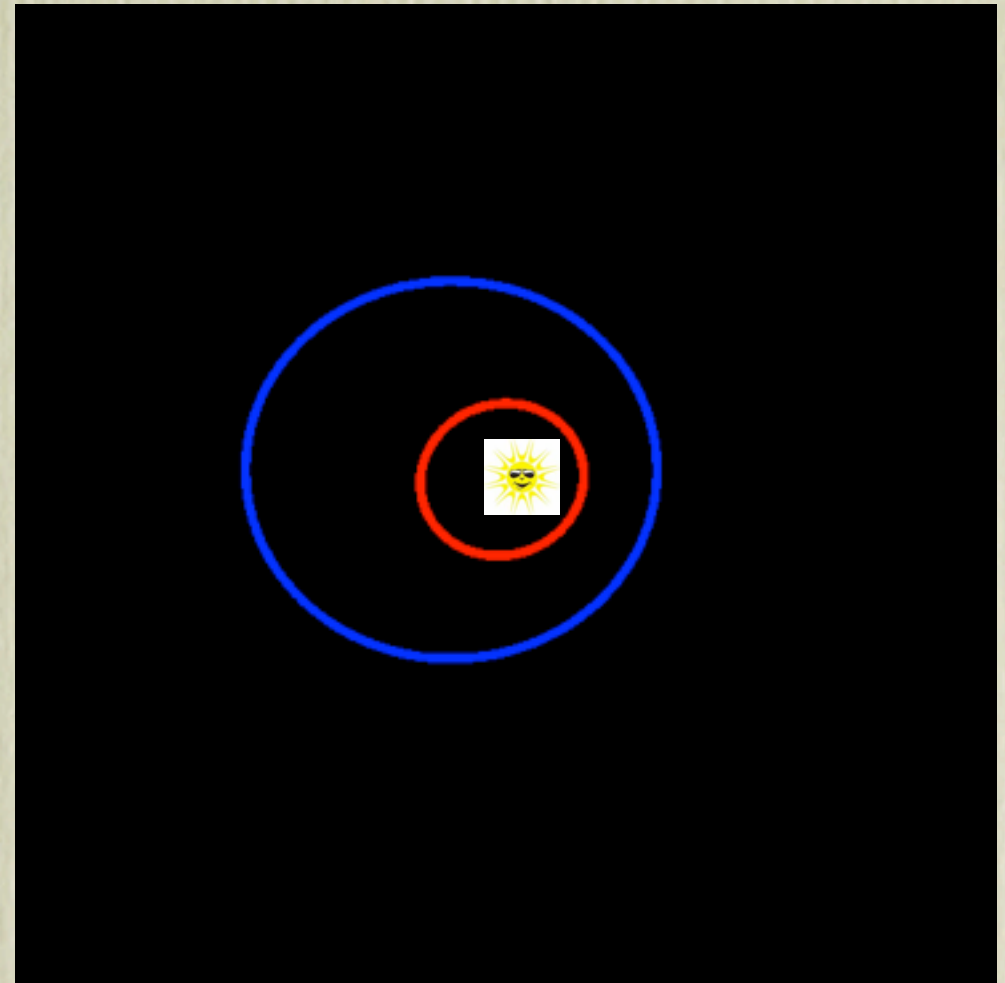
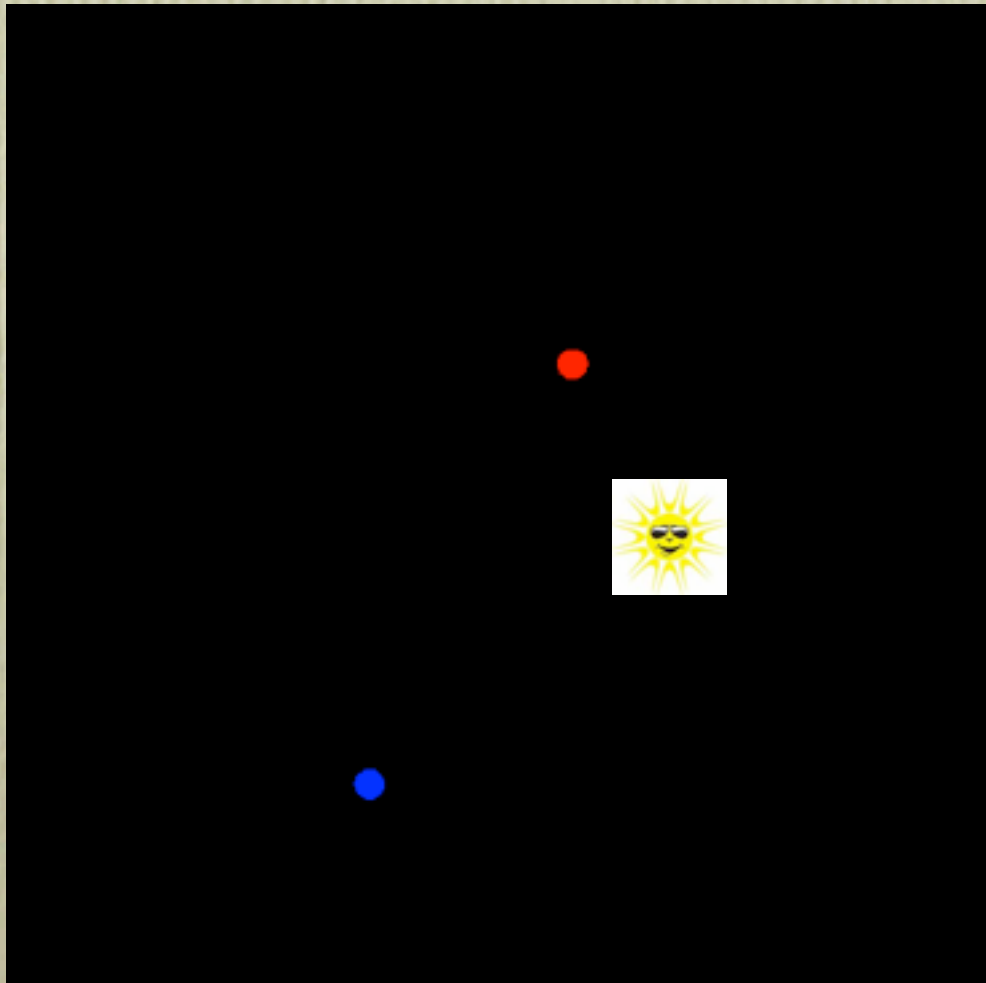




# A New Migration Mechanism: Secular Chaos

(Wu & Lithwick 2011)

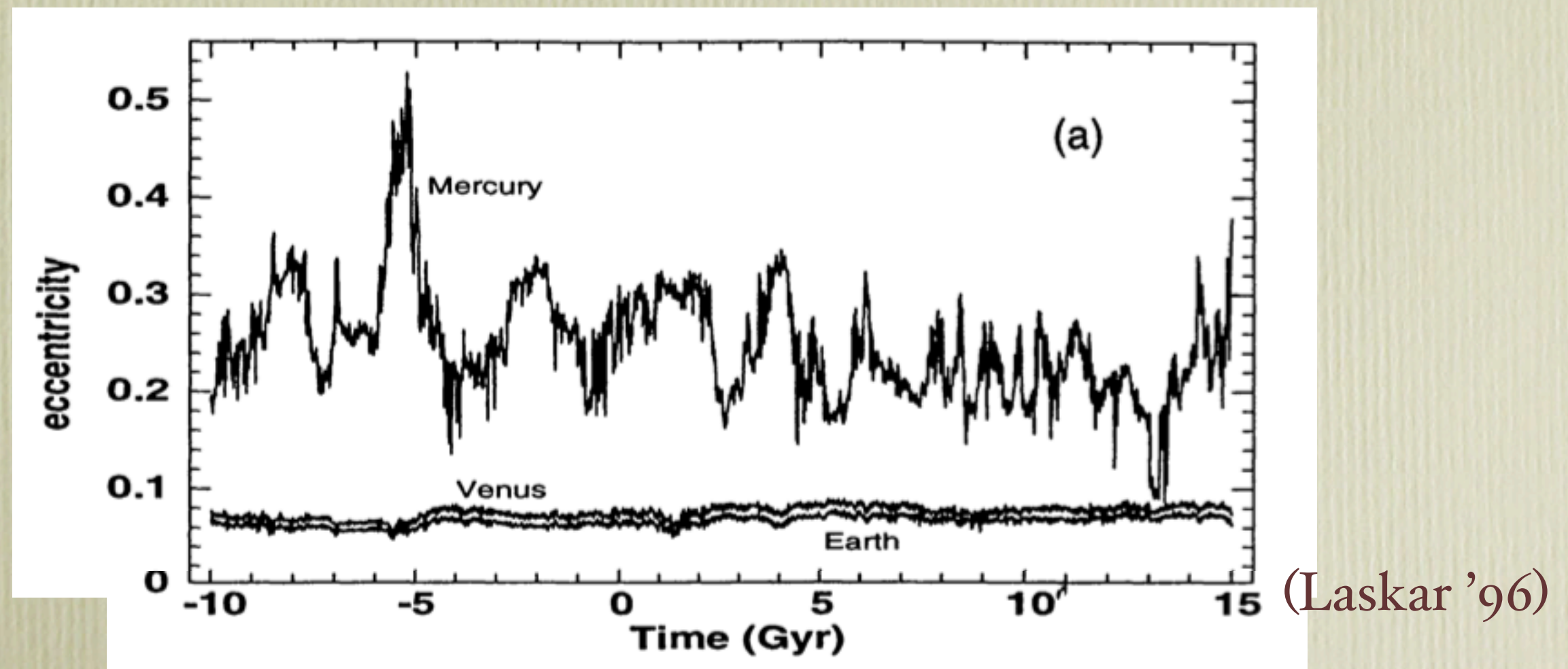
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# A New Migration Mechanism: Secular Chaos

- secular interactions can lead to chaos
- e.g., terrestrial planets' orbits driven by secular chaos

