Dynamics in Young Star Clusters

From Planets to Massive Stars

Christoph Olczak

ARI & MPIA, Heidelberg; NAOC, Beijing

Collaborators:

Rainer Spurzem (NAOC & KIAA, Beijing; ARI, Heidelberg)

Thomas Henning, Wolfgang Brandner (MPIA, Heidelberg)

Susanne Pfalzner, Thomas Kaczmarek (MPIfR Bonn)

Andrea Stolte, Benjamin Hussmann (AIfA Bonn)

Simon Portegies Zwart, Stefan Harfst (Leiden Univ., TU Berlin)

ISIMA 2011: Star and Planet Formation

Outline

Some facts about star and planet formation

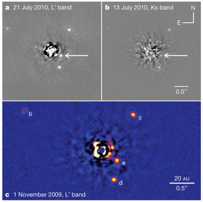
Planets and their hosts:

- stars form with dusty discs
  $\Rightarrow$ *protoplanetary discs*
- protoplanetary discs serve as hosts of planet formation
- protopl. discs last for $\lesssim 10\,\mathrm{Myr}$

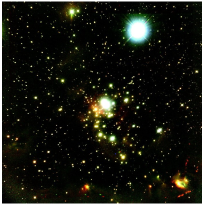
Stars and their hosts:

- up to 90 % of all stars form in clusters (Lada & Lada, 2003; Evans et al., 2009)
- 50 % of all stars form in *massive* clusters ($N > 1000$)
- star clusters last for $\gtrsim 10\,\mathrm{Myr}$

Ori 114-426
O'Dell & Beckwith (1997)



HR 8799
Marois et al. (2010)



IC 348
Muench et al. (2003)

NGC 3603
Brandl et al. (2001)

Some facts about star and planet formation

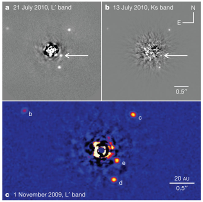
Planets and their hosts:

- stars form with dusty discs
  $\Rightarrow$ *protoplanetary discs*
- protoplanetary discs serve as hosts of planet formation
- protopl. discs last for $\lesssim 10\,\mathrm{Myr}$

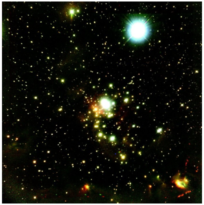
Stars and their hosts:

- up to 90 % of all stars form in clusters (Lada & Lada, 2003; Evans et al., 2009)
- 50 % of all stars form in *massive* clusters ($N > 1000$)
- star clusters last for $\gtrsim 10\,\mathrm{Myr}$

Ori 114-426
O'Dell & Beckwith (1997)



HR 8799
Marois et al. (2010)



IC 348
Muench et al. (2003)

NGC 3603
Brandl et al. (2001)

$\Rightarrow$ **star and planet formation is affected by the cluster environment**

Some facts about star and planet formation

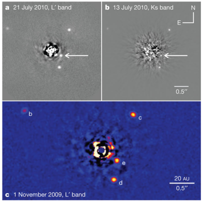
Planets and their hosts:

- stars form with dusty discs
  $\Rightarrow$ *protoplanetary discs*
- protoplanetary discs serve as hosts of planet formation
- protopl. discs last for $\lesssim 10\,\mathrm{Myr}$

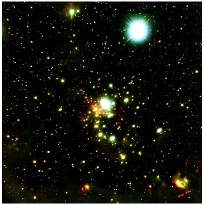
Stars and their hosts:

- up to 90 % of all stars form in clusters (Lada & Lada, 2003; Evans et al., 2009)
- 50 % of all stars form in *massive* clusters ($N > 1000$)
- star clusters last for $\gtrsim 10\,\mathrm{Myr}$

Ori 114-426
O'Dell & Beckwith (1997)



HR 8799
Marois et al. (2010)



IC 348
Muench et al. (2003)

NGC 3603
Brandl et al. (2001)

$\Rightarrow$ star and planet formation is affected by the cluster environment

$\Rightarrow$ investigation of the effect of stellar encounters on protoplanetary discs

The dynamically outstanding role of massive stars

The effect of stellar encounters is dominated by massive stars twofold:

Gravitational focusing

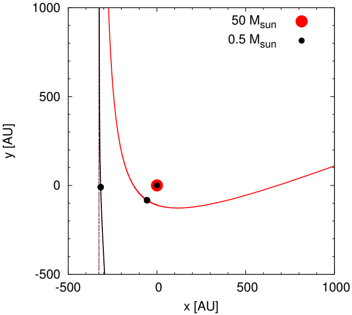
Mass-ratio dependent perturbation

The dynamically outstanding role of massive stars

The effect of stellar encounters is dominated by massive stars twofold:

Gravitational focusing

$$b^2 = r_{\mathrm{enc}}^2 \left(1 + \frac{2GMm}{\mu r_{\mathrm{enc}} v^2}\right) = r_{\mathrm{enc}}^2 (1+\Theta)$$



Orion: $M = 50\,\mathrm{M}_\odot$, $m = 0.5\,\mathrm{M}_\odot$
$\Rightarrow b \approx 330\,\mathrm{AU} \rightarrow r_{\mathrm{enc}} = 100\,\mathrm{AU}$

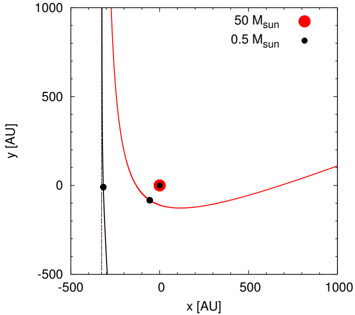
Mass-ratio dependent perturbation

The dynamically outstanding role of massive stars

The effect of stellar encounters is dominated by massive stars twofold:

Gravitational focusing

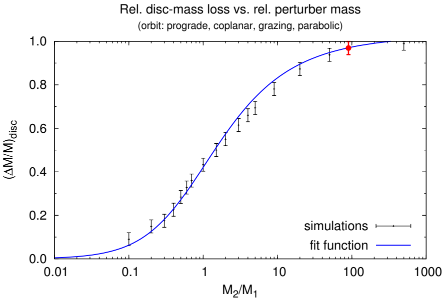
$$b^2 = r_{\mathrm{enc}}^2 \left(1 + \frac{2GMm}{\mu r_{\mathrm{enc}} v^2}\right) = r_{\mathrm{enc}}^2 (1+\Theta)$$



Orion: $M = 50\,\mathrm{M}_\odot$, $m = 0.5\,\mathrm{M}_\odot$

$\Rightarrow b \approx 330\,\mathrm{AU} \rightarrow r_{\mathrm{enc}} = 100\,\mathrm{AU}$

Mass-ratio dependent perturbation



Equivalent disc-mass loss:

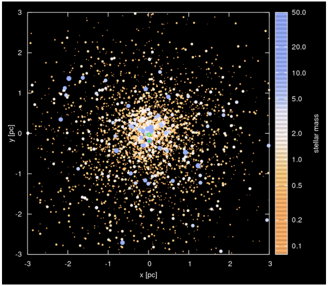
$50\,\mathrm{M}_\odot$ perturber at $r_{\mathrm{enc}} = 500\,\mathrm{AU}$

$0.5\,\mathrm{M}_\odot$ perturber at $r_{\mathrm{enc}} = 100\,\mathrm{AU}$

Disc destruction (97 % mass loss):

$50\,\mathrm{M}_\odot$ perturber at $r_{\mathrm{enc}} = 100\,\mathrm{AU}$.