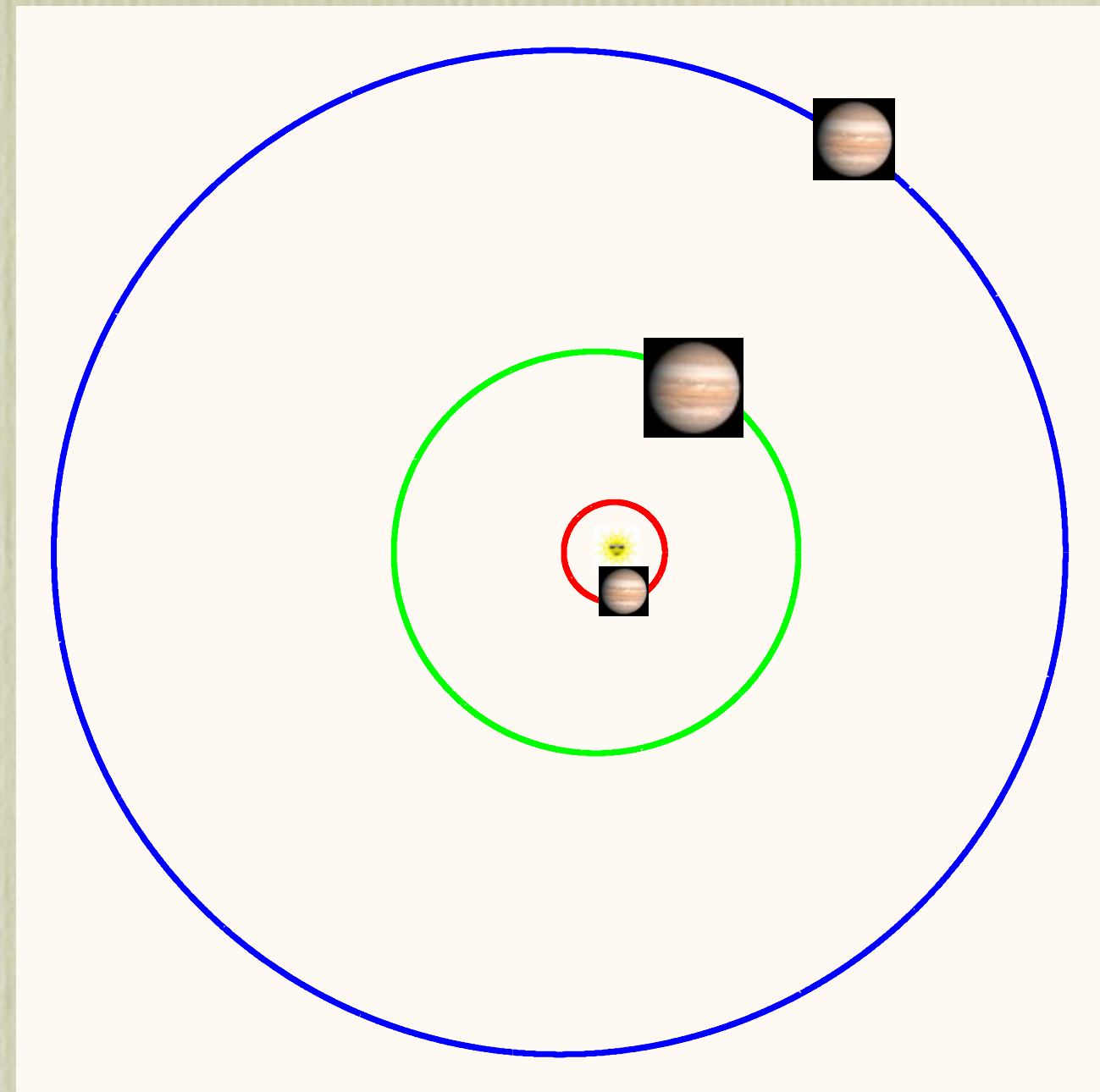




# An Example N-body Simulation

## ● Initial conditions:

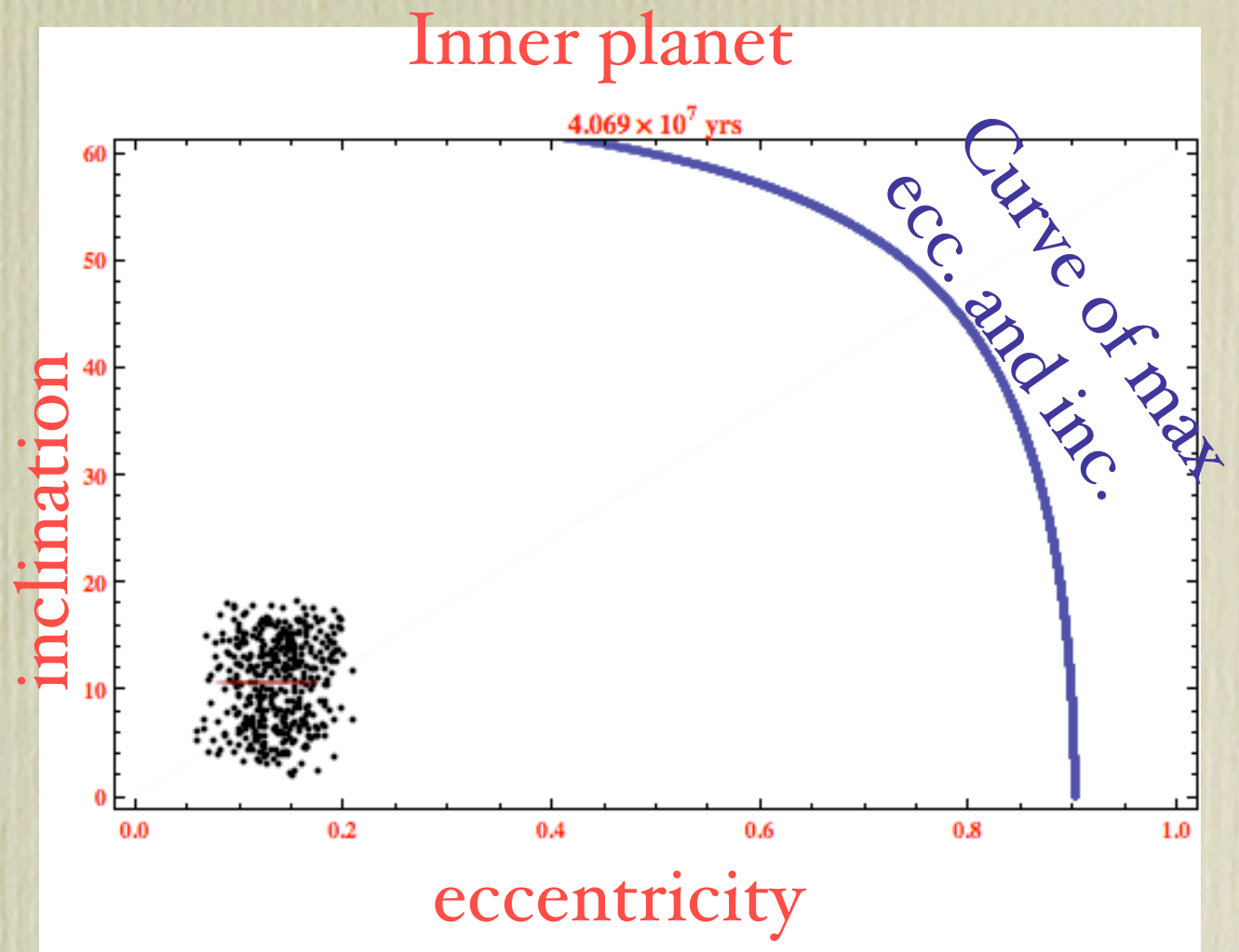
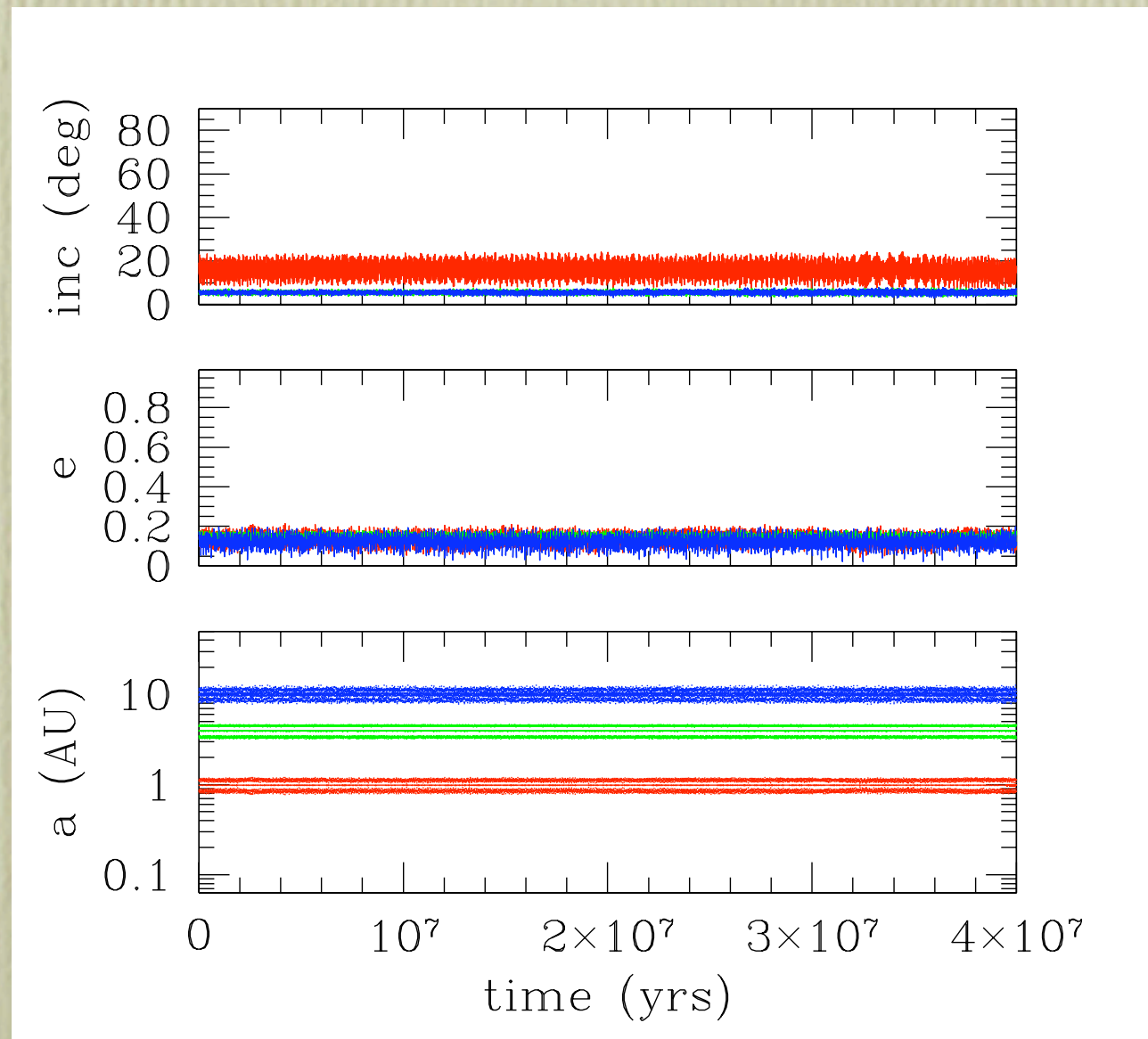
$a(AU)$	$ecc.$	$inc.$ (deg)	$mass$ ( $M_{jup}$ )
I	0.12	10	0.5
4	0.12	5	10
10	0.12	5	5





# Secular Instability

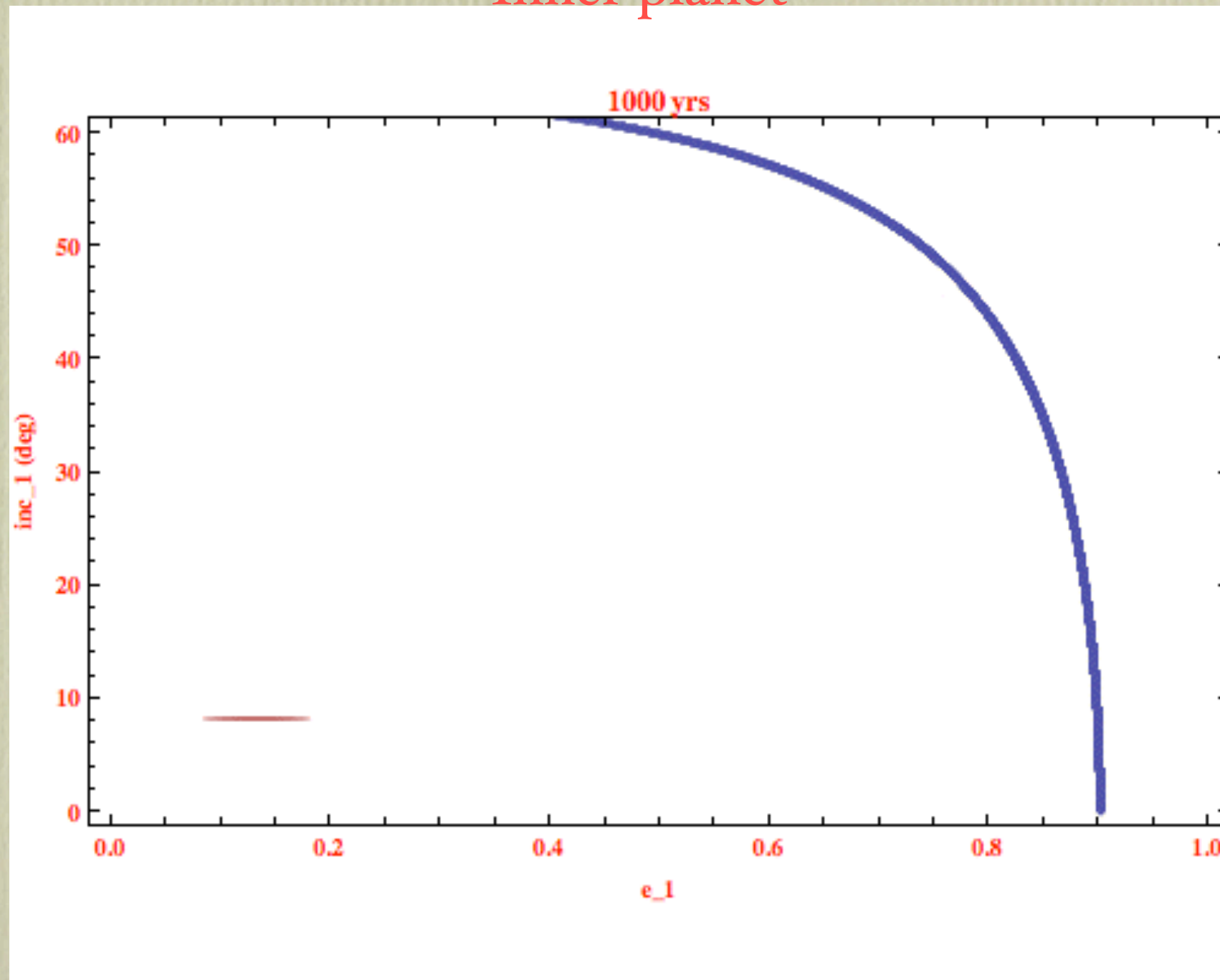
Nothing happens for a long time...





# Secular Instability

Inner planet



- Inner planet can reach high eccentricities & inclinations, given enough time



# Secular Instability

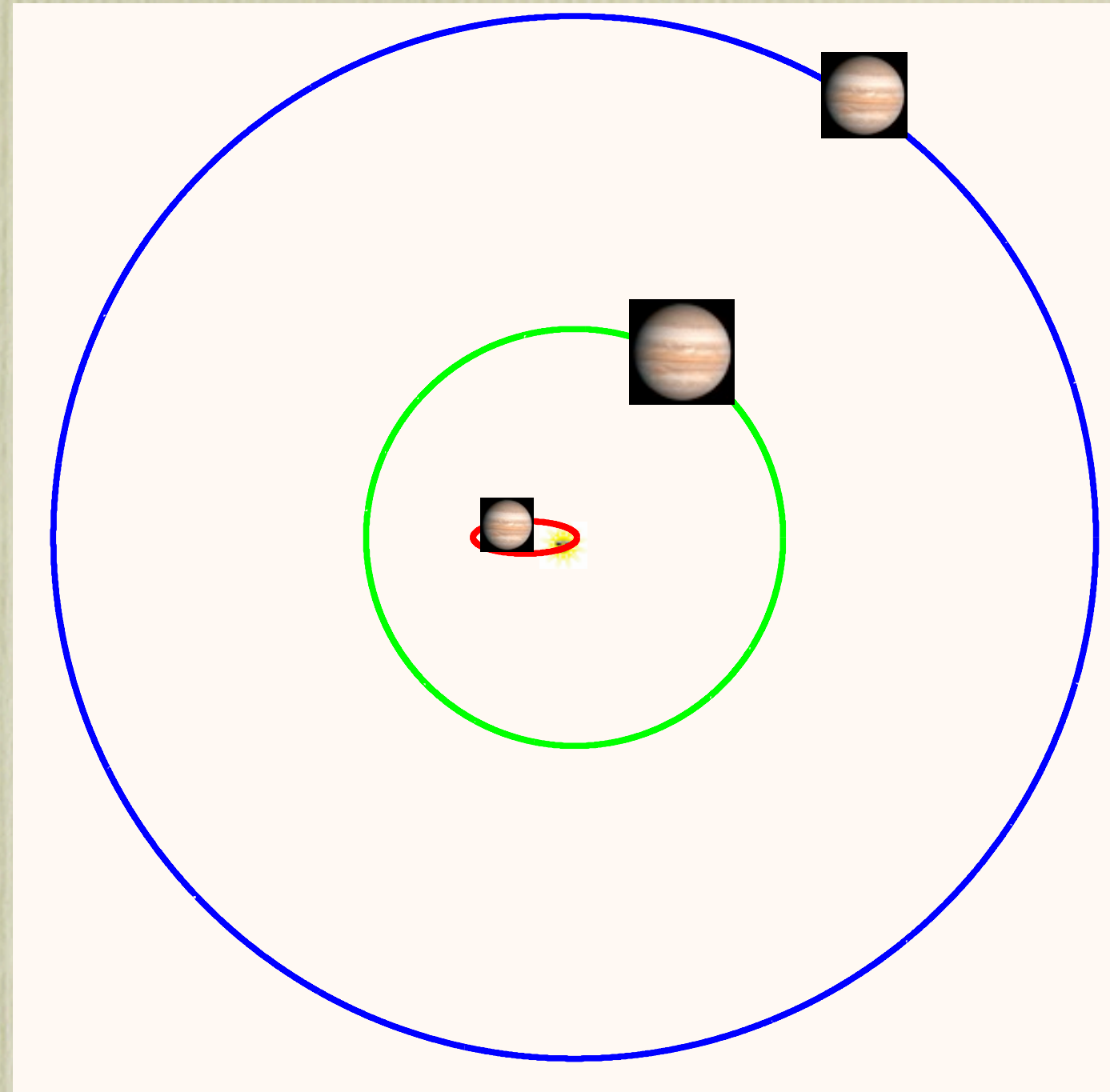
When inner planet  
acquires high  $e$ ,