Realization of the numerical simulations



simulations of star cluster dynamics
(pure particle model, ∼1000 simulations)
→ direct N-body codes NBODY6++, NBODY6-GPU

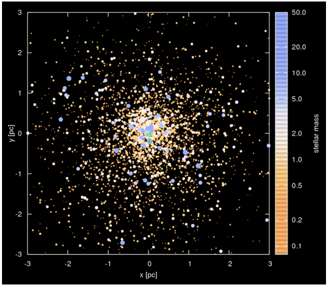
↓

tracking of encounters

↓

record of encounter parameters

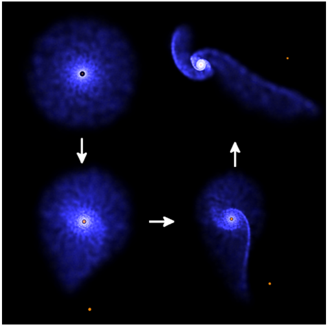
Realization of the numerical simulations



simulations of star cluster dynamics
(pure particle model, ∼1000 simulations)
→ direct N-body codes NBODY6++, NBODY6-GPU

tracking of encounters

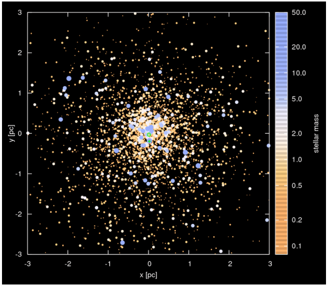
record of encounter parameters

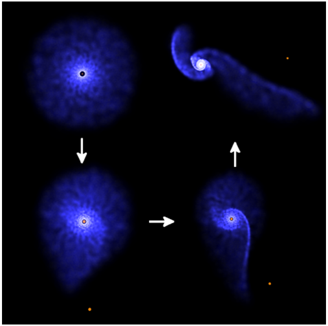


parameterized fit formula for effect of encounters:
- disc mass
- angular momentum

parameter study of star-disc encounters
(pure particle model, ∼2000 simulations)
→ tree code

Realization of the numerical simulations





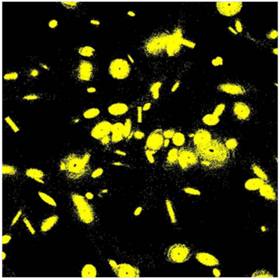
simulations of star cluster dynamics
(pure particle model, ~1000 simulations)
→ direct N-body codes NBODY6++, NBODY6-GPU

tracking of encounters

record of encounter parameters

parameterized fit formula for effect of encounters:
- disc mass
- angular momentum

parameter study of star-disc encounters
(pure particle model, ~2000 simulations)
→ tree code



encounter-induced evolution of star-disc systems in a cluster environment

Performance: CPU vs. GPU

The hardware revolution of N-body simulations: **GPU**s!

		CPU	**GPU**	GRAPE
I	s	NBODY1-3,5[1]		NBODY4[1,2]
	p			
II	s	NBODY6[1]		
	p			

[1] S. Aarseth, [2] J. Makino, [3] R. Spurzem, [4] S. Harfst, [5] P. Berczik, [6] K. Nitadori

Performance: CPU vs. GPU

The hardware revolution of N-body simulations: **GPU**s !

		CPU	**GPU**	GRAPE
I	s	NBODY1-3,5[1]		NBODY4[1,2]
	p		φGPU[4]	φGRAPE[3]
II	s	NBODY6[1]	NBODY6-GPU[1,5]	
	p	NBODY6++[2]		

[1] S. Aarseth, [2] J. Makino, [3] R. Spurzem, [4] S. Harfst, [5] P. Berczik, [6] K. Nitadori

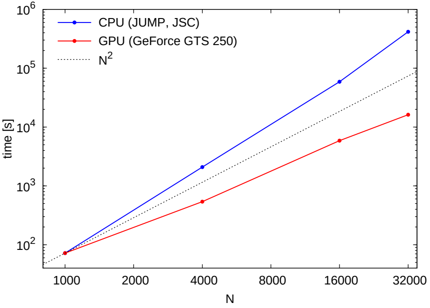
Performance: CPU vs. GPU

The hardware revolution of N-body simulations: **GPUs**!

→ More than one order of magnitude gain for large systems!

		CPU	GPU	GRAPE
I	s	NBODY1-3,5[1]		NBODY4[1,2]
	p		φGPU[4]	φGRAPE[3]
II	s	NBODY6[1]	NBODY6-GPU[1,5]	
	p	NBODY6++[2]		

[1] S. Aarseth, [2] J. Makino, [3] R. Spurzem, [4] S. Harfst, [5] P. Berczik, [6] K. Nitadori



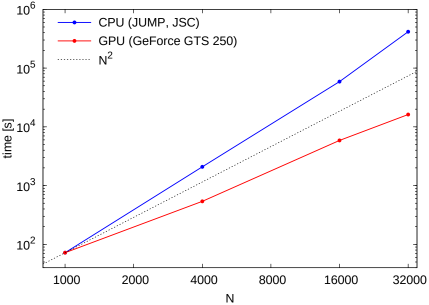
Performance: CPU vs. GPU

The hardware revolution of N-body simulations: **GPU**s !

→ More than one order of magnitude gain for large systems!

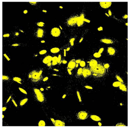
		CPU	GPU	GRAPE
I	s	NBODY1-3,5[1]		NBODY4[1,2]
	p		φGPU[4]	φGRAPE[3]
II	s	NBODY6[1]	NBODY6-GPU[1,5]	
	p	NBODY6++[2]	NBODY6++GPU[1,2]	

[1] S. Aarseth, [2] J. Makino, [3] R. Spurzem, [4] S. Harfst, [5] P. Berczik, [6] K. Nitadori



The next step: parallelization (β-stage)

NBODY6++GPU: simulations of clusters of star-discs systems with $\mathbf{10^8}$ particles.

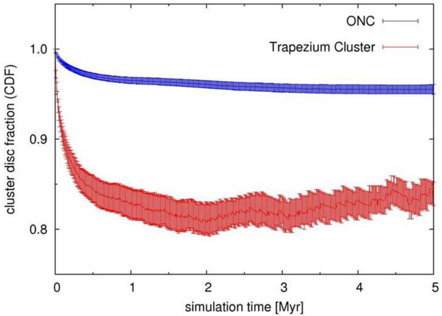


Encounter-induced disc destruction in the ONC

Numerical evolution of the dynamical model of the ONC ($t \approx 1\,\text{Myr}$).
Investigation of the disc-mass loss over time (destruction: $> 90\,\%$ mass loss).