- pericenter approaches star

- tides raised by star can  
circularize planet  
 $\Rightarrow$  hot Jupiter

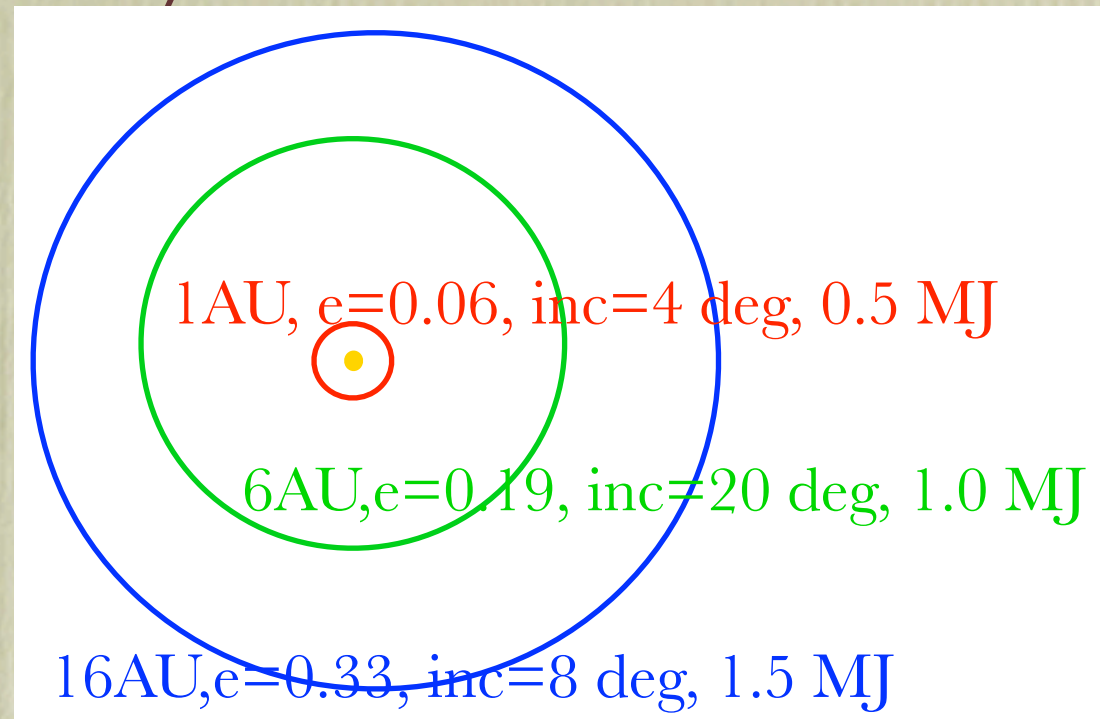
- Inner planet has smallest  
“inertia”  $\Rightarrow$  most likely to be  
excited

- Note also: remaining planets “cooled”

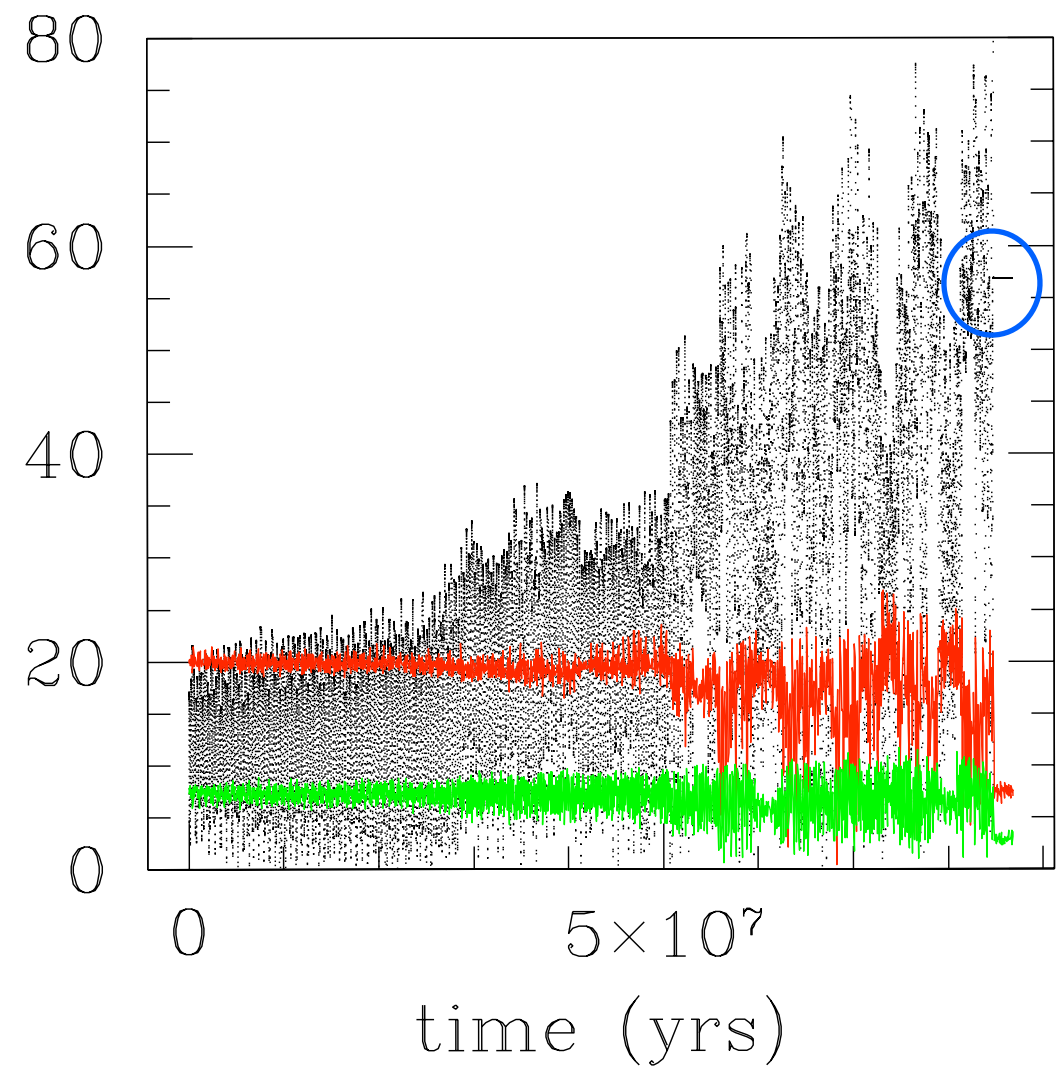
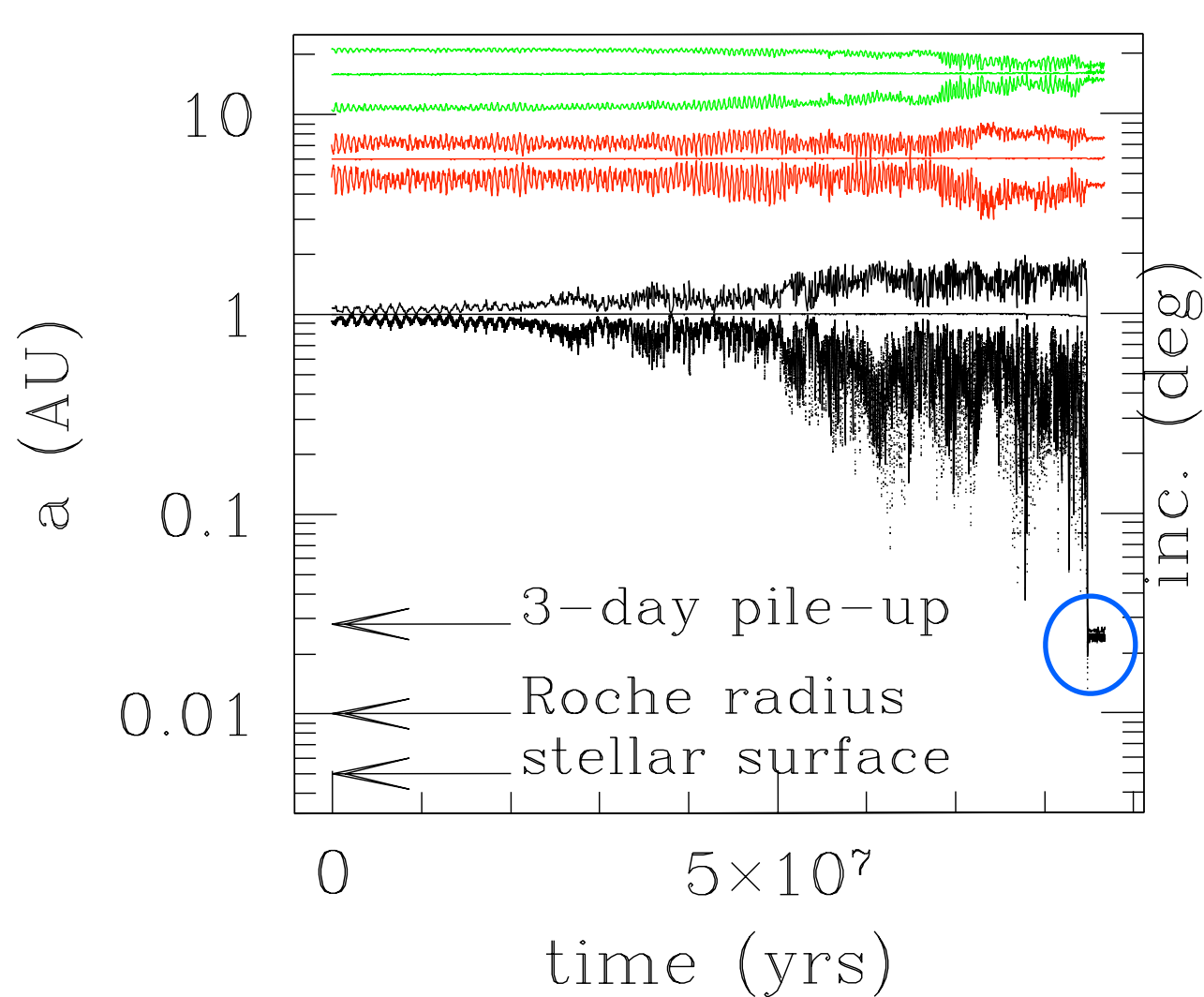