→ **Stellar encounters lead to significant disc destruction** (Olczak et al., 2006):

- $\sim$ 5 % discs destroyed in entire cluster ($R = 2.5\,\text{pc}$)
- $\sim$ 20 % discs destroyed in cluster core ($R = 0.3\,\text{pc}$, "Trapezium Cluster")



Encounter-induced disc destruction in the ONC

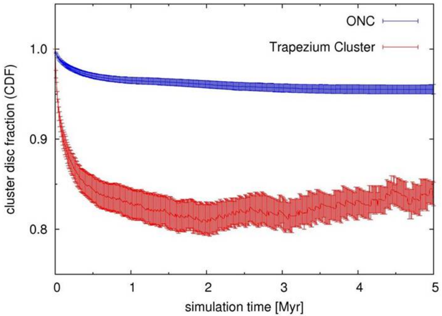
Numerical evolution of the dynamical model of the ONC ($t \approx 1$ Myr).

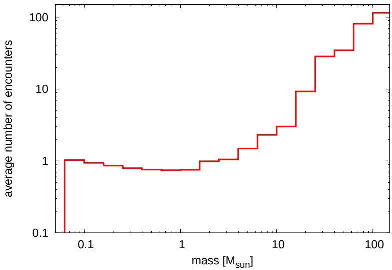
Investigation of the disc-mass loss over time (destruction: $> 90\,\%$ mass loss).

→ **Stellar encounters lead to significant disc destruction** (Olczak et al., 2006):

- ∼ 5 % discs destroyed in entire cluster ($R = 2.5$ pc)
- ∼ 20 % discs destroyed in cluster core ($R = 0.3$ pc, "Trapezium Cluster")

→ **High-mass stars dominate interactions: "gravitational foci"** (Pfalzner et al., 2006).





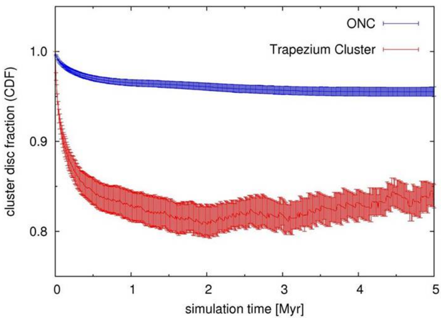
Encounter-induced disc destruction in the ONC

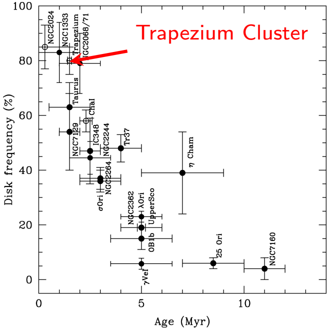
Numerical evolution of the dynamical model of the ONC ($t \approx 1$ Myr).
Investigation of the disc-mass loss over time (destruction: $> 90\,\%$ mass loss).

→ **Stellar encounters lead to significant disc destruction** (Olczak et al., 2006):

- $\sim$ 5 % discs destroyed in entire cluster ($R = 2.5$ pc)
- $\sim$ 20 % discs destroyed in cluster core ($R = 0.3$ pc, "Trapezium Cluster")

→ **High-mass stars dominate interactions: "gravitational foci"** (Pfalzner et al., 2006).





Encounter-induced disc destruction in the ONC

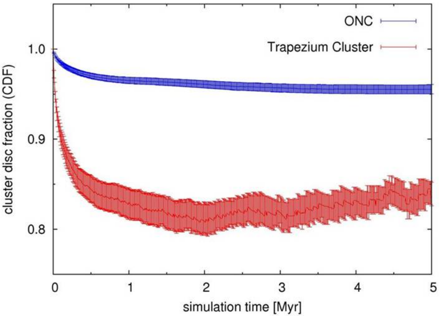
Numerical evolution of the dynamical model of the ONC ($t \approx 1\,\text{Myr}$).

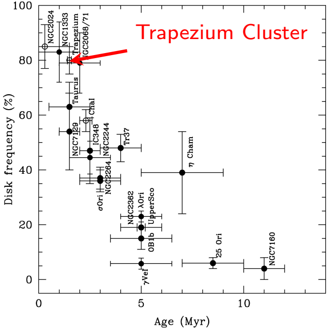
Investigation of the disc-mass loss over time (destruction: $> 90\,\%$ mass loss).

Conclusion

Gravitational interactions in star clusters

1. **cause very rapid disc destruction,**
2. **lower disc frequency close to massive stars** (independent of photoevaporation!),
3. **make planet formation around massive stars improbable.**



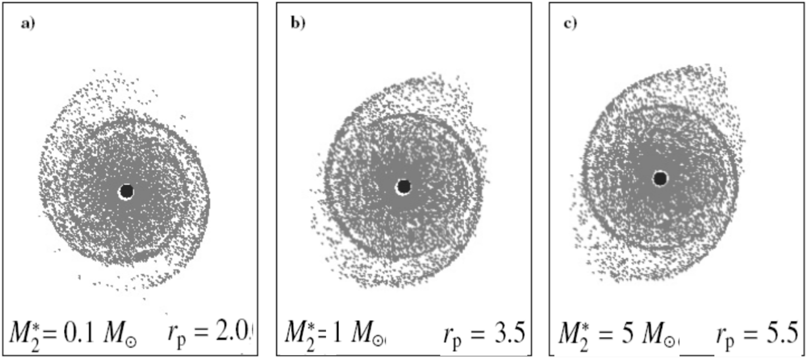


Encounter-induced angular momentum loss in the ONC

Investigation of the angular momentum loss (AML) in the ONC over time ($t \approx 1$ Myr).

→ **Stellar encounters lead to 3-5 % average AML in the ONC** (Pfalzner & Olczak, 2007).

⇒ Pronounced spiral arm structure triggered by encounters in most of the cluster stars.



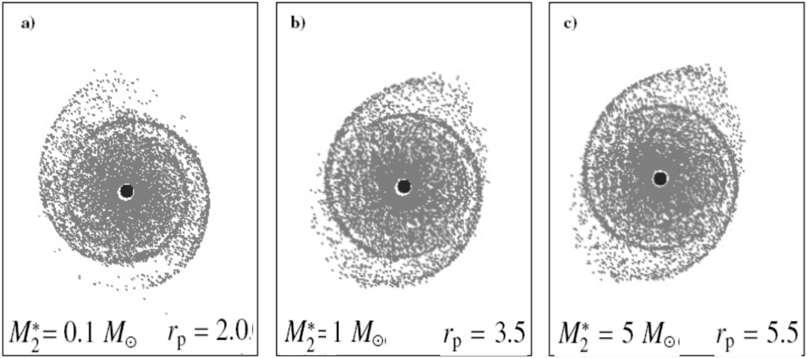
Encounter-induced angular momentum loss in the ONC

Investigation of the angular momentum loss (AML) in the ONC over time ($t \approx 1$ Myr).