# Another system (with tides & GR)

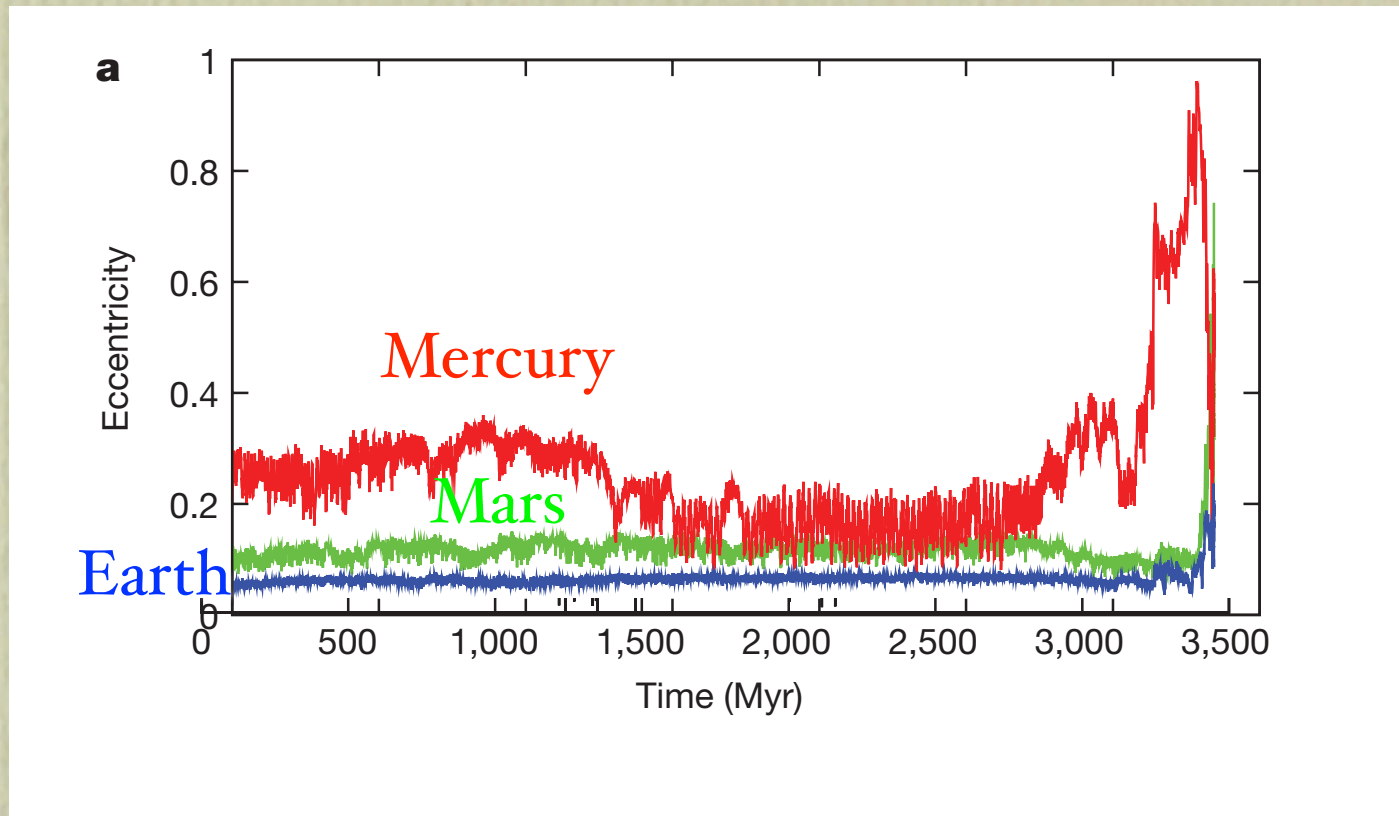


constant  $a \Rightarrow$  secular





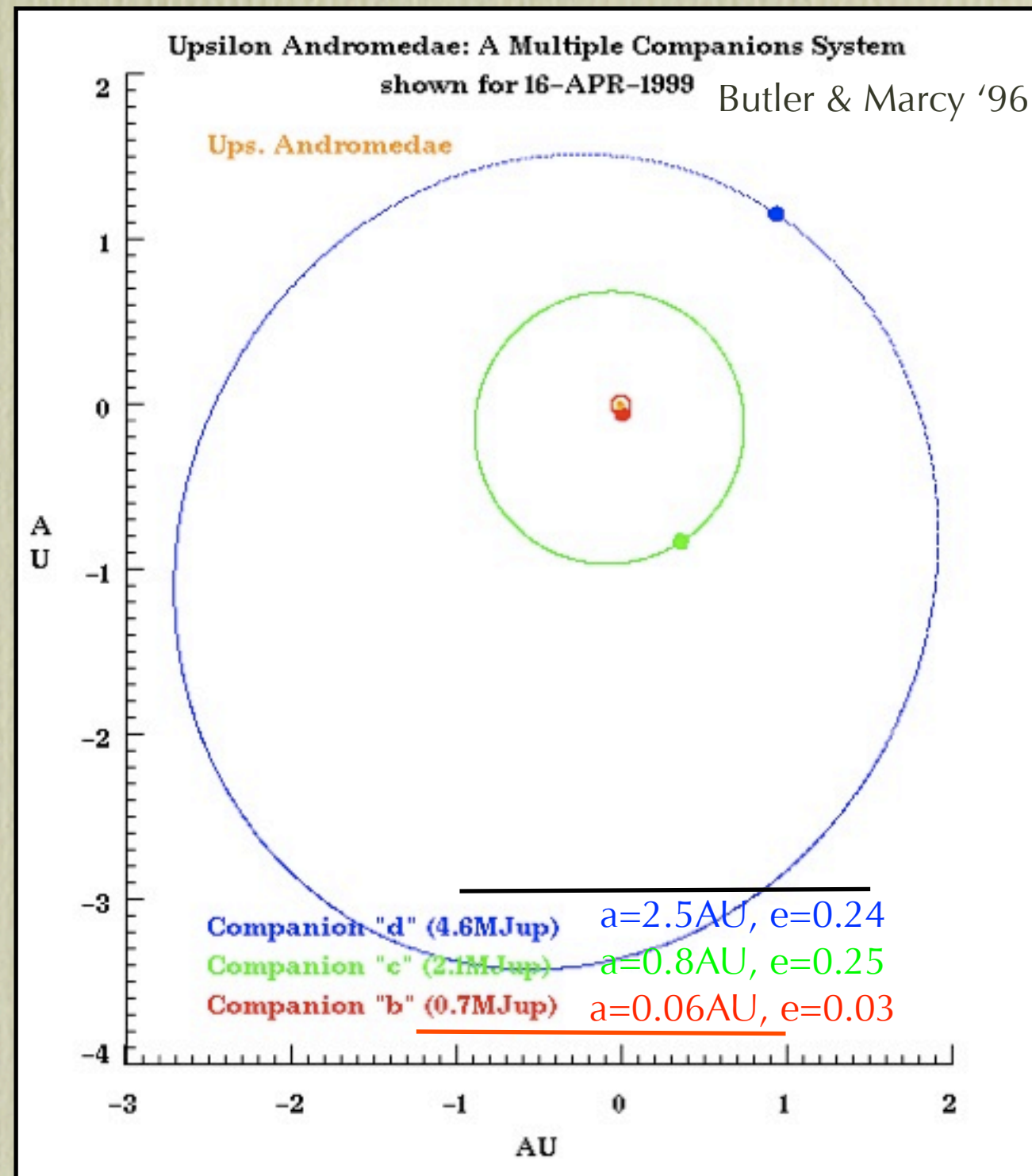
# Similar to Mercury's orbital chaos



(Laskar & Gastineau '09)



Has it really  
happened?



Michtchenko & Malhotra '04  
Migaszewski & Gozdzewski '08,'09



# Comparison with Observations

observation		explanation
3-day pile-up	✓	gradual e-growth (timescale $> 10^6$ yrs) + tidal dissipation
range of stellar obliquities (R-M)	✓	excite both e and i
lack of close companions	✓	predict: no TTV for hot Jupiters more Jupiters beyond a few AU
Masses lower than average	✓	easier to excite low mass planets

Also predict that fraction of hot Jupiters increases with stellar age  $\Rightarrow$  no hot Jupiter around T Tauri

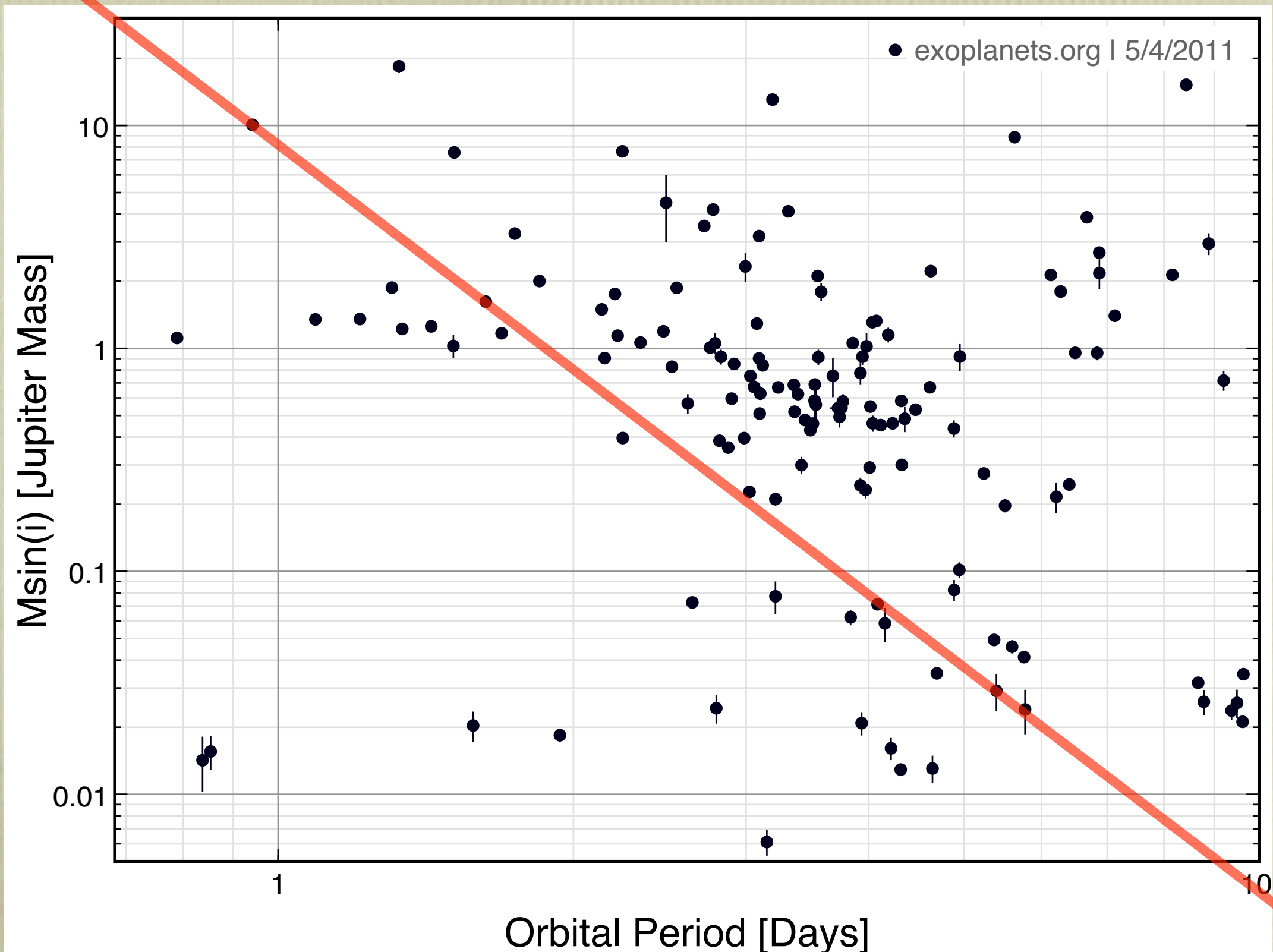


observation	secular chaos	disk migration	Kozai migration	planet scattering	
3-day pile-up	✓	✗?	✓	✗	
range of stellar obliquities (R-M)	✓	✗?	✓	✓	
lack of close companions	✓	✗	✓	✓	
Masses lower than average	✓	?	✗	✗	
			Also, cannot produce frequency of hot Jupiters (Wu et al.)	Also, initial conditions are artificial	



# Secular chaos predicts

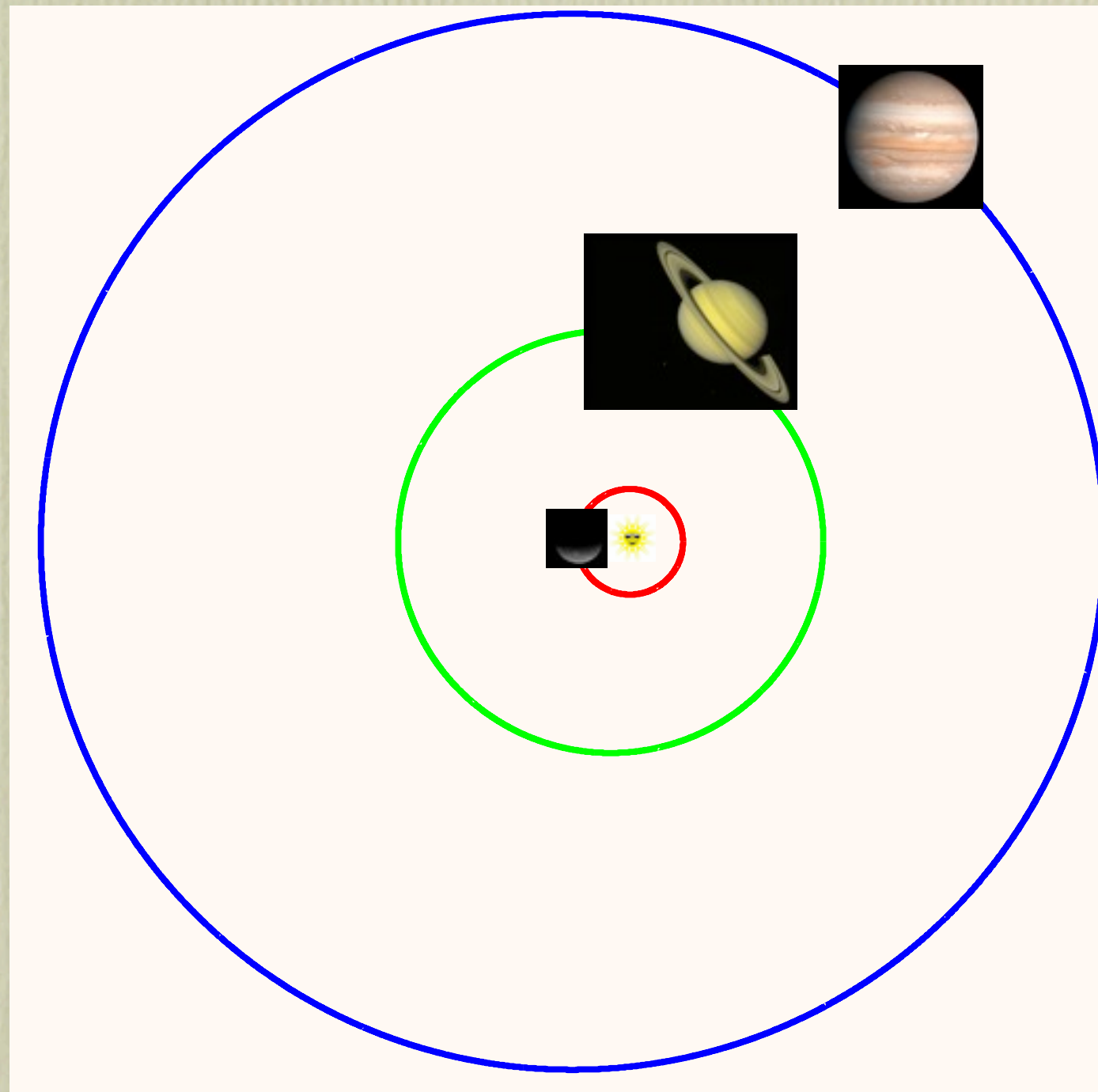
$$M \simeq M_J \left( \frac{\text{orbital period}}{1.9 \text{ days}} \right)^{-10/3}$$





# Theory of Secular Chaos (Lithwick & Wu 2011)

Simple example of instability: two massive & one massless planet (“Jupiter”, “Saturn”, and “Mercury”). Assume **coplanar**.