→ **Stellar encounters lead to 3-5 % average AML in the ONC** (Pfalzner & Olczak, 2007).

⇒ Pronounced spiral arm structure triggered by encounters in most of the cluster stars.

→ **Planet formation in triggered overdensities might be common**.
(see Rice et al., 2004, 2006; Clarke & Lodato, 2009)



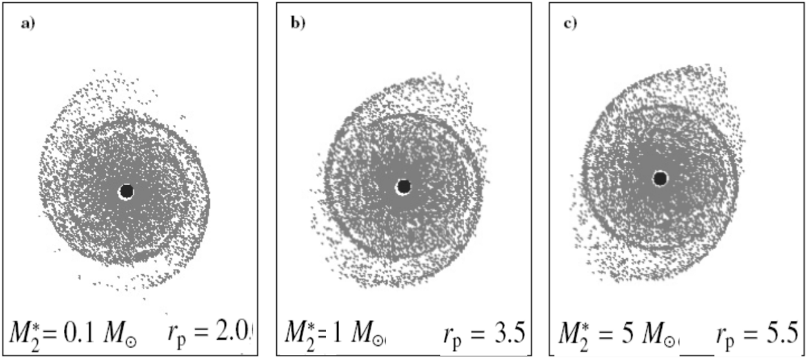
Encounter-induced angular momentum loss in the ONC

Investigation of the angular momentum loss (AML) in the ONC over time ($t \approx 1\,\text{Myr}$).

Conclusion

Gravitational interactions in star clusters

1. **cause significant perturbations of most protoplanetary discs,**
2. **potentially trigger "synchronous" planet formation.**



Numerical models of ONC-like star clusters

Using standard ONC-model for construction of additional models.

→ variation of size (R), density (ρ), and particle number (N)

Two families of models:

1. Size-scaled: $R \propto N$ $(\rho = \text{const})$
2. Density-scaled: $\rho \propto N$ $(R = \text{const})$

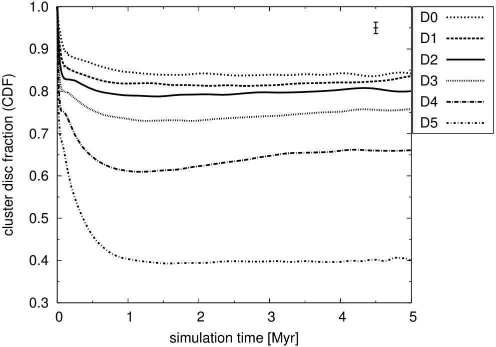
In total 11 cluster models with 1k, 2k, 4k, 8k, 16k, and 32k particles:

family	1k	2k	4k	8k	16k	32k
size-scaled	S0	S1	S2/D2 (ONC)	S3	S4	S5
density-scaled	D0	D1		D3	D4	D5

Disc destruction in different cluster environments

CDF evolution of density-scaled cluster models (within $R = 0.3\,\text{pc}$, "Trapezium Cluster").

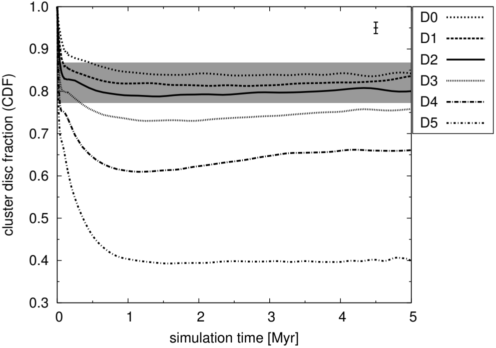
$\longrightarrow$ trend as expected: CDF decreases with higher density



Disc destruction in different cluster environments

CDF evolution of density-scaled cluster models (within $R = 0.3\,\mathrm{pc}$, "Trapezium Cluster").

→ trend as expected: CDF decreases with higher density



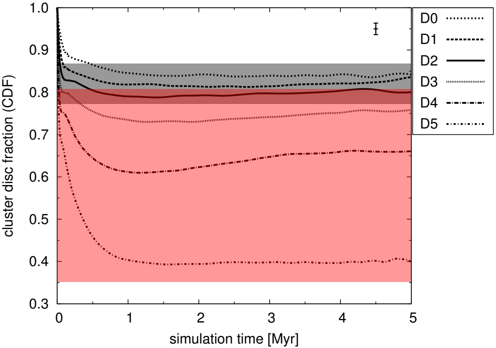
Disc destruction in different cluster environments

CDF evolution of density-scaled cluster models (within $R = 0.3\,\mathrm{pc}$, "Trapezium Cluster").

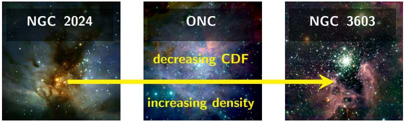
→ trend as expected: CDF decreases with higher density

→ "critical density" of ONC: 2-4 times denser systems show much higher disc destruction

⇒ in agreement with observations?



below critical density:
high-mass stars dominate disc-mass loss
⇒ **focusing !**



above critical density:
low-mass stars dominate disc-mass loss
⇒ **no focusing !**

Towards an extreme environment: the Arches Cluster.

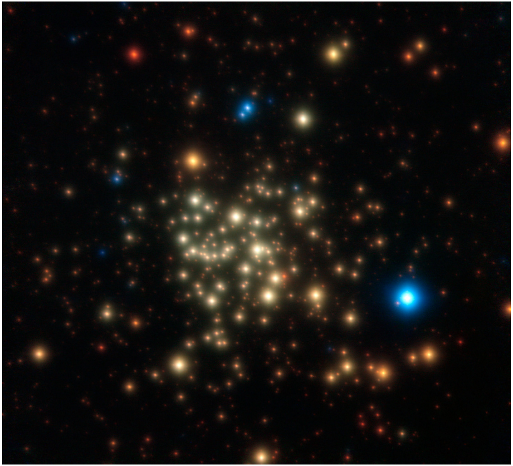


Figure: Espinoza et al. (2009)

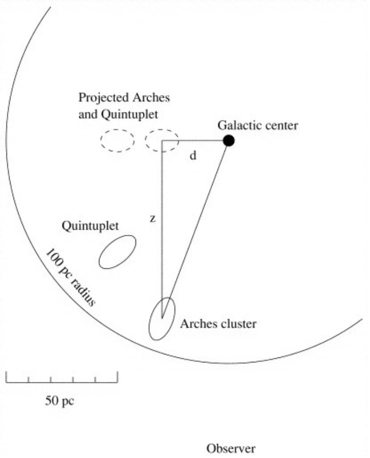


Figure: Portegies Zwart et al. (2002)

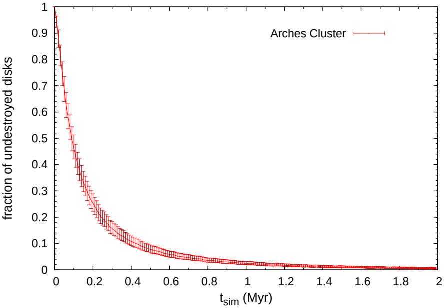
The Arches Cluster is one of the densest and most massive young star clusters in the Milky Way: $M \gtrsim 2 \cdot 10^4\,\mathrm{M}_\odot$, $\rho \gtrsim 10^5\,\mathrm{M}_\odot\,\mathrm{pc}^{-3}$, $t \approx 2\,\mathrm{Myr}$

Star-disc encounters in the Arches Cluster

Simulations provided by S. Harfst and S. Portegies Zwart (Harfst et al., 2009).

Analysis of encounter-induced disc-mass loss (after 2 Myr of numerical evolution):

→ only few discs are not destroyed via encounters



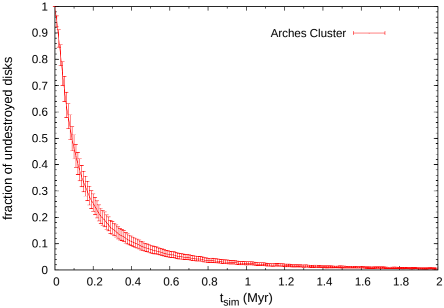
Star-disc encounters in the Arches Cluster

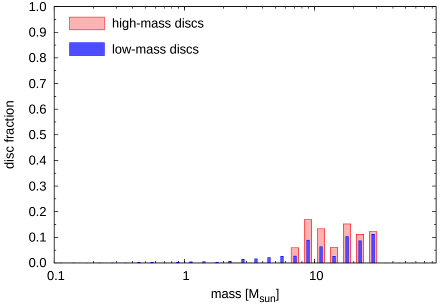
Simulations provided by S. Harfst and S. Portegies Zwart (Harfst et al., 2009).

Analysis of encounter-induced disc-mass loss (after 2 Myr of numerical evolution):

$\longrightarrow$ only few discs are not destroyed via encounters

$\longrightarrow$ discs survive preferentially around stars of $\sim 10\,M_\odot$ (spectral type B)





Star-disc encounters in the Arches Cluster

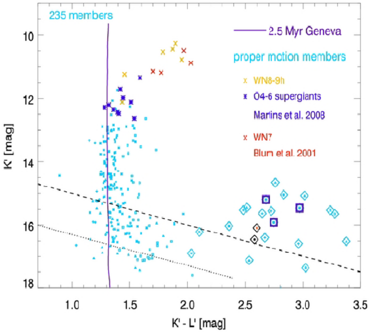
Simulations provided by S. Harfst and S. Portegies Zwart (Harfst et al., 2009).

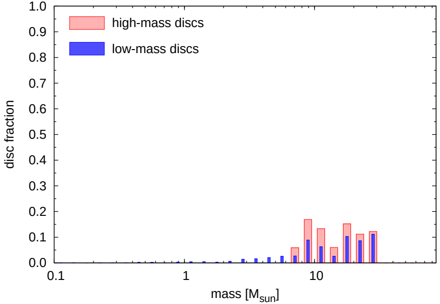
Analysis of encounter-induced disc-mass loss (after 2 Myr of numerical evolution):

→ only few discs are not destroyed via encounters

→ discs survive preferentially around stars of $\sim 10\,M_\odot$ (spectral type B)

⇒ agreement with observations by Stolte et al. (2010)





Star-disc encounters in the Arches Cluster

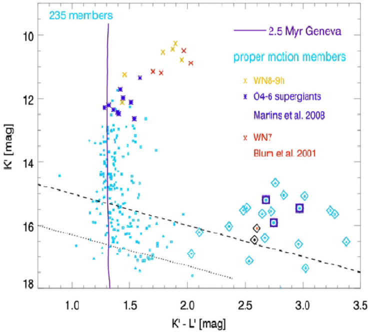
Simulations provided by S. Harfst and S. Portegies Zwart (Harfst et al., 2009).

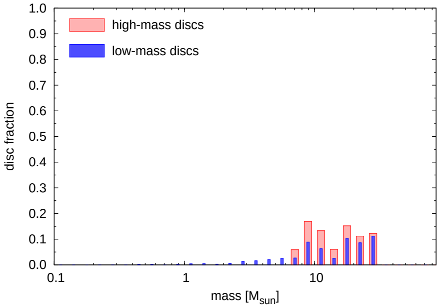
Analysis of encounter-induced disc-mass loss (after 2 Myr of numerical evolution):

Conclusion

Gravitational interactions in starburst clusters

1. **destroy nearly all environments of planet formation,**
2. **make B-type stars the most probable hosts of planetary systems.**





A new efficient measure of mass segregation

Problem

- Do young star clusters really show evidence for mass segregation?
- Is the observed mass segregation in young clusters due to initial conditions (i.e. *primordial*)?
- Does the observed degree of (*dynamical*) mass segregation in old clusters agree with theory?

A new efficient measure of mass segregation

Problem

- Do young star clusters really show evidence for mass segregation?
- Is the observed mass segregation in young clusters due to initial conditions (i.e. *primordial*)?
- Does the observed degree of (*dynamical*) mass segregation in old clusters agree with theory?

Goal

Efficient measure of mass segregation for observational and numerical data.

- Geometrically independent.
- Independence of quantitative mass measurement.
- Numerical robustness.
- Simple, intuitive measure.

A new efficient measure of mass segregation

Problem

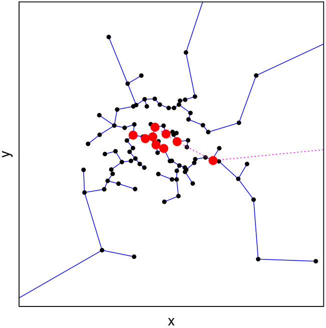
- Do young star clusters really show evidence for mass segregation?
- Is the observed mass segregation in young clusters due to initial conditions (i.e. *primordial*)?
- Does the observed degree of (*dynamical*) mass segregation in old clusters agree with theory?

Goal

Efficient measure of mass segregation for observational and numerical data.

- Geometrically independent.
- Independence of quantitative mass measurement.
- Numerical robustness.
- Simple, intuitive measure.

⇒ **Minimum Spanning Tree (MST)**



Definition

MST ≡ shortest connecting graph of all vertices without closed loops.

Measuring mass segregation via the MST

Quantifying mass segregation: $\Lambda_{\rm MST}$

The *length* of the MST, $l_{\rm MST}$, as a measure of mass segregation (Allison et al., 2009):

1. Calculate $l_{\rm MST}$ of the n most massive stars: $l_{\rm MST}^{\rm mass} = \sum_{i=1}^{n} e_i$
2. Calculate $< l_{\rm MST} >$ of k sets of n random stars: $l_{\rm MST}^{\rm ref}, \Delta l_{\rm MST}^{\rm ref}$
3. Normalization: $\Lambda_{\rm MST} = \frac{l_{\rm MST}^{\rm ref}}{l_{\rm MST}^{\rm mass}}$

   $\Lambda_{\rm MST} > 1$: massive stars more concentrated than reference sample.