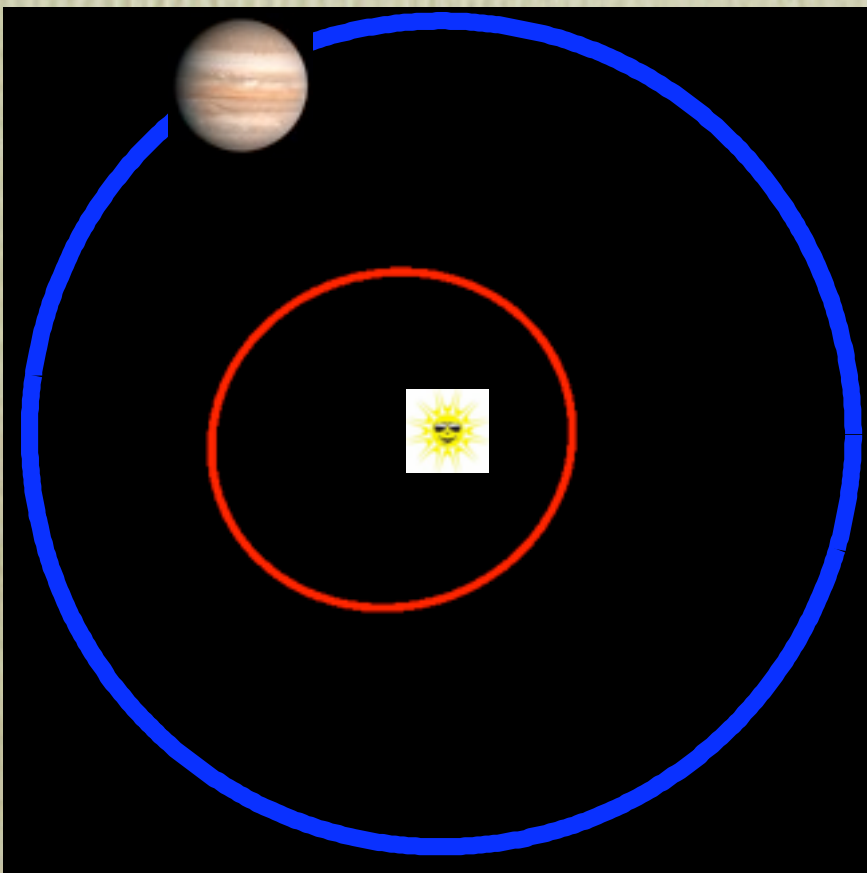




# I. Circular Jupiter, no Saturn

⇒ Mercury precesses at const. rate, with const. eccentricity

$$\text{precession time / orbital time} \sim \frac{M_{\odot}}{M_J} \frac{a_J^3}{a_M^3} \sim 10^6$$

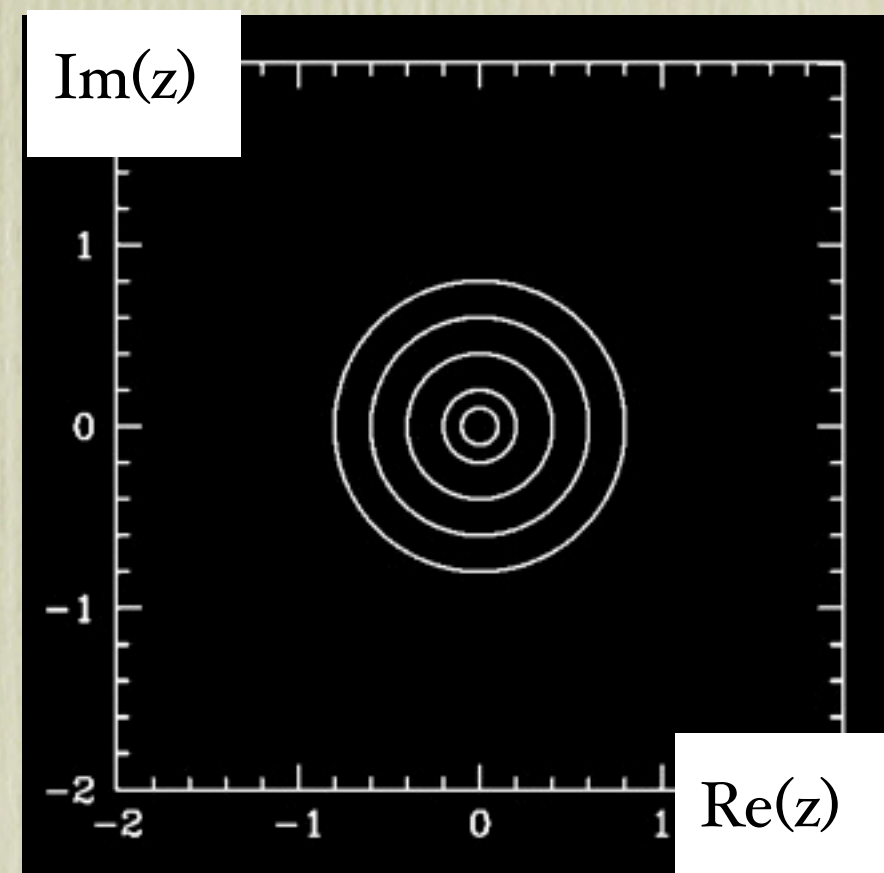
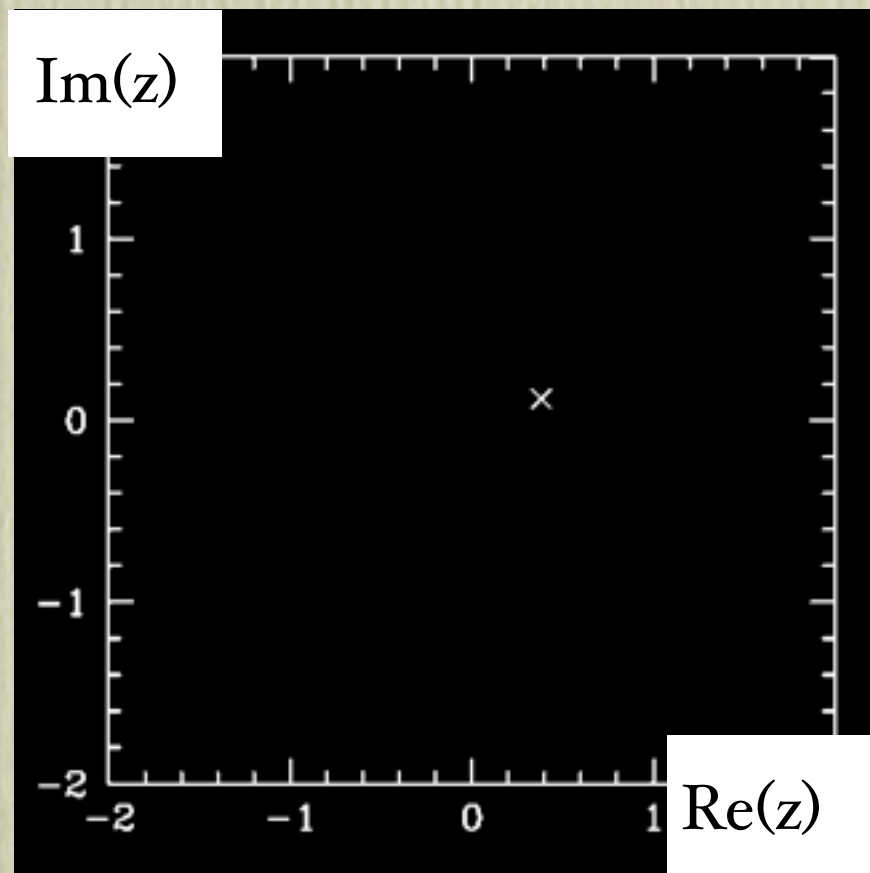
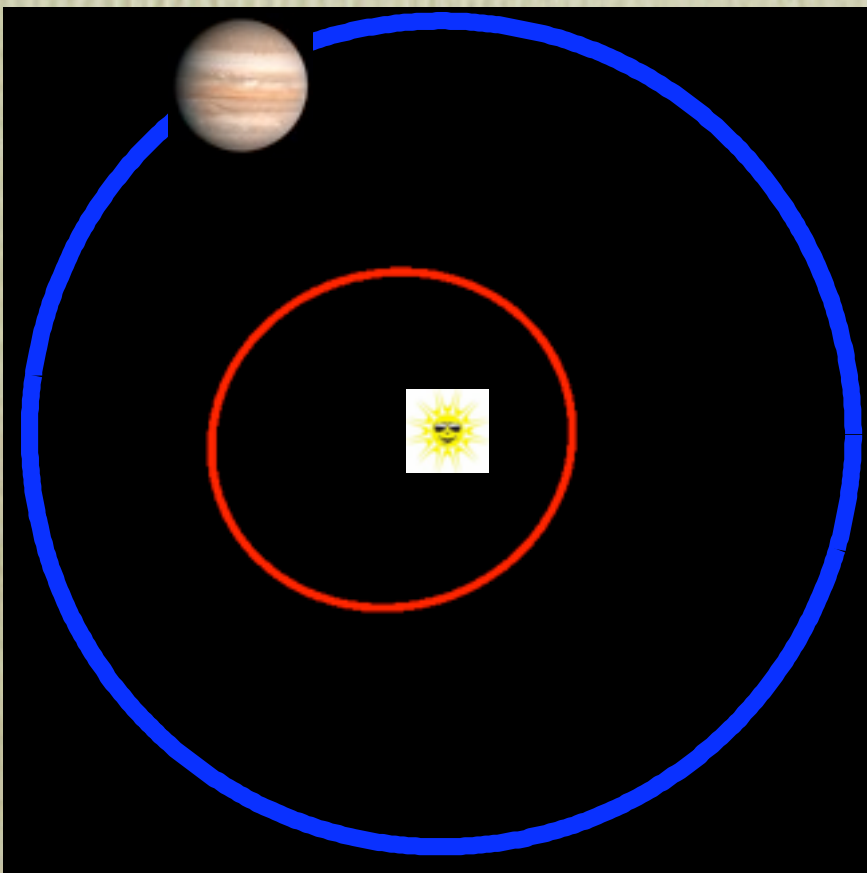




# I. Circular Jupiter, no Saturn

⇒ Mercury precesses at const. rate, with const. eccentricity

$$z \equiv \underset{\substack{\uparrow \\ \text{eccentricity}}}{e} \cdot \underset{\substack{\uparrow \\ \text{peri. angle}}}{e^{i\varpi}}$$



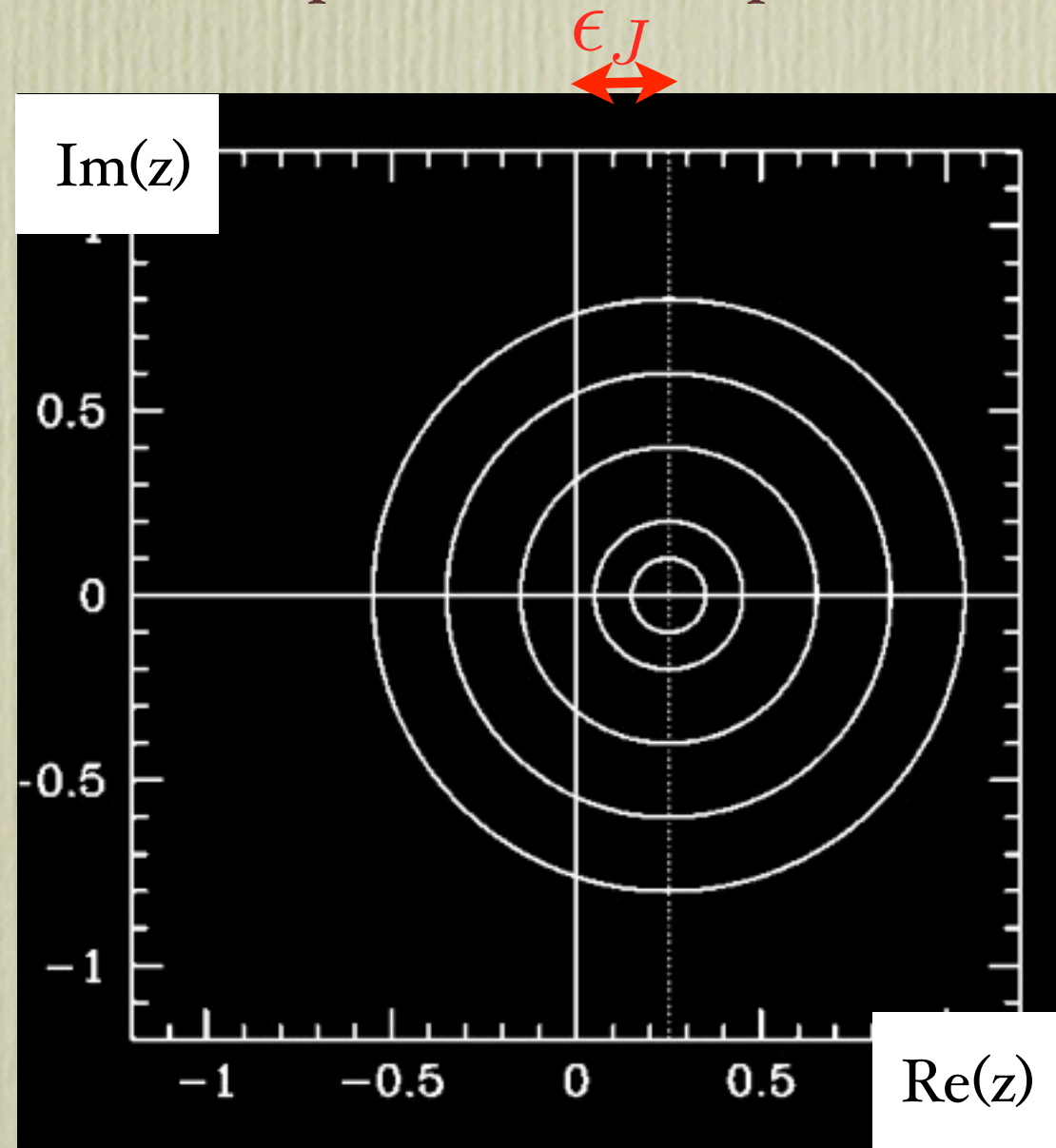


2. **Eccentric Jupiter:**  $\frac{d}{dt}z = i(z - \epsilon_J)$

$\uparrow$   
 ~Jupiter's eccentricity  $\times a_M/a_J$

solution:  $z = \underset{\substack{\uparrow \\ \text{free}}}{C}e^{it} + \underset{\substack{\uparrow \\ \text{forced}}}{\epsilon_J}$

(Time in units of the free secular precession freq.)

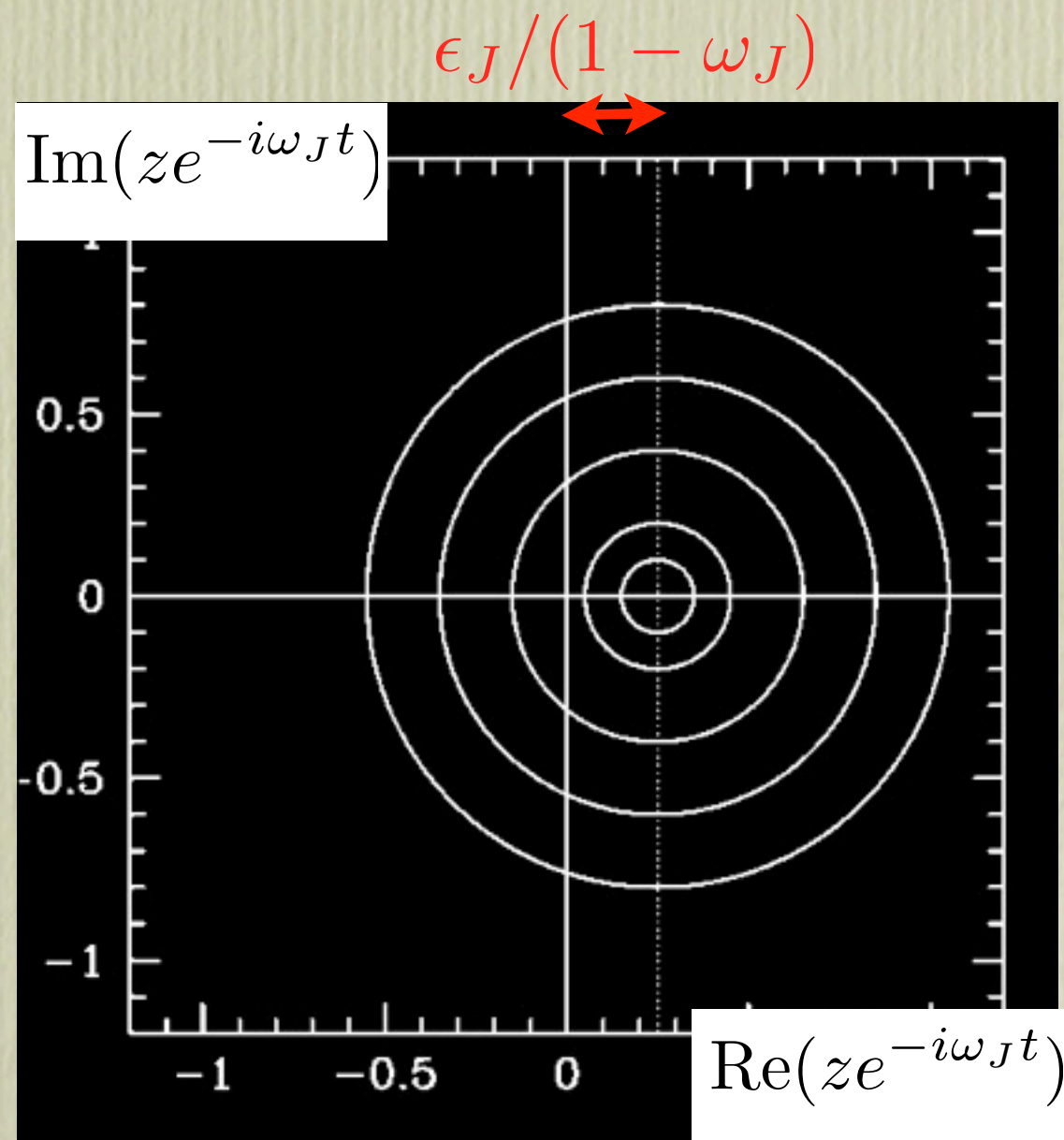




3. Eccentric **Precessing** Jupiter:  $\frac{d}{dt}z = i(z - \epsilon_J e^{i\omega_J t})$

Jupiter's precession frequency

solution:  $z = Ce^{it} + \frac{\epsilon_J}{1 - \omega_J} e^{i\omega_J t}$



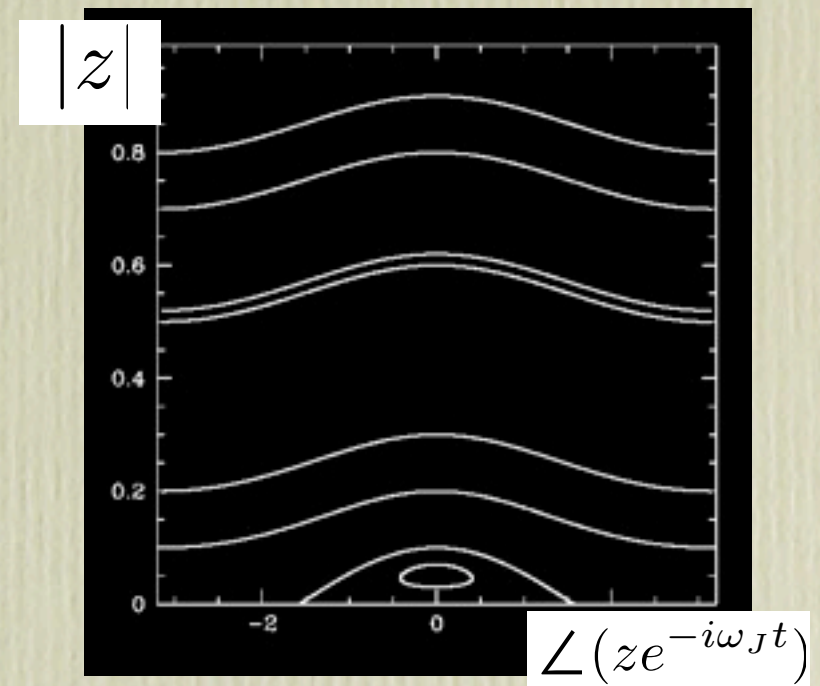
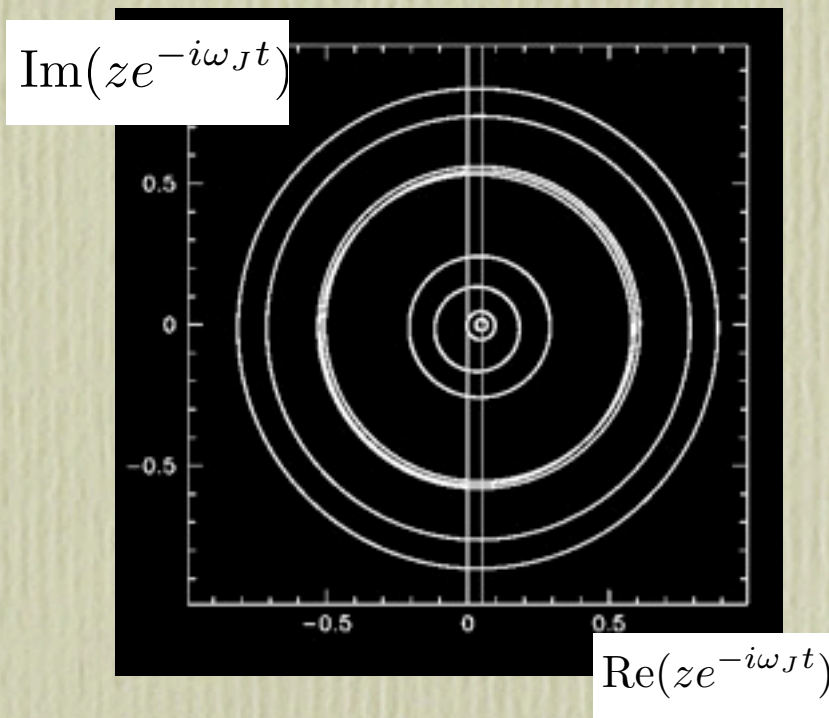


## 4. Eccentric Precessing Jupiter & Nonlinear Mercury

$$\frac{d}{dt}z = i \left[ \left( 1 - \frac{|z|^2}{2} \right) z - \epsilon_J e^{i\omega_J t} \right]$$

$$\epsilon_J = 0.01, \quad \omega_J = 0.8$$

Linear:



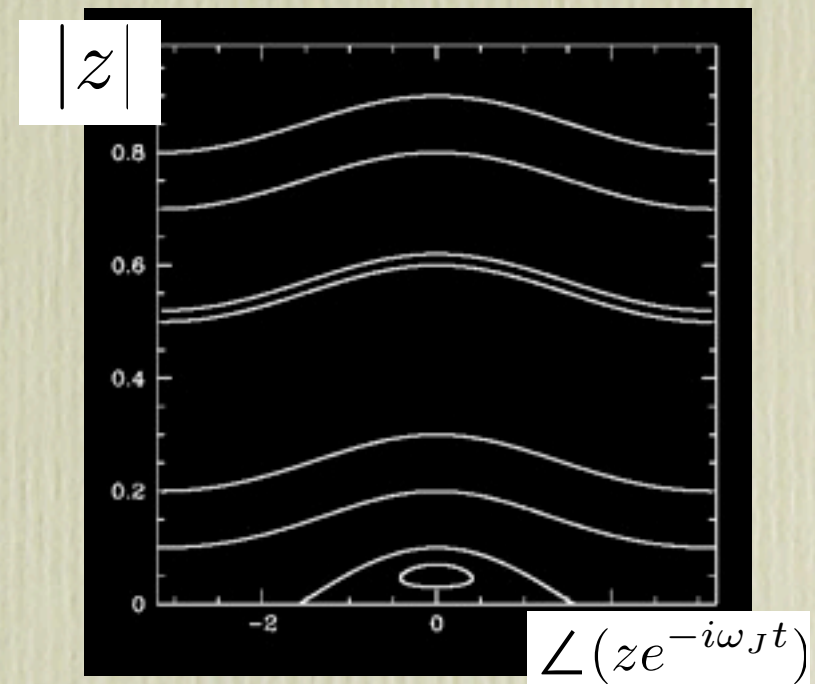
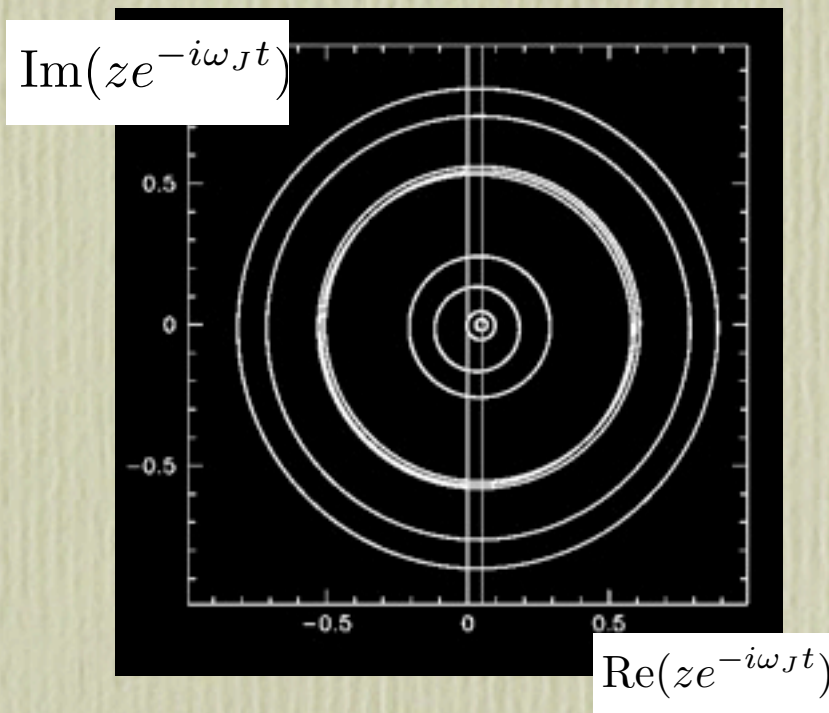