   $\Rightarrow$ **Quantitative measure of the degree of mass segregation**.
4. Standard deviation: $\Delta\Lambda_{\rm MST} = \frac{\Delta l_{\rm MST}^{\rm ref}}{l_{\rm MST}^{\rm mass}}$

   $\Rightarrow$ **Quantitative measure of the significance of the result**.

Measuring mass segregation via the MST

Quantifying mass segregation: $\Lambda_{\rm MST}$

The *length* of the MST, $l_{\rm MST}$, as a measure of mass segregation (Allison et al., 2009):

1. Calculate $l_{\rm MST}$ of the n most massive stars: $l_{\rm MST}^{\rm mass} = \sum_{i=1}^{n} e_i$ | $\gamma_{\rm MST}^{\rm mass} = \sqrt[n]{\prod_{i=1}^{n} e_i}$
2. Calculate $< l_{\rm MST} >$ of k sets of n random stars: $l_{\rm MST}^{\rm ref}, \Delta l_{\rm MST}^{\rm ref}$ | $\gamma_{\rm MST}^{\rm ref}, \Delta\gamma_{\rm MST}^{\rm ref}$
3. Normalization: $\Lambda_{\rm MST} = \frac{l_{\rm MST}^{\rm ref}}{l_{\rm MST}^{\rm mass}}$ | $\Gamma_{\rm MST} = \frac{\gamma_{\rm MST}^{\rm ref}}{\gamma_{\rm MST}^{\rm mass}}$

   $\Lambda_{\rm MST} > 1$: massive stars more concentrated than reference sample.
   $\Rightarrow$ **Quantitative measure of the degree of mass segregation**.

4. Standard deviation: $\Delta\Lambda_{\rm MST} = \frac{\Delta l_{\rm MST}^{\rm ref}}{l_{\rm MST}^{\rm mass}}$ | $\Delta\Gamma_{\rm MST} = \Delta\gamma_{\rm MST}^{\rm ref}$

   $\Rightarrow$ **Quantitative measure of the significance of the result**.

An improved measure of mass segregation: $\Gamma_{\rm MST}$

Use the *geometric mean* $\Gamma_{\rm MST}$ of the edges rather than their sum $\Lambda_{\rm MST}$ (Olczak et al., 2011).
$\Rightarrow$ Acts as an intermediate pass that damps contributions from extreme edge lengths.

Measuring the degree of mass segregation in model star clusters

Star cluster with single stars and Kroupa (2001) mass function in the range $0.08 - 150\,M_\odot$.

Initial mass segregation due to prescription of Šubr et al. (2008): parametrization via $S \in (1, 0]$.

Number of stars: $N = 1k$
Index of mass segregation: $S = 0.3$

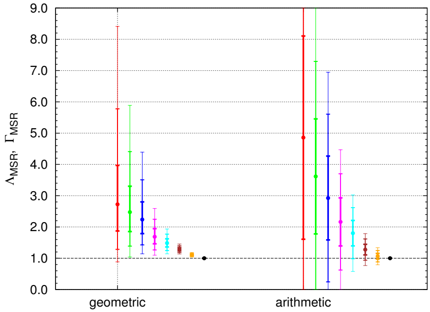


Figure: 5, 10, 20, 50, 100, 200, 500, 1000 most massive stars.

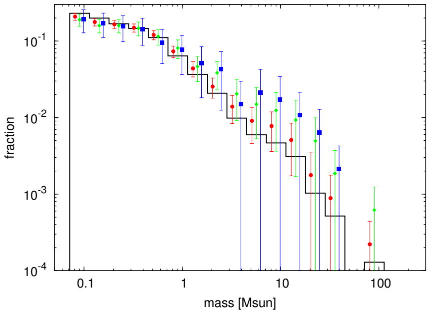


Figure: R_{hm} (red), $1/2R_{\mathrm{hm}}$ (green), $1/4R_{\mathrm{hm}}$ (blue).

Mass-segregation in the ONC

Application of Γ_{MST} to observational data of the ONC obtained by Hillenbrand (1997).

- Analysis via cumulative and differential mass groups:
  → Very strong segregation of the five most massive stars.

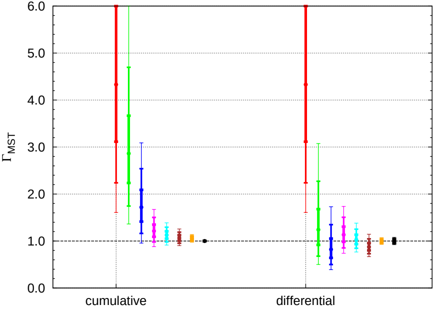


Figure: 5, 10, 20, 50, 100, 200, 500, 929 most massive stars.

Mass-segregation in the ONC

Application of $\Gamma_{\rm MST}$ to observational data of the ONC obtained by Hillenbrand (1997).

- Analysis via cumulative and differential mass groups:
  → Very strong segregation of the five most massive stars.
- Effect of incompleteness ($N = 929 \rightarrow 485$):

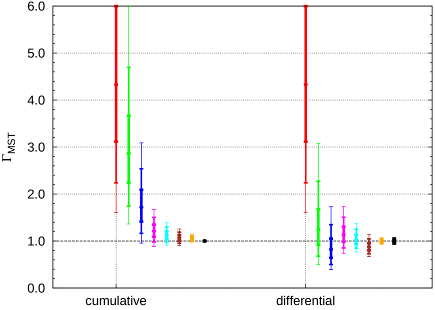


Figure: 5, 10, 20, 50, 100, 200, 500, 929 most massive stars.

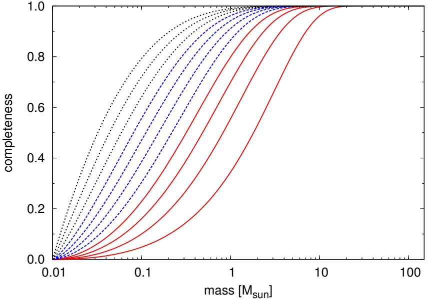


Figure: Completeness model as a function of stellar mass and radial position.

Mass-segregation in the ONC

Application of Γ_{MST} to observational data of the ONC obtained by Hillenbrand (1997).

- Analysis via cumulative and differential mass groups:
  → Very strong segregation of the five most massive stars.
- Effect of incompleteness ($N = 929 \rightarrow 485$): Γ_{MST} rises for most massive stars.

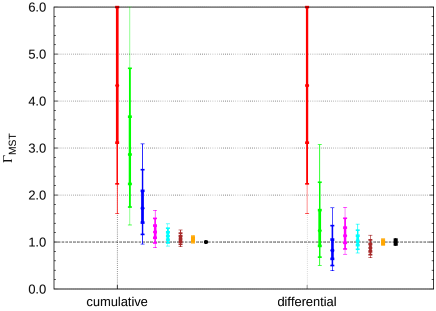


Figure: 5, 10, 20, 50, 100, 200, 500, 929 most massive stars.

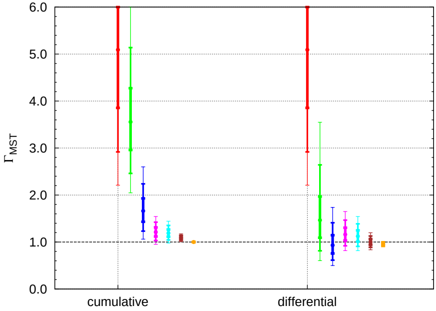


Figure: Incomplete sample: 485 stars.

Mass-segregation in the ONC

Application of Γ_{MST} to observational data of the ONC obtained by Hillenbrand (1997).

- Analysis via cumulative and differential mass groups:
  → Very strong segregation of the five most massive stars.
- Effect of incompleteness ($N = 929 \rightarrow 485$): Γ_{MST} rises for most massive stars.
- Sample reconstruction ($N = 485 \rightarrow 830$) via inverse individual completeness.

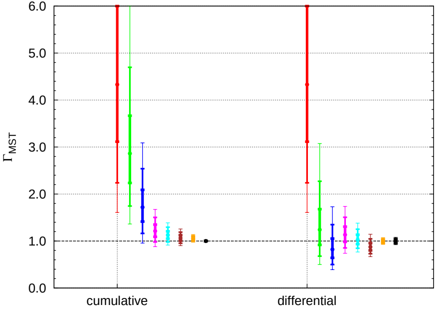


Figure: 5, 10, 20, 50, 100, 200, 500, 929 most massive stars.

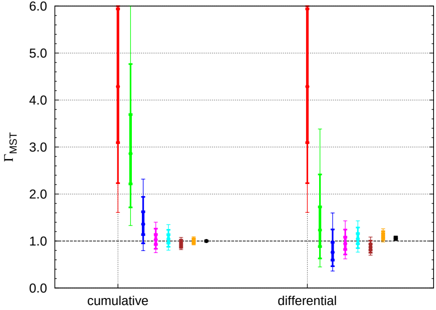


Figure: Reconstructed sample: 830 stars.

Dynamical evolution of mass segregation

Single star cluster (spherically symmetric, no substructure):

- density distribution: isothermal
- velocity distribution: Maxwell
- virial ratio: $Q = 0.1$
- particle numbers: $N = \{1k, 10k\}$

→ Very rapid dynamical mass segregation within few $t_{\rm dyn}$.

(As expected: $t_{\rm seg} \approx \frac{<m>}{m} \frac{N}{8 \ln N} t_{\rm dyn} \approx t_{\rm dyn}$.)

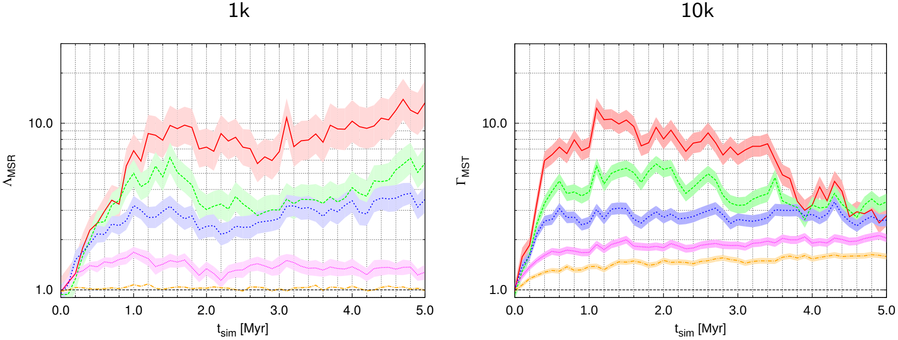


Figure: 5, 10, 20, 50, 500 most massive stars.

Summary

Stellar interactions in young star clusters

Stellar encounters affect the star and planet formation process in a huge variety:

- Massive stars (in the ONC) act as gravitational foci.
- Most star-disc systems are (weakly) perturbed: triggering of planet formation?
- Critical density of ONC: transition of dominant mode of disc destruction.
- Arches cluster: potential planet formation around B-type stars.

Mass segregation in young star clusters

Mass segregation in young star clusters is a key observable of the star formation process:

- New measure of mass segregation: $\Gamma_{\rm MST}$ = Minimum Spanning Tree + geometrical mean. $\rightarrow$ $\Gamma_{\rm MST}$ highly advantageous over classical $\Lambda_{\rm MST}$ method.
- ONC shows significant segregation of massive members.
- Very rapid mass segregation of young star clusters.

Numerical implementation of the MST

A three-step procedure

1. 2D Delaunay triangulation of a three-dimensional set of vertices (GEOMPACK: Joe, 1991)
2. Sorting of triangles' edges in ascending order (and removal of duplicates).
3. Construction of MST via Kruskal's algorithm (with an efficient union-find-algorithm).

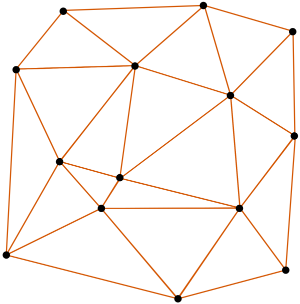
Numerical implementation of the MST

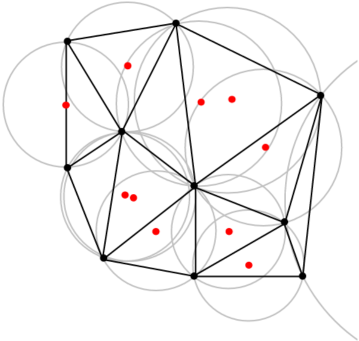
A three-step procedure

1. 2D Delaunay triangulation of a three-dimensional set of vertices (GEOMPACK: Joe, 1991)
2. Sorting of triangles' edges in ascending order (and removal of duplicates).
3. Construction of MST via Kruskal's algorithm (with an efficient union-find-algorithm).

Delaunay triangulation (in the plane)

No point in set of points P is inside the circumcircle of any triangle in DT(P).





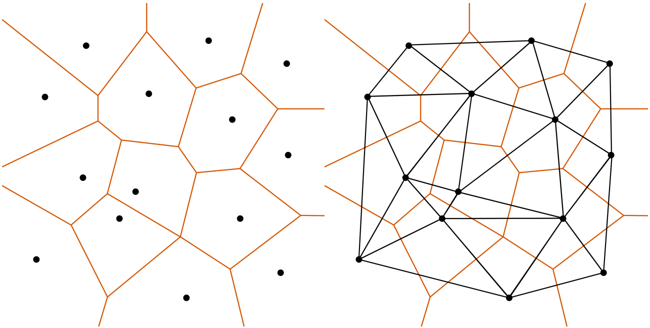
Numerical implementation of the MST

A three-step procedure

1. 2D Delaunay triangulation of a three-dimensional set of vertices (GEOMPACK: Joe, 1991)
2. Sorting of triangles' edges in ascending order (and removal of duplicates).
3. Construction of MST via Kruskal's algorithm (with an efficient union-find-algorithm).

Delaunay triangulation (in the plane)

No point in set of points P is inside the circumcircle of any triangle in DT(P).



Numerical implementation of the MST

A three-step procedure

1. 2D Delaunay triangulation of a three-dimensional set of vertices (GEOMPACK: Joe, 1991)
2. Sorting of triangles' edges in ascending order (and removal of duplicates).
3. Construction of MST via Kruskal's algorithm (with an efficient union-find-algorithm).

Kruskal's algorithm

1. Remove the next shortest edge from the graph.
2. Check whether it forms a close loop with the edges of the MST.
3. If not, add it to the MST.

An efficient union-find algorithm

- Union-by-rank: merge smaller tree of nodes into larger tree.
- Path compression: connect nodes with the tree root.

Computational cost of the MST

Definition

$|E|$: number of edges

$|V|$: number of vertices

1. Delaunay triangulation:

$$\mathcal{O}(|V| \cdot \log |V|) \,. \tag{1}$$

2. Sorting of edges:

$$\mathcal{O}\big(|E| \cdot \log(|E|)\big) \,. \tag{2}$$

3. Union-find algorithm:

$$\mathcal{O}(|E| \cdot \log^* |V|) \,, \tag{3}$$

where

$$\log^*(n) = \min\Big\{ s \in \mathbb{N} \mid \underbrace{\log\big(\log(\ldots \log(n) \ldots)\big)}_{s \text{ times}} \leq 1 \Big\}$$

$\Rightarrow$ In practice constant (though in principle unlimited).

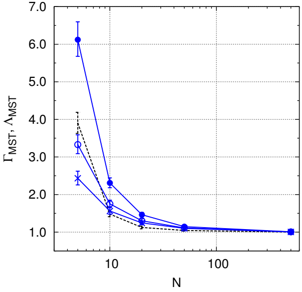
The total computational cost is $\mathcal{O}(|V| \cdot \log |V|)$.

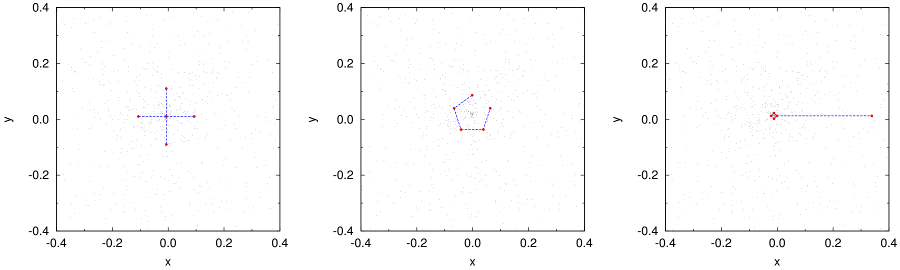
Artificial configurations of mass segregation

The power of Γ_{MST} for some simple setups of artificial mass segregation.

Three artificial configurations of massive stars with identical Λ_{MST} – but different Γ_{MST} - in a model star cluster:

- "cross"
- "ring"
- "clump"





Measuring the degree of mass segregation in model star clusters

Star cluster with single stars and Kroupa (2001) mass function in the range $0.08 - 150\,M_{\odot}$.

Initial mass segregation due to prescription of Šubr et al. (2008): parametrization via $S \in (1, 0]$.

Number of stars: $N = 1k$
Index of mass segregation: $S = 0.3$

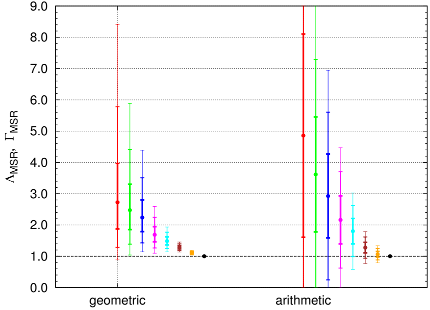


Figure: 5, 10, 20, 50, 100, 200, 500, 1000 most massive stars.

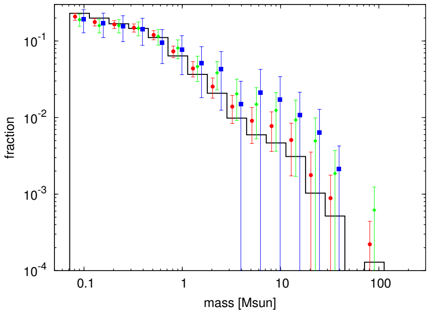


Figure: $R_{\rm hm}$ (red), $1/2R_{\rm hm}$ (green), $1/4R_{\rm hm}$ (blue).

Allison, R. J., Goodwin, S. P., Parker, R. J., et al. 2009, MNRAS, 395, 1449

Brandl, B., The Ngc 3603 Team, & The 30 Doradus Team. 2001, in Astronomical Society of the Pacific Conference Series, Vol. 243, From Darkness to Light: Origin and Evolution of Young Stellar Clusters, ed. T. Montmerle & P. André, 505–+

Clarke, C. J. & Lodato, G. 2009, MNRAS, 398, L6

Espinoza, P., Selman, F. J., & Melnick, J. 2009, A&A, 501, 563

Evans, N. J., Dunham, M. M., Jørgensen, J. K., et al. 2009, ApJS, 181, 321

Harfst, S., Portegies Zwart, S., & Stolte, A. 2009, ArXiv e-prints

Hillenbrand, L. A. 1997, AJ, 113, 1733

Joe, B. 1991, Advances in Engineering Software, 13, 325

Kroupa, P. 2001, MNRAS, 322, 231

Lada, C. J. & Lada, E. A. 2003, ARA&A, 41, 57

Marois, C., Zuckerman, B., Konopacky, Q. M., Macintosh, B., & Barman, T. 2010, Nature, 468, 1080

Muench, A. A., Lada, E. A., Lada, C. J., et al. 2003, AJ, 125, 2029

O'Dell, C. R. & Beckwith, S. V. W. 1997, Science, 276, 1355

Olczak, C., Pfalzner, S., & Spurzem, R. 2006, ApJ, 642, 1140

Pfalzner, S. & Olczak, C. 2007, A&A, 462, 193

Pfalzner, S., Olczak, C., & Eckart, A. 2006, A&A, 454, 811

Portegies Zwart, S. F., Makino, J., McMillan, S. L. W., & Hut, P. 2002, ApJ, 565, 265

Rice, W. K. M., Lodato, G., Pringle, J. E., Armitage, P. J., & Bonnell, I. A. 2004, MNRAS, 355, 543

Rice, W. K. M., Lodato, G., Pringle, J. E., Armitage, P. J., & Bonnell, I. A. 2006, MNRAS, 372, L9

Stolte, A., Morris, M. R., Ghez, A. M., et al. 2010, ApJ, 718, 810

Šubr, L., Kroupa, P., & Baumgardt, H. 2008, MNRAS, 385, 1673