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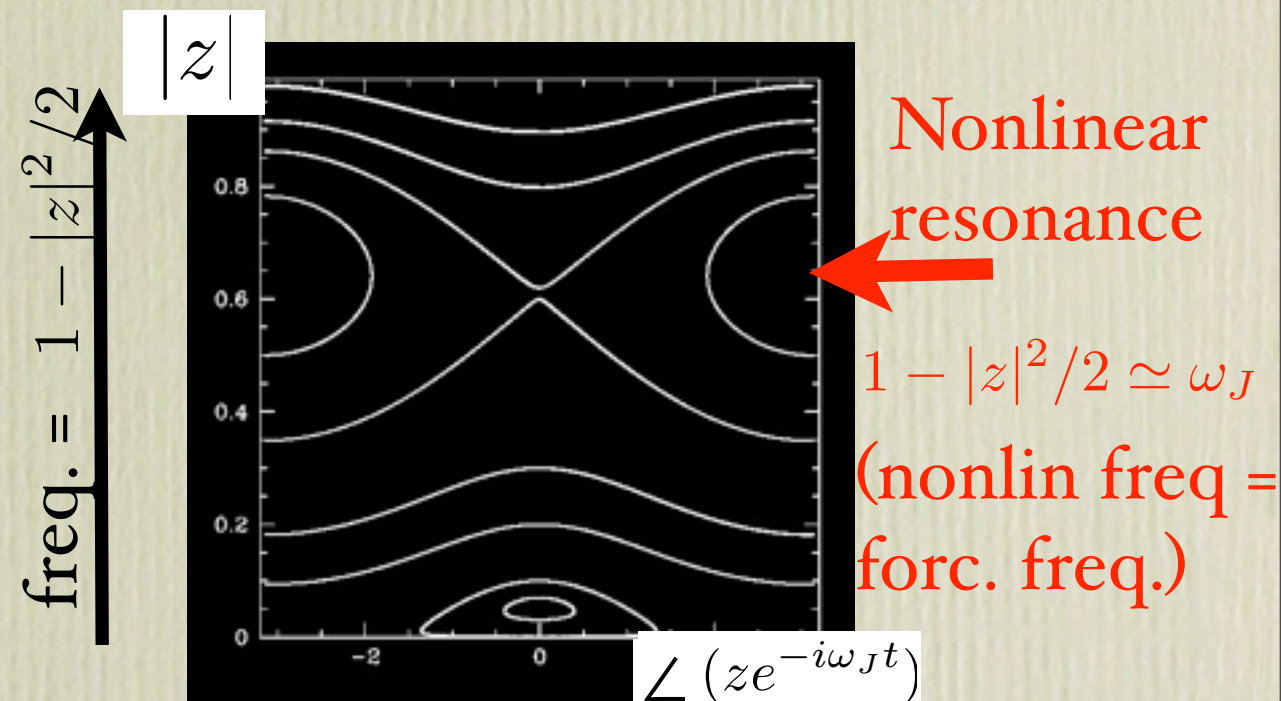
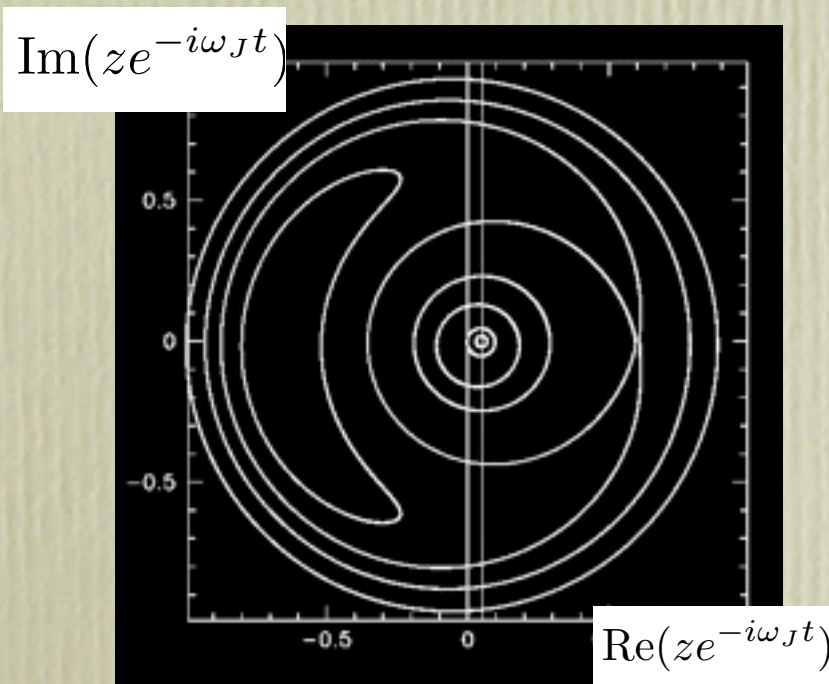
$$\frac{d}{dt}z = i \left[ \left(1 - \frac{|z|^2}{2}\right) z - \epsilon_J e^{i\omega_J t} \right]$$

$$\epsilon_J = 0.01, \quad \omega_J = 0.8$$

Linear:



Nonlinear:



freq. =  $1 - |z|^2/2$

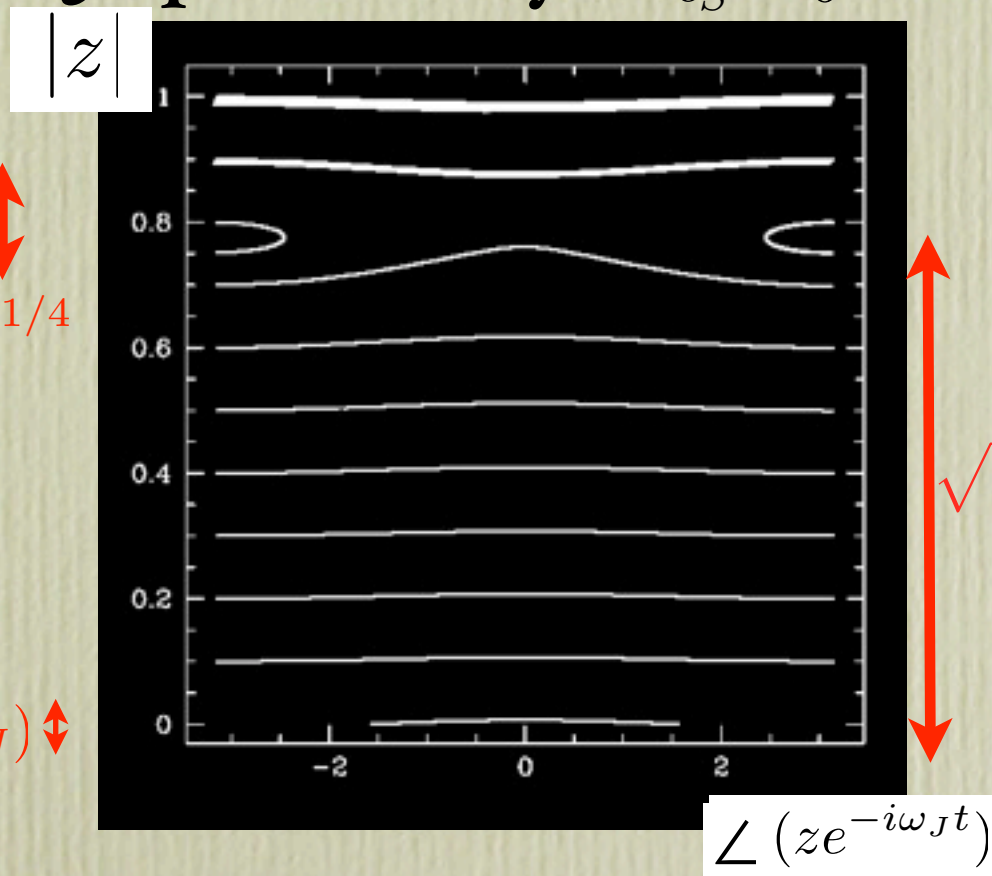


# 5. Eccentric Precessing Jupiter & Saturn & Nonlinear Mercury

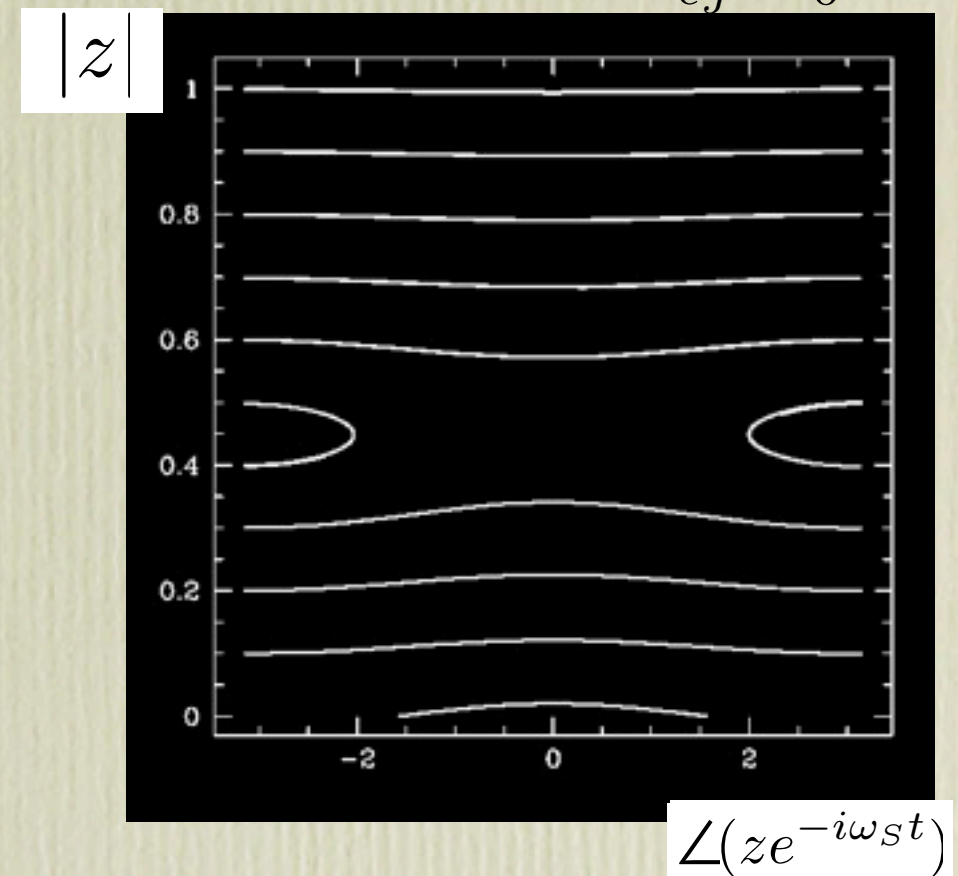
$$\frac{d}{dt}z = i \left[ \left(1 - \frac{|z|^2}{2}\right)z - \epsilon_J e^{i\omega_J t} - \epsilon_S e^{i\omega_S t} \right]$$

Case 1: nonoverlapping resonances

Jupiter only ( $\epsilon_J = .001$   
 $\omega_J = .7$   
 $\epsilon_S = 0$ )



Saturn only ( $\epsilon_S = .001$   
 $\omega_S = .9$   
 $\epsilon_J = 0$ )

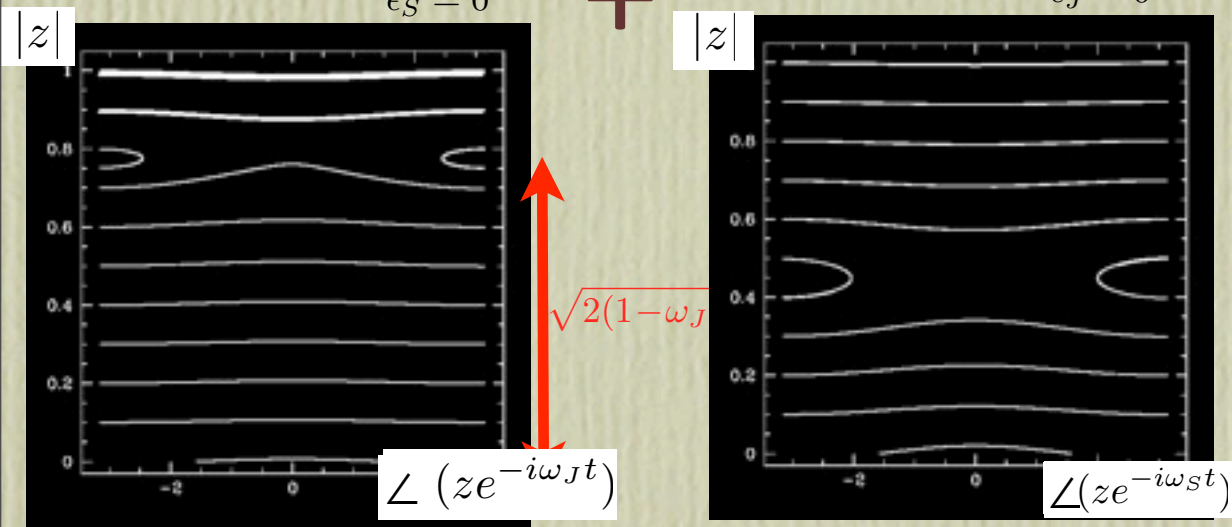




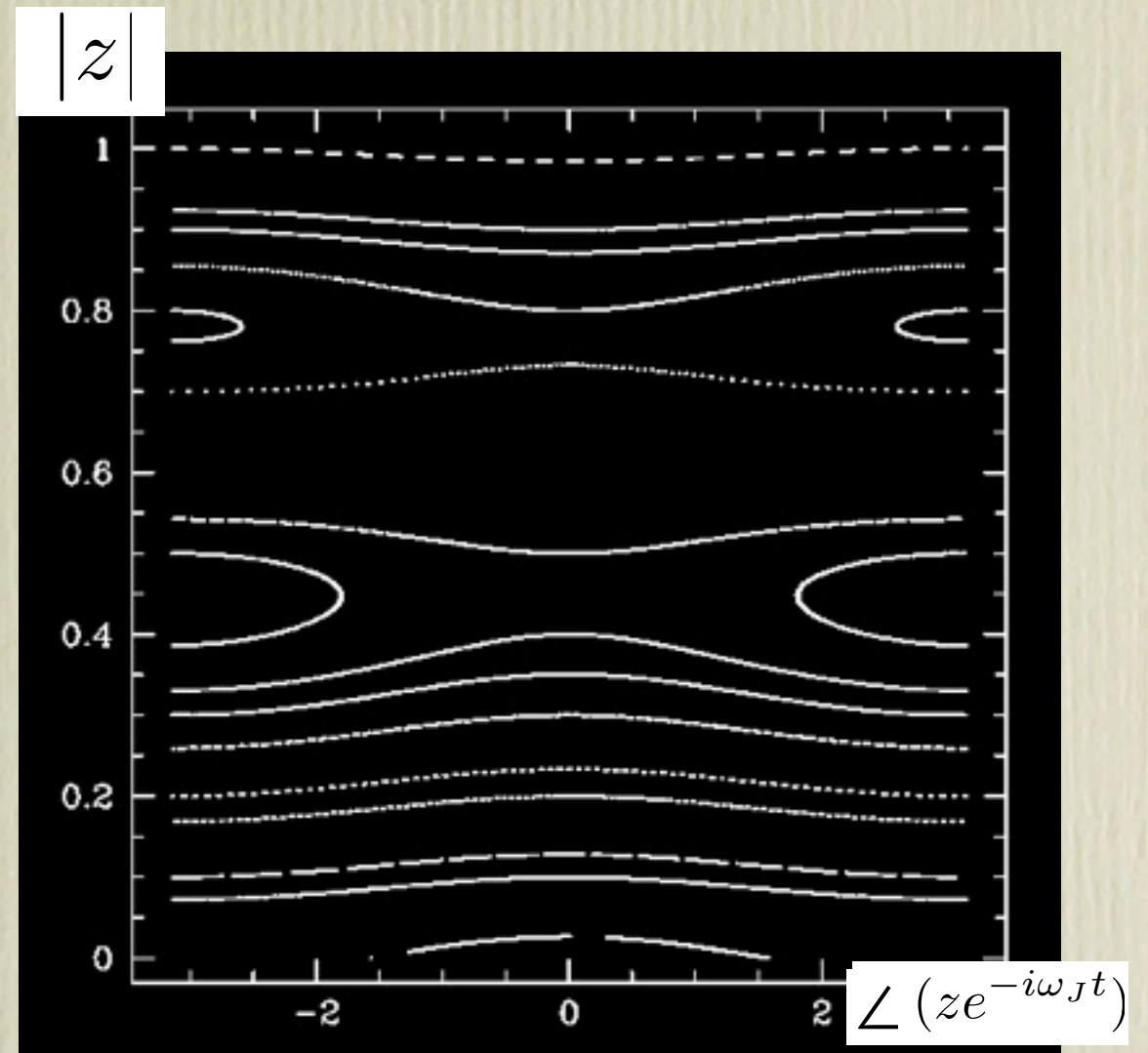
$$\frac{d}{dt} z = i \left[ \left(1 - \frac{|z|^2}{2}\right) z - \epsilon_J e^{i\omega_J t} - \epsilon_S e^{i\omega_S t} \right]$$

## Case 1: nonoverlapping resonances

Jupiter only  $\epsilon_J = .001$   
 $\omega_J = .7$   
 $\epsilon_S = 0$     +    Saturn only  $\epsilon_S = .001$   
 $\omega_S = .9$   
 $\epsilon_J = 0$     =    Jupiter & Saturn together



$\epsilon_J = .001$   
 $\epsilon_S = .001$   
 $\omega_J = .7$   
 $\omega_S = .9$



(surface of section,  
 plotted when  
 $e^{it(\omega_J - \omega_S)} = 1$ )



$$\frac{d}{dt}z = i \left[ \left(1 - \frac{|z|^2}{2}\right)z - \epsilon_J e^{i\omega_J t} - \epsilon_S e^{i\omega_S t} \right]$$

## Case 2: overlapping resonances

Jupiter only

+

Saturn only

=

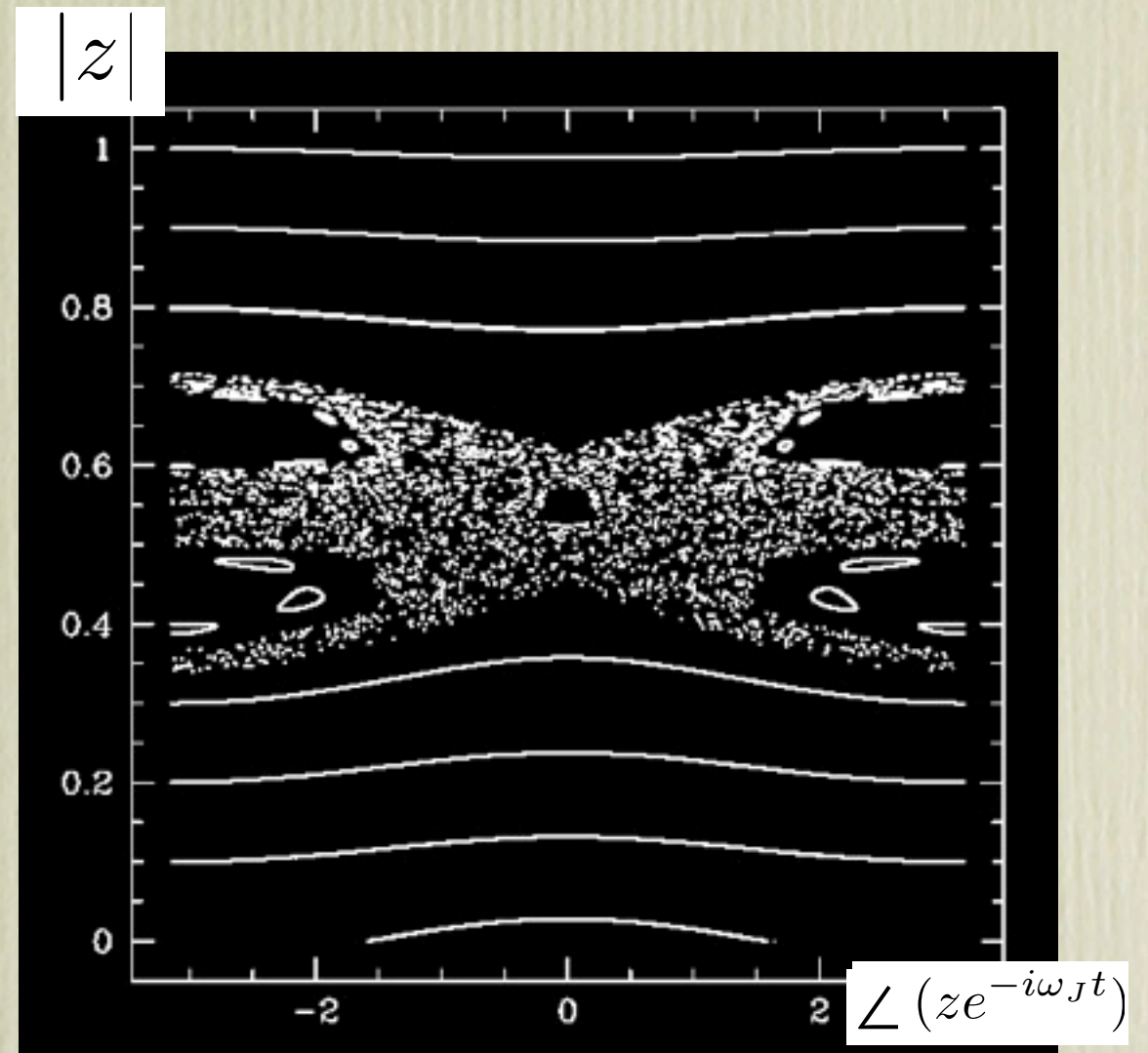
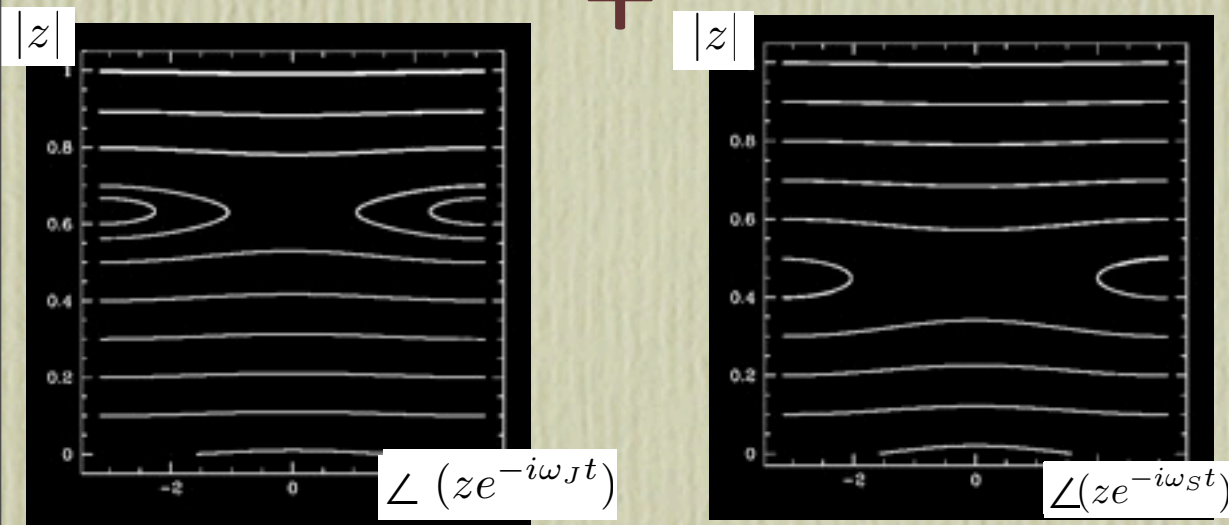
Jupiter & Saturn together

$$\epsilon_J = .001$$

$$\epsilon_S = .001$$

$$\omega_J = .7 \rightarrow \omega_J = .8$$

$$\omega_S = .9$$



(surface of section,  
plotted when  
 $e^{it(\omega_J - \omega_S)} = 1$ )



# Inclination

Mercury perturbed by eccentric Jupiter and inclined Venus:

$$\sigma \equiv i e^{i\Omega}$$

Mercury's inclination      orientation of inclined orbital plane  
("longitude of ascending node")

$$\frac{d}{dt} z = i \left[ \left( 1 - \frac{|z|^2}{2} - 2|\sigma|^2 \right) z + \frac{5}{2} z^* \sigma^2 - \epsilon_J e^{i\omega_J t} \right]$$

$$\frac{d}{dt} \sigma = i \left[ \left( -1 + \frac{|\sigma|^2}{2} - 2|z|^2 \right) \sigma + \frac{5}{2} \sigma^* z^2 - i_V e^{i\omega_V t} \right]$$

(some terms omitted)

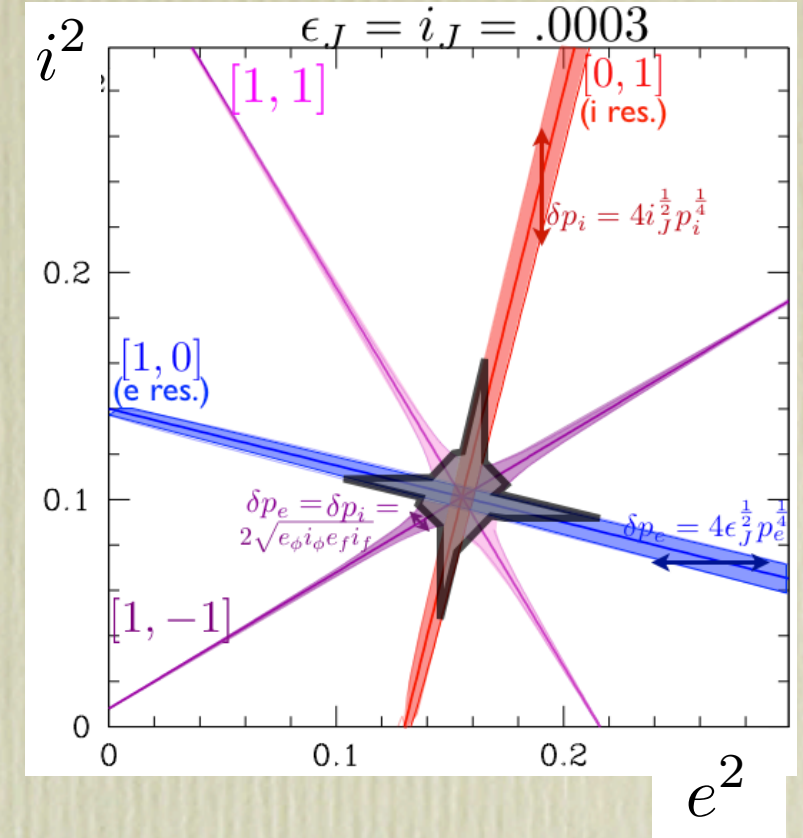
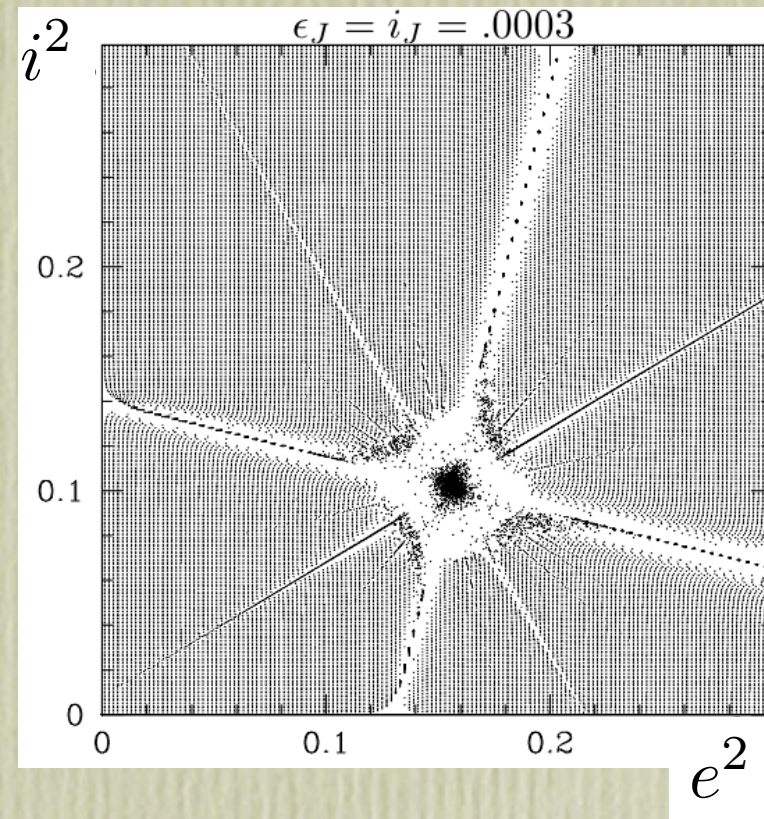


# Inclination

## Simulation

## Theory

low forcing:



true forcing:

