MIXING IN STARS (BY FINGERING CONVECTION)

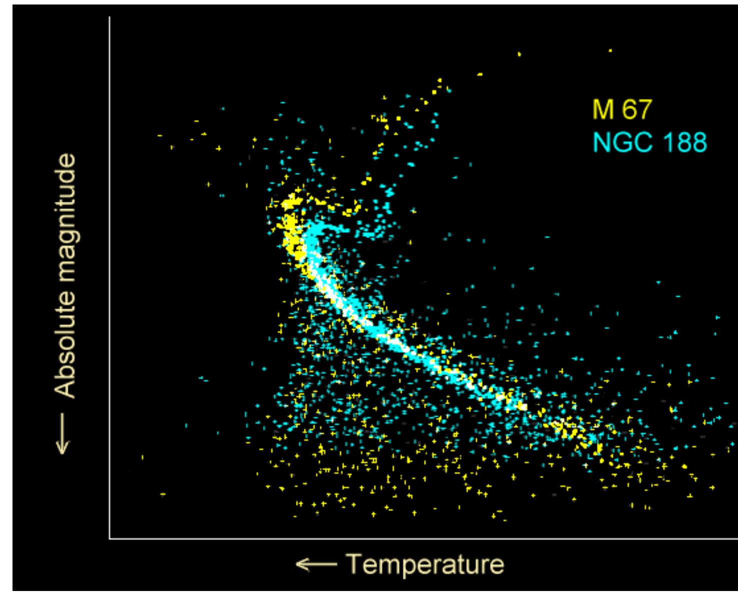
Pascale Garaud, UC Santa Cruz

Outline

- The success of stellar evolution models
- Missing mixing in RGB stars
- How to model “missing mixing”?
- Fingering (thermohaline) Convection
- Implications for RGB observations
- Fingering convection and the effect of planetary infall.

The success of standard stellar models

- Stellar evolution theory is incredibly successful at explaining stellar observations, as for instance:
- Properties of HR diagrams



Source: Wikimedia Commons

The success of standard stellar models

- Stellar evolution theory is incredibly successful at explaining stellar observations, as for instance:
- Properties of HR diagrams
- Helioseismic observations
- ….

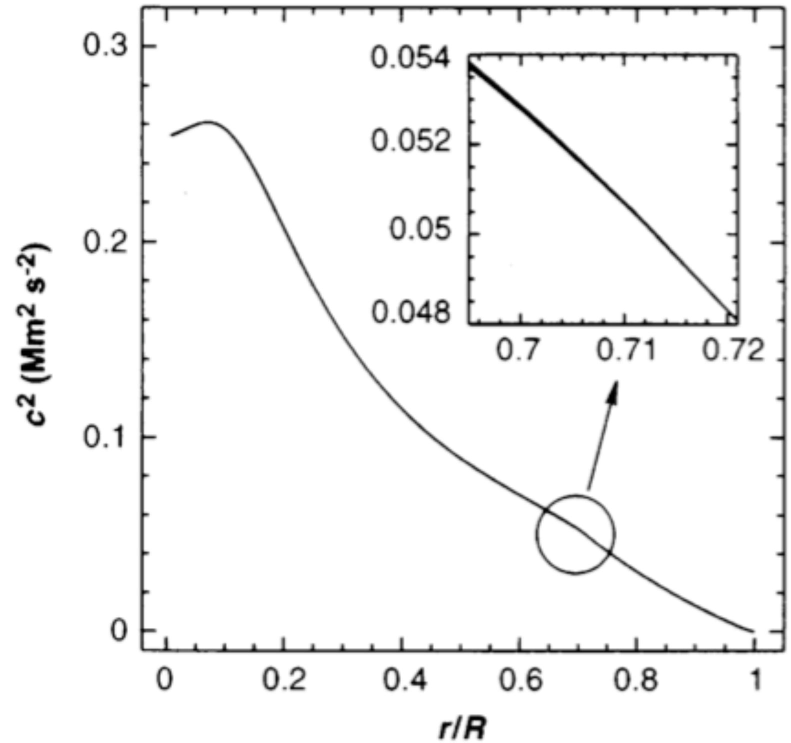


Fig. 3. The dashed curve is the square of the spherically averaged sound speed in the sun. The solid curve corresponds to a standard theoretical model.

Gough et al, 1996

The success of standard stellar models

- Stellar evolution theory is incredibly successful at explaining stellar observations, as for instance:
- Properties of HR diagrams
- Helioseismic observations
- ….

Today, stellar evolution models are sufficiently reliable to be used as tools for other purposes in physics/astrophysics.

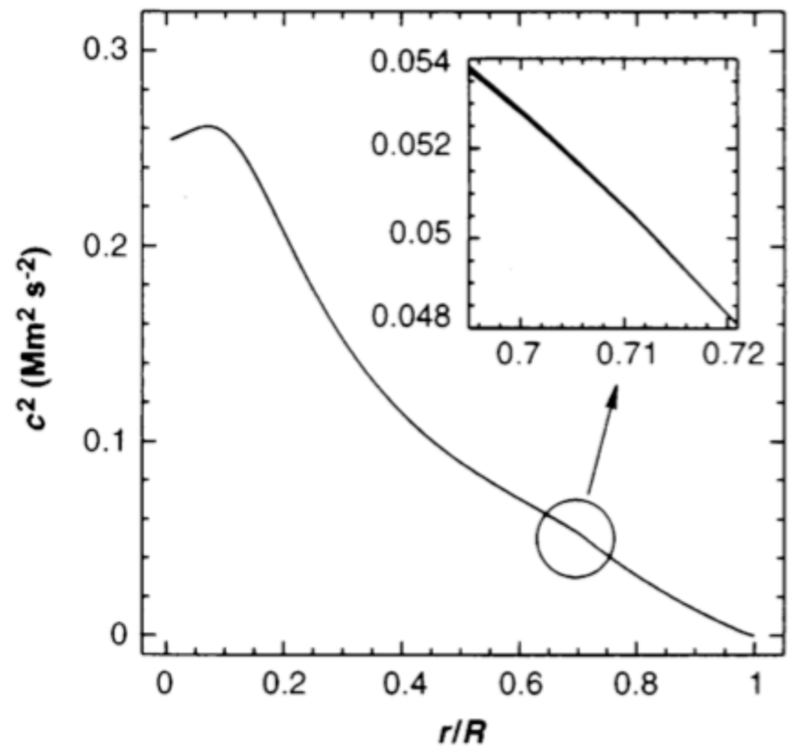


Fig. 3. The dashed curve is the square of the spherically averaged sound speed in the sun. The solid curve corresponds to a standard theoretical model.

Gough et al, 1996

The success of standard stellar models

- This success is perhaps surprising given the "simplicity" of the majority of stellar models:
  - Spherically symmetric, hydrostatic equilibrium
  - Convective zones are chemically homogeneous, energy transport is modeled with mixing-length theory
  - Radiative zones are quiescent, no (or little) chemical mixing, energy transport is radiative.

➔ Most salient properties of stellar evolution lie in the microphysics, which are well-represented in models:
  - Equation of state
  - Nuclear reaction rates
  - Opacities
  - Surface boundary conditions/atmosphere model

The success of standard stellar models

- Equations of stellar structure (for given abundance profile)

$$\frac{dm}{dr} = 4\pi r^2 \rho$$

$$\frac{dp}{dr} = -\frac{Gm}{r^2}\rho$$

$$\frac{dL}{dr} = 4\pi r^2 \rho\left(\varepsilon - \varepsilon_\nu\right)$$

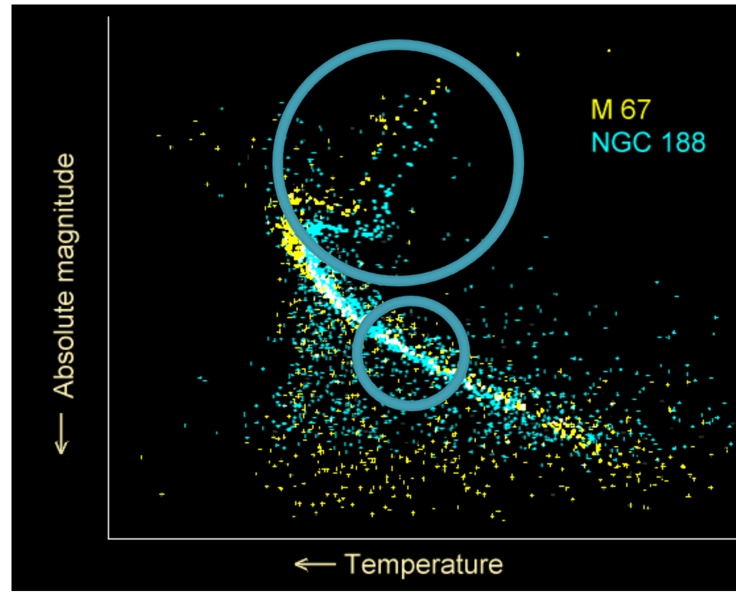
$$L = 4\pi r^2\left(F_{rad} + F_{turb}\right) = 4\pi r^2\left(-\frac{4acT^3}{3\kappa\rho}\frac{dT}{dr} + F_{turb}\right)$$

\+ an equation of state.

- Most of the difficulty lies in determining $\varepsilon\left(\rho, T; \mathbf{X}\right)$, $\kappa\left(\rho, T; \mathbf{X}\right)$ and $F_{turb}(\rho, T; \mathbf{X})$ as well as the equation of state.
- Evolution (in quasi-steady state, e.g. on main sequence) occurs because of evolution of abundance profile (through nuclear reactions, and gravitational settling)

However…

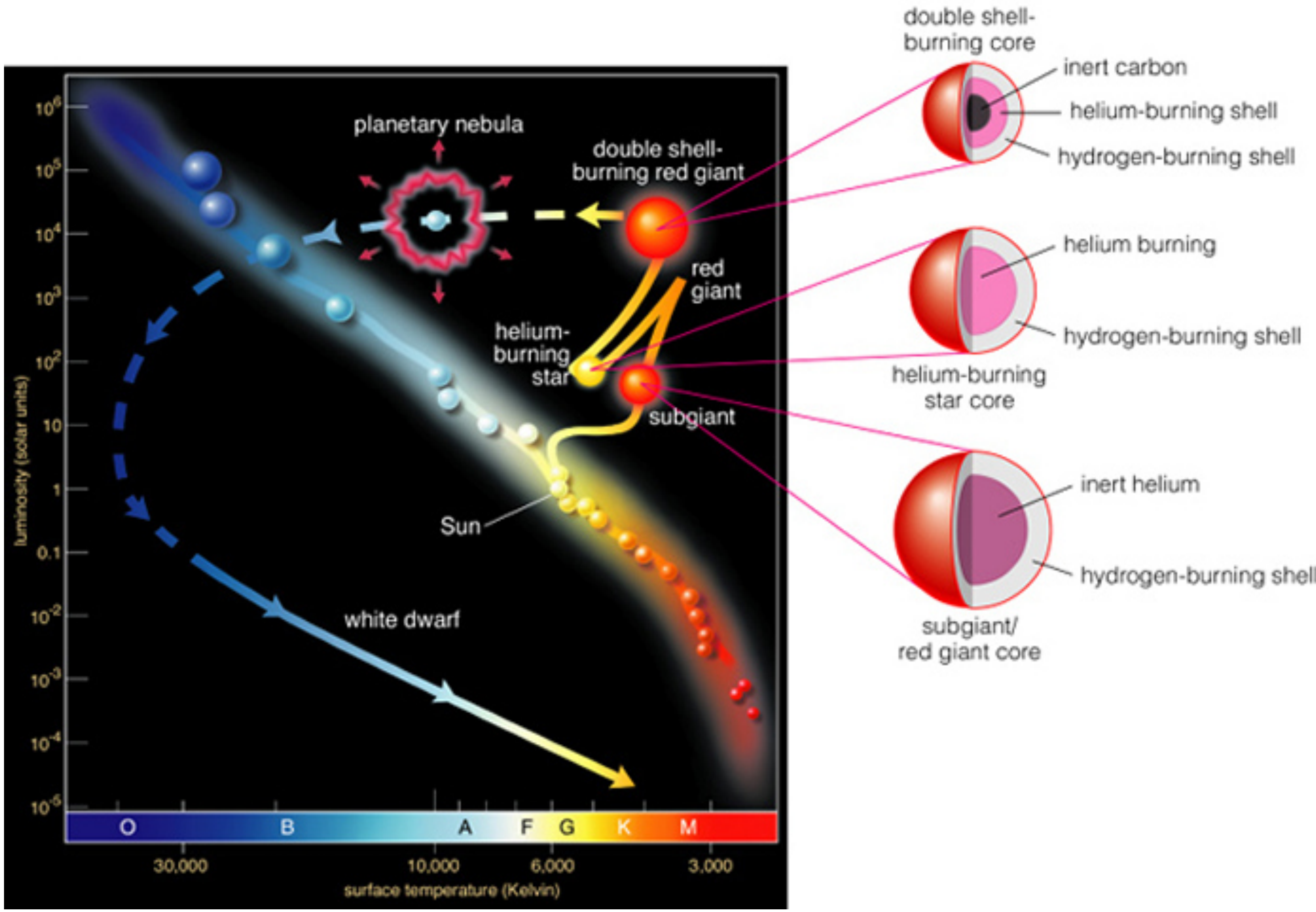
- Currently used models for the *macroscopic* transport of chemical species are very crude.
- Discrepancies between models and observations remain, suggesting need for improvement.
- These manifest themselves both on the Main Sequence and in the Post-MS phase.



Outline

- The success of stellar evolution models
- Missing mixing in RGB stars
- How to model missing mixing
- Fingering (thermohaline) Convection
- Implications for RGB observations
- Fingering convection and the effect of planetary infall.

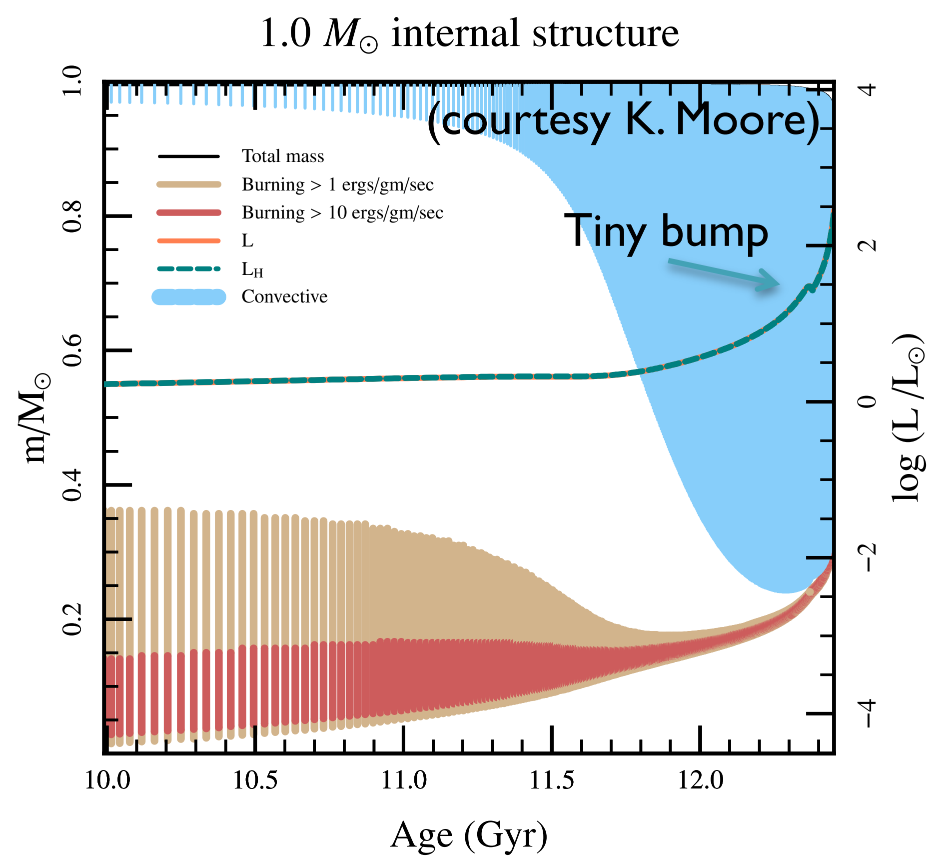
The Red Giant Branch



The Red Giant Branch

- Moving up on the RGB, the outer convection zone expands and dredges up material from deep within the star: first dredge-up.

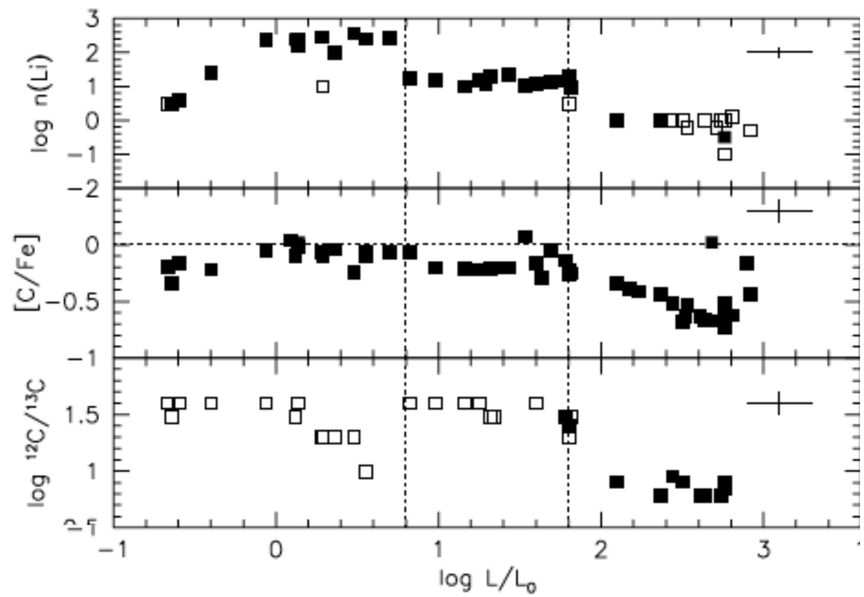
- After this event, the base of the convection zone retreats again as the Hydrogen burning shell moves outwards. The two never overlap.



- Because of the prior compositional homogenization, no more changes in surface element abundances are expected on the RGB.

Evidence for missing mixing on the RGB

- However, surface abundance data does not support this claim.
- What could cause additional mixing?



Gratton et al. 2000

1st dredge-up: convective mixing

2nd dip in abundances: ???

Outline

- The success of stellar evolution models
- Examples of Missing mixing
- How to model missing mixing
    - Mathematical modeling
    - Canonical models in “standard” stellar evolution codes.
- Fingering (thermohaline) convection
- Implications for RGB observations
- Fingering convection and the effect of planetary infall.

How to model mixing mathematically

- Modeling transport in stellar interiors always starts from the basic conservation equations of fluid mechanics:

$$\underbrace{\frac{D\rho}{Dt}}_{1} = \underbrace{-\rho\nabla\cdot\mathbf{u}}_{2}, \quad \text{and} \quad \underbrace{\frac{D\rho_S}{Dt}}_{1} = \underbrace{-\rho_S\nabla\cdot\mathbf{u}}_{2} \underbrace{- \nabla\cdot\mathbf{F}_s}_{3} + \underbrace{\left(\frac{D\rho_S}{Dt}\right)_{nucl}}_{4}$$

1. Lagrangian change in local density "following the fluid"
2. Effect of compression or expansion of the fluid
3. Additional flux of chemical species in or out of fluid blob
4. Nuclear reactions

How to model mixing mathematically

- Modeling transport in stellar interiors always starts from the basic conservation equations of fluid mechanics:

$$\frac{D\rho}{Dt} = -\rho\nabla\cdot\mathbf{u}, \quad \text{and} \quad \frac{D\rho_S}{Dt} = -\rho_S\nabla\cdot\mathbf{u} - \nabla\cdot\mathbf{F}_s + \left(\frac{D\rho_S}{Dt}\right)_{nucl}$$

- Usually we are interested in the mass fraction of a particular species:

$$X_s = \frac{\rho_s}{\rho}$$

$$\rightarrow \frac{DX_s}{Dt} = \frac{1}{\rho}\frac{D\rho_s}{Dt} - \frac{\rho_s}{\rho^2}\frac{D\rho}{Dt}$$

$$= \frac{1}{\rho}\left[-\rho_s\nabla\cdot\mathbf{u} - \nabla\cdot\mathbf{F}_s\right] - \frac{\rho_s}{\rho^2}\left[-\rho\nabla\cdot\mathbf{u}\right] + \frac{1}{\rho}\left(\frac{D\rho_S}{Dt}\right)_{nucl}$$

$$\rightarrow \frac{DX_s}{Dt} = -\frac{1}{\rho}\nabla\cdot\mathbf{F}_s + \frac{1}{\rho}\left(\frac{D\rho_S}{Dt}\right)_{nucl}$$

How to model mixing mathematically

$$\frac{DX_s}{Dt} = -\frac{1}{\rho}\nabla \cdot \mathbf{F}_s + \frac{1}{\rho}\left(\frac{D\rho_S}{Dt}\right)_{nucl}$$

So what is the flux F_s?

- It is common to assume that $\mathbf{F}_s = -D\rho\nabla X_s$
  - D is a diffusivity, and has units of cm^2/s (in cgs).
  - This is called Fick's Law.

- However, other cases can also arise, where the flux is proportional to the pressure gradient, or to the temperature gradient, etc… These are important for "atomic diffusion" (not the subject of this lecture, however).

How to model mixing mathematically

- Combining these equations, we get

$$\frac{DX_s}{Dt} = \frac{1}{\rho}\nabla\cdot(\rho D\nabla X_s) + \frac{1}{\rho}\left(\frac{D\rho_S}{Dt}\right)_{nucl}$$

$$= \frac{1}{\rho r^2}\frac{d}{dr}\left(r^2\rho D\frac{dX_s}{dr}\right) + \frac{1}{\rho}\left(\frac{D\rho_S}{Dt}\right)_{nucl}$$

- The only question left is:

What is D?

How to model mixing mathematically

- The mixing coefficient D can be due to
  - Basic collisional processes (i.e. microscopic)
  - Turbulent processes (i.e. macroscopic).

- Since D has units of length^2/time, or length x velocity, it is often (but not always) estimated from

$$D = vl \quad \text{or} \quad D = \frac{l^2}{t}$$

Characteristic velocity

Characteristic lengthscale

Characteristic lengthscale

Characteristic timescale

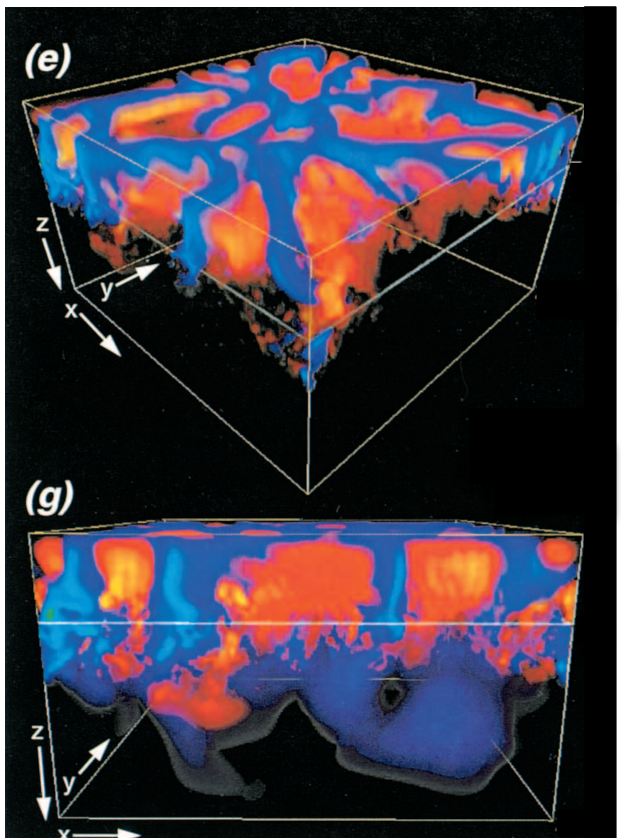
- In general, the turbulent processes (if present) lead to much larger values of D than microscopic processes.

Outline

- The success of stellar evolution models
- Examples of Missing mixing
- How to model missing mixing
  - Mathematical modeling
  - Canonical models in “standard” stellar evolution codes.
- Fingering (thermohaline) convection
- Implications for RGB observations
- Fingering convection and the effect of planetary infall.

“Canonical” mixing (standard stellar model)

- In “canonical” mixing models, the only two mixing processes taken into account are convection and overshoot.



Simulations from Brummell et al. 2002

Convection zone

Base of convection zone

Overshoot region

“Canonical” mixing (standard stellar model)

- In “canonical” mixing models, the only two mixing processes taken into account are convection and overshoot.

- Mixing of chemical species by convection is usually done by assuming that :

$$D_{conv} = \frac{1}{3} v_{conv} l_{conv}$$

  - v_{conv} is the mean velocity of the convective eddies calculated using mixing-length theory (cf. Cox & Giuli for instance)
  - l_{conv} is the mean travel distance of the eddies (the so-called “mixing length”).

“Canonical” mixing (standard stellar model)

- In “canonical” mixing models, the only two mixing processes taken into account are convection and overshoot.

- There are many possible overshoot models in the literature. Furthermore, there are different models depending on the location/ nature of the overshoot layer (above/below a CZ, etc..). For instance (cf. Herwig, 2000)

$$D_{oversht}(r) = D_{conv,edge} \exp\left(-\frac{2\left|r - r_{edge}\right|}{f_{oversht} H_p} \right)$$

where:

  - $D_{conv,edge}$ is the convective diffusion coefficient just inside the convection zone
  - r_{edge} is the position of the edge of the convection zone
  - H_p is the local pressure scaleheight
  - $f_{oversht}$ is a proportionality constant.

“Canonical” mixing (standard stellar model)

- In “canonical” mixing models, the only two mixing processes taken into account are convection and overshoot.

The RGB “abundance problem” described earlier cannot be solved by canonical mixing alone, hence the need for “extra mixing”.

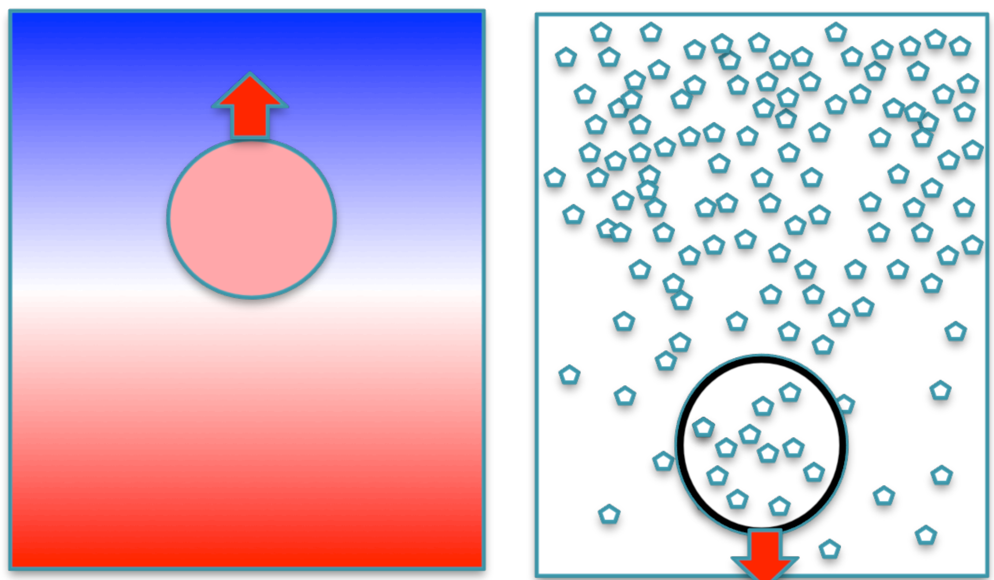
- In what follows, I will present possible processes that may play a role in RGB stars.

Outline

- The success of stellar evolution models
- Examples of Missing mixing
- How to model missing mixing
- Fingering (thermohaline) convection
    - What is fingering convection ?
    - Mathematical & Numerical model
- Implications for RGB observations
- Fingering convection and the effect of planetary infall.

What fingering convection isn't.

- *Overturning* convection is a linear instability of stratified fluids with "top-heavy" density profiles, and occurs whenever $\frac{\partial \rho}{\partial p} < \left(\frac{\partial \rho}{\partial p} \right)_{ad}$
- Different possibilities:
  - Thermal convection
  - Compositional convection



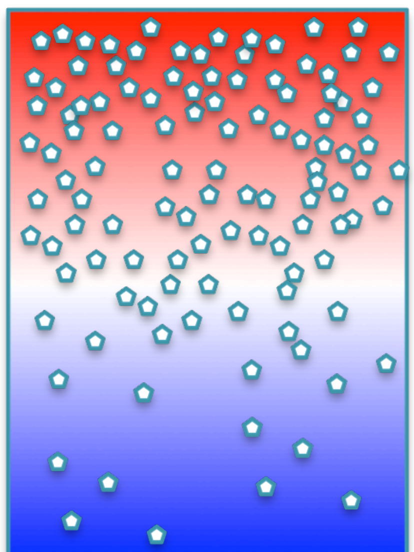
- The instability grows rapidly, resulting mixing is very efficient.

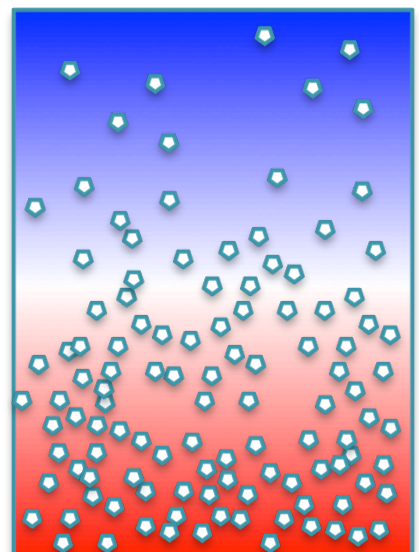
What fingering convection isn't.

- In stellar interiors, density usually depend on two or more components (temperature, mean molecular weight)

- **Question**: what happens when the two stratifications compete?

Stable s gradient
Unstable μ gradient





Unstable s gradient
Stable μ gradient

What fingering convection isn't.

The answer is superficially simple:

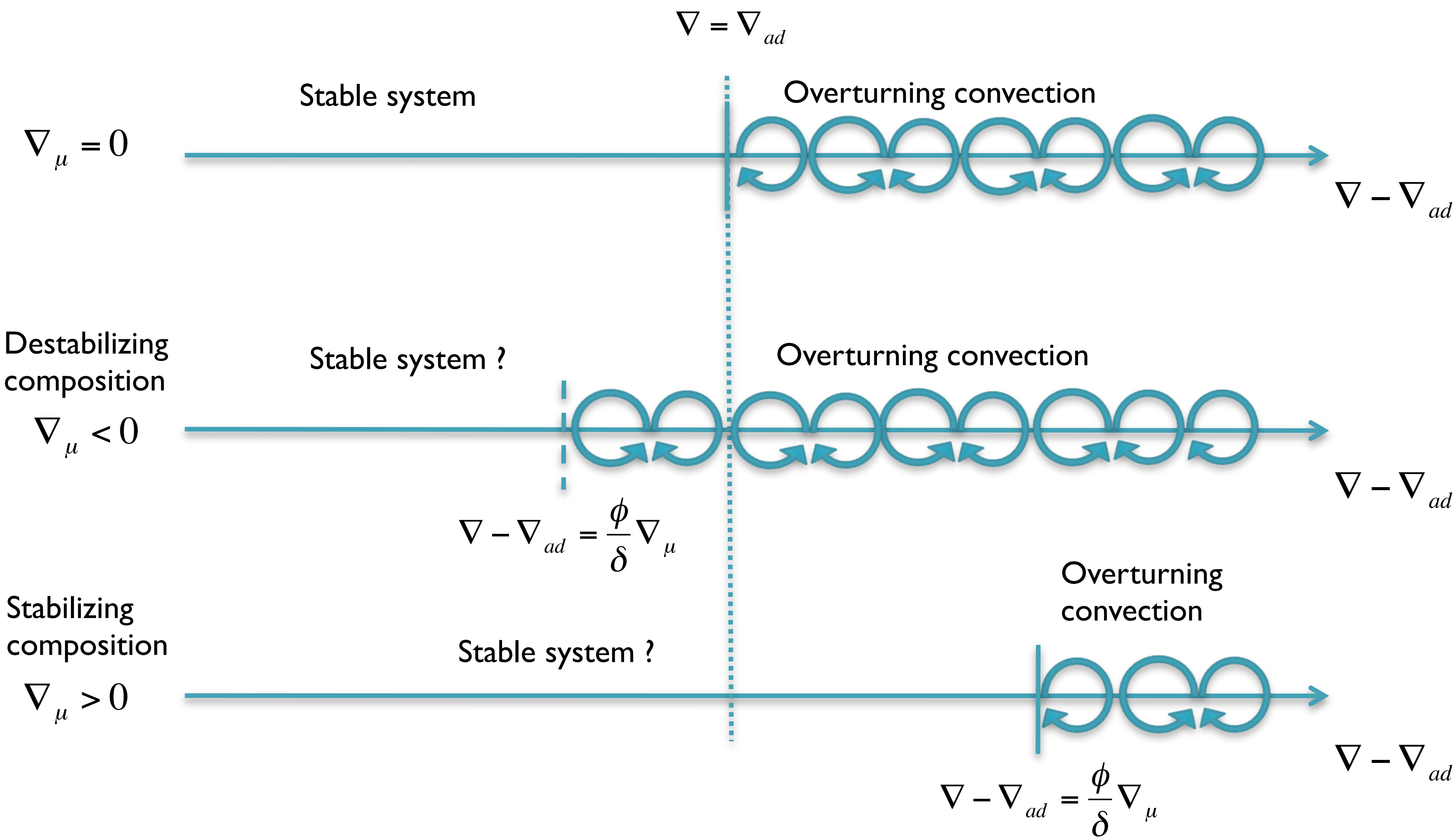
- As before, the system is *overturning* unstable when $\frac{\partial \rho}{\partial p} < \left(\frac{\partial \rho}{\partial p}\right)_{ad}$
- This is expressed as the **Ledoux criterion**

$$\nabla - \nabla_{ad} > \frac{\phi}{\delta} \nabla_{\mu}$$

where

$$\nabla = \frac{d\ln T}{d\ln P},\quad \nabla_{ad} = \left(\frac{d\ln T}{d\ln P}\right)_{ad},\quad \nabla_{\mu} = \frac{d\ln \mu}{d\ln P},\quad \delta = -\left(\frac{\partial \ln \rho}{d\ln T}\right)_{P,\mu},\quad \phi = \left(\frac{\partial \ln \rho}{d\ln \mu}\right)_{P,T}$$

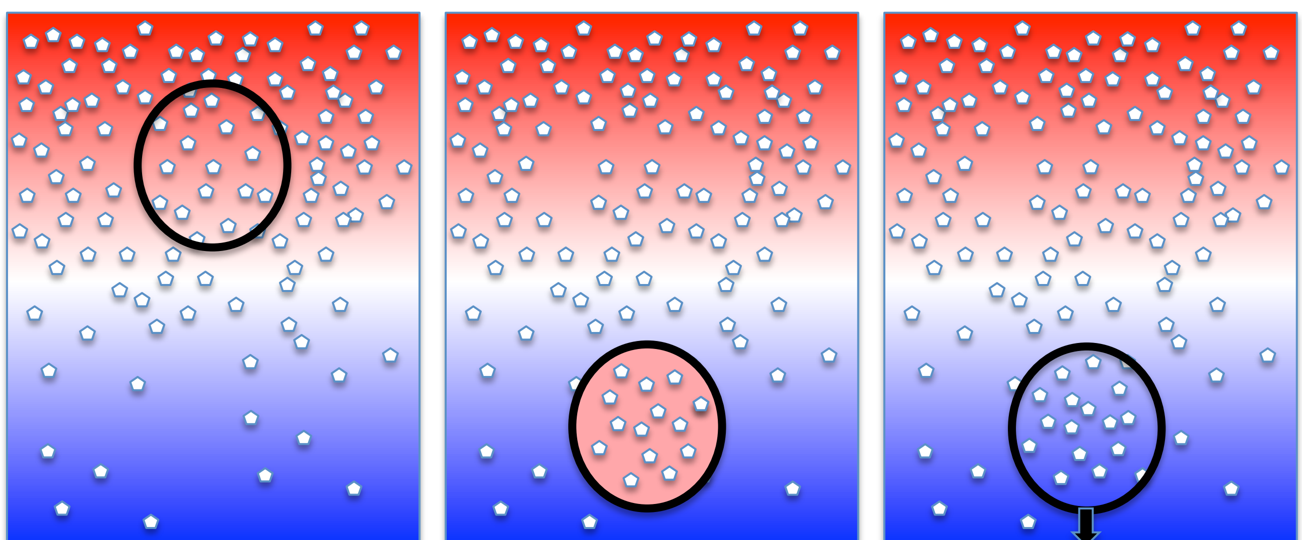
What fingering convection isn't.



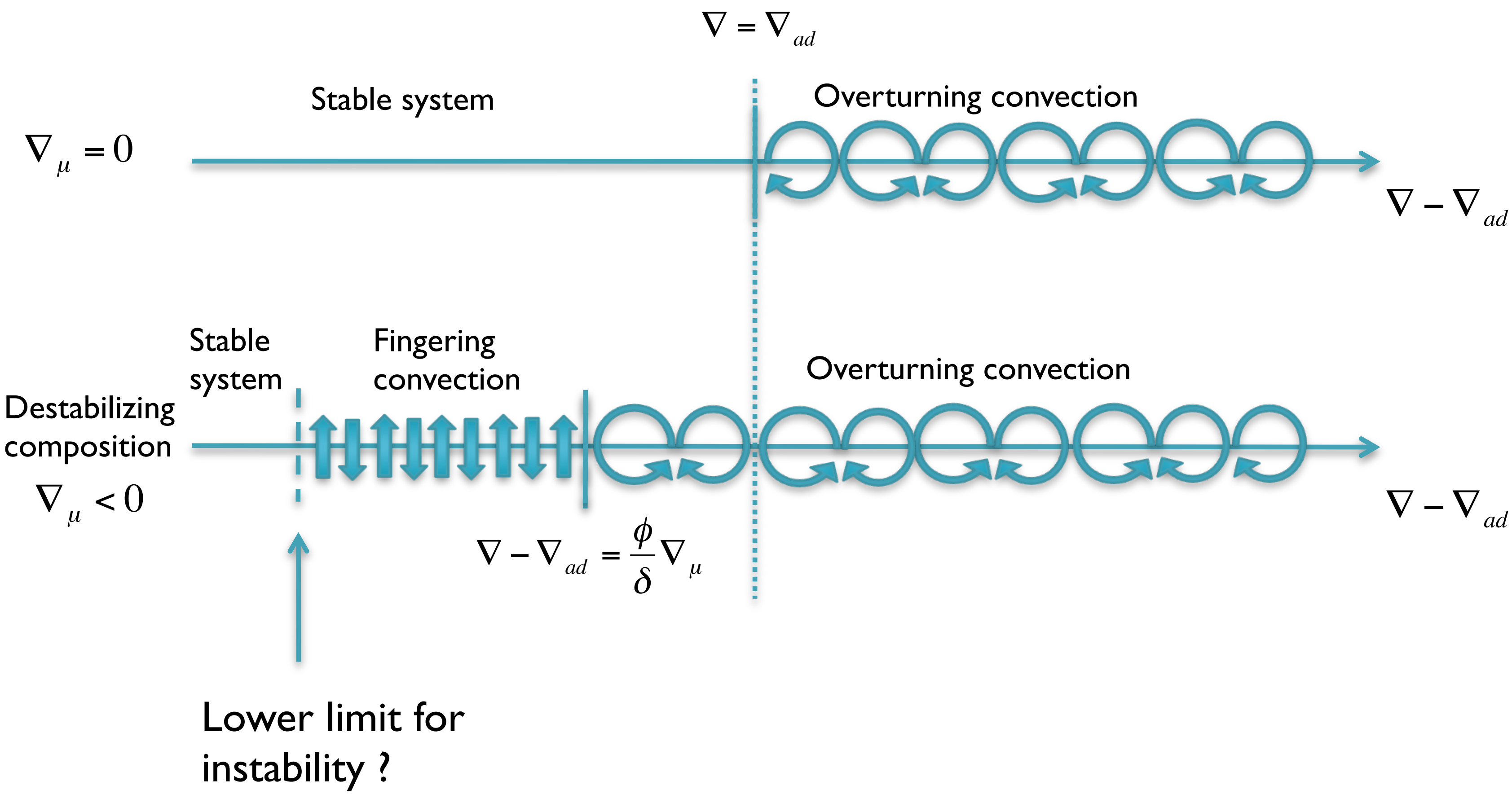
Fingering convection

- However, heat & composition usually diffuse at different rates. When this is the case, new linear instabilities can occur *even in the case of Ledoux-stable profiles*.

- Weak inverse μ - gradients can trigger **fingering** convection (often called thermohaline convection by analogy with case of similar instability in salt water)



Fingering convection

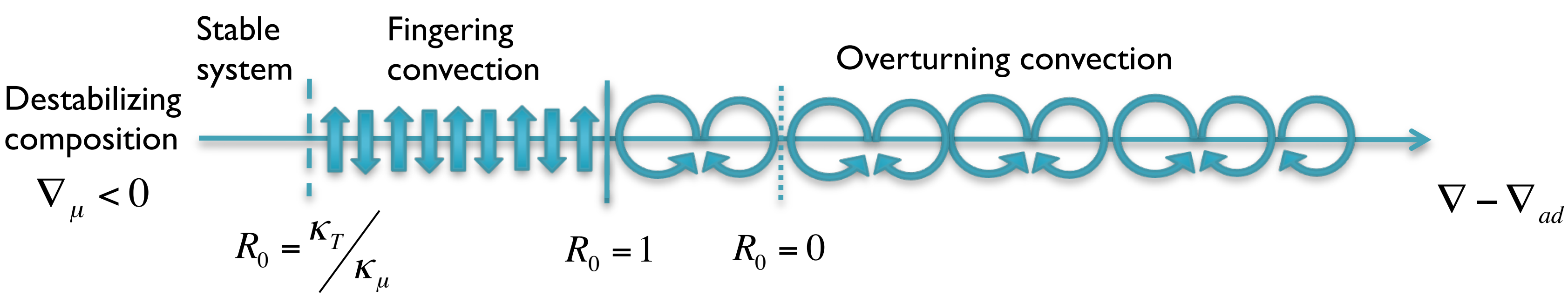


Fingering convection

- The dynamics of the fingering instability depends on the non-dimensional **density ratio** (Stern, 1960, Ulrich, 1972)

$$R_0 = \frac{\nabla - \nabla_{ad}}{\frac{\phi}{\delta}\nabla_\mu}$$

R_0 = 1 corresponds to Ledoux criterion
R_0 = 0 corresponds to Schwartzschild criterion



- Ulrich (1972), Kippenhahn et al. (1980) proposed that $D_\mu = \frac{C}{R_0}\kappa_T$

- However, this is a model based on dimensional analysis, and the value of C is arbitrary. Can we do better?

Outline

- The success of stellar evolution models
- Missing mixing in RGB stars
- How to model missing mixing
- Fingering (thermohaline) convection
  - What is fingering convection
  - Mathematical & numerical models
- Implications for RGB observations
- Fingering convection and the effect of planetary infall.

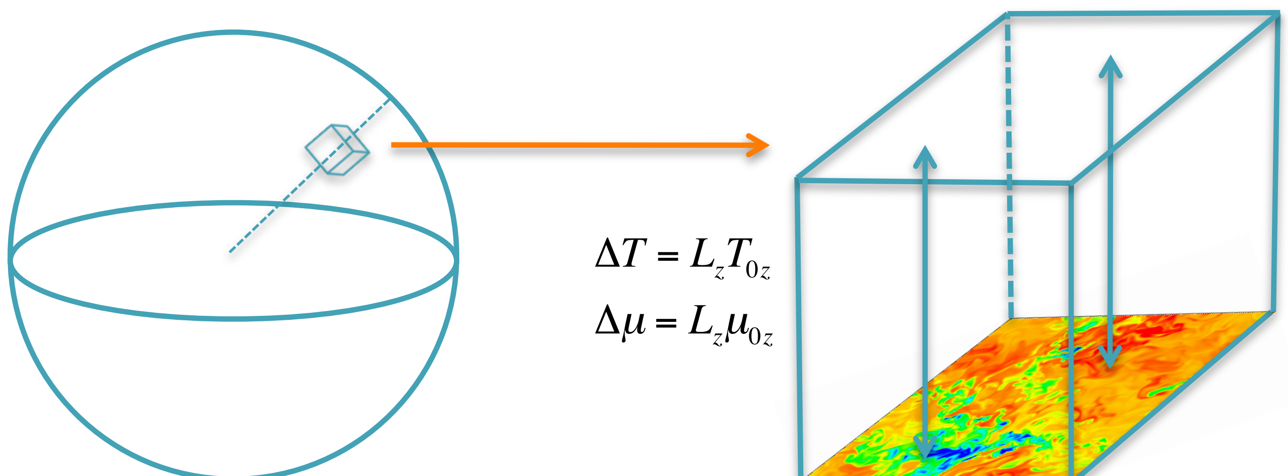
Mathematical modeling

Model considered:

- Assume **background** temperature and concentration profiles are linear (constant gradients $T_{0z}, T_{0z}^{ad}, \mu_{0z}$)
- Assume that all **perturbations** are triply-periodic in domain (L_x,L_y,L_z):

$$q(x,y,z,t) = q(x+L_x,y,z,t) = q(x,y+L_y,z,t) = q(x,y,z+L_z,t)$$

- This enables us to study the phenomenon with little influence from boundaries.
- (Diagram not to scale)



Mathematical modeling

Governing non-dimensional equations:

$$\frac{1}{\Pr}\left(\frac{\partial u}{\partial t}+u\cdot\nabla u\right)=-\nabla p+(T-\mu)\mathbf{e}_{\mathbf{z}}+\nabla^2 u$$

$$\frac{\partial T}{\partial t}+u\cdot\nabla T+w=\nabla^2 T$$

$$\frac{\partial \mu}{\partial t}+u\cdot\nabla \mu+\frac{w}{R_0}=\tau\nabla^2 \mu$$

$$\nabla\cdot u=0$$

Questions:
How efficient is mixing by fingering convection in stars ?
How does this depend on the local stellar properties?

Governing parameters:

$$\Pr=\frac{\nu}{\kappa_T}\approx 10^{-5\to -7}$$

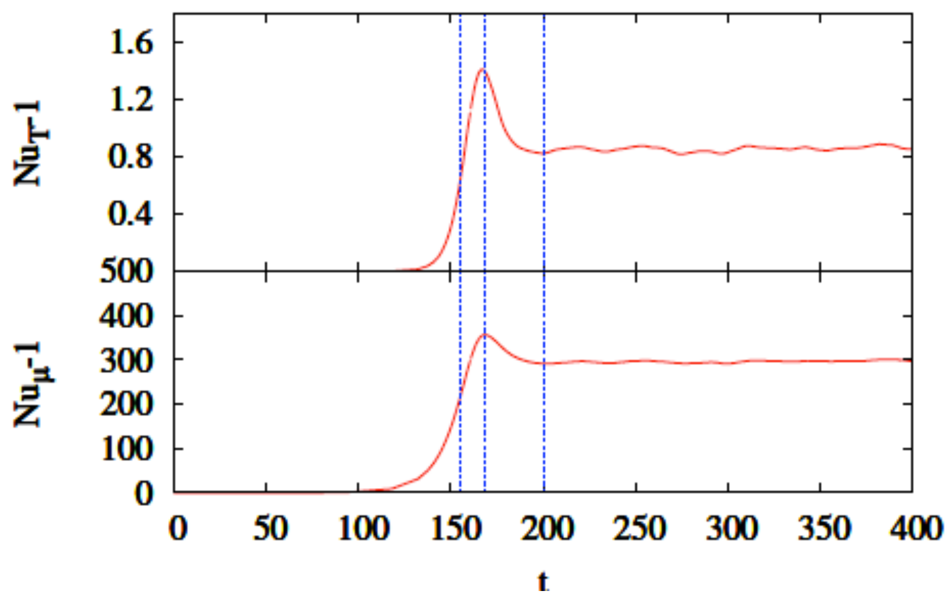
$$\tau=\frac{\kappa_\mu}{\kappa_T}\approx 10^{-6\to -8}$$

$$1<R_0=\frac{\nabla-\nabla_{ad}}{\frac{\phi}{\delta}\nabla_\mu}<\tau^{-1}$$

Numerical modeling

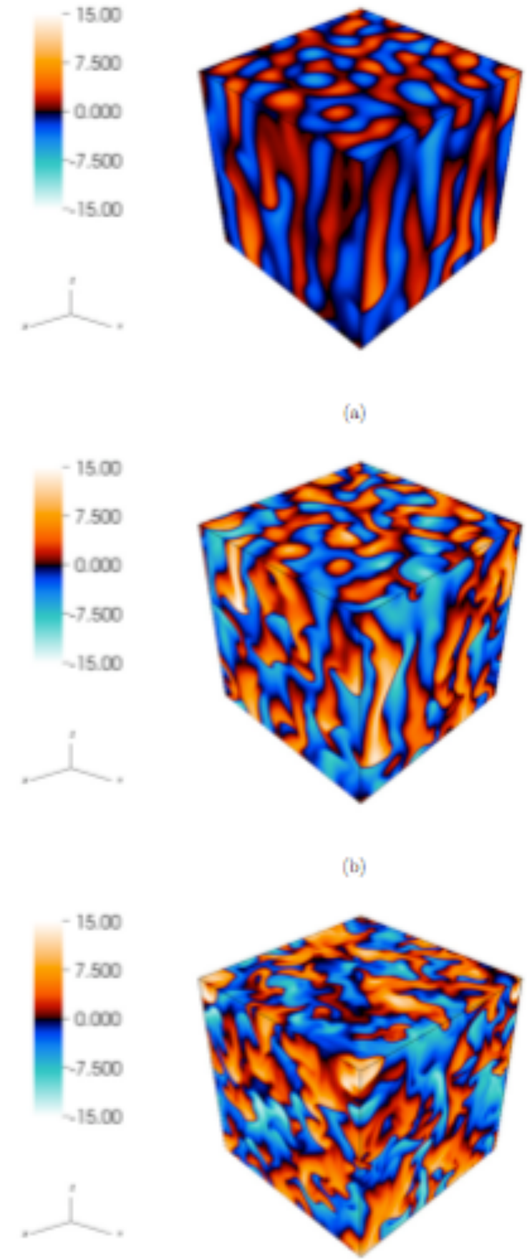
Sample simulation:

$$Nu - 1 = \frac{D_{\rm turb}}{\kappa_{\rm micro}}$$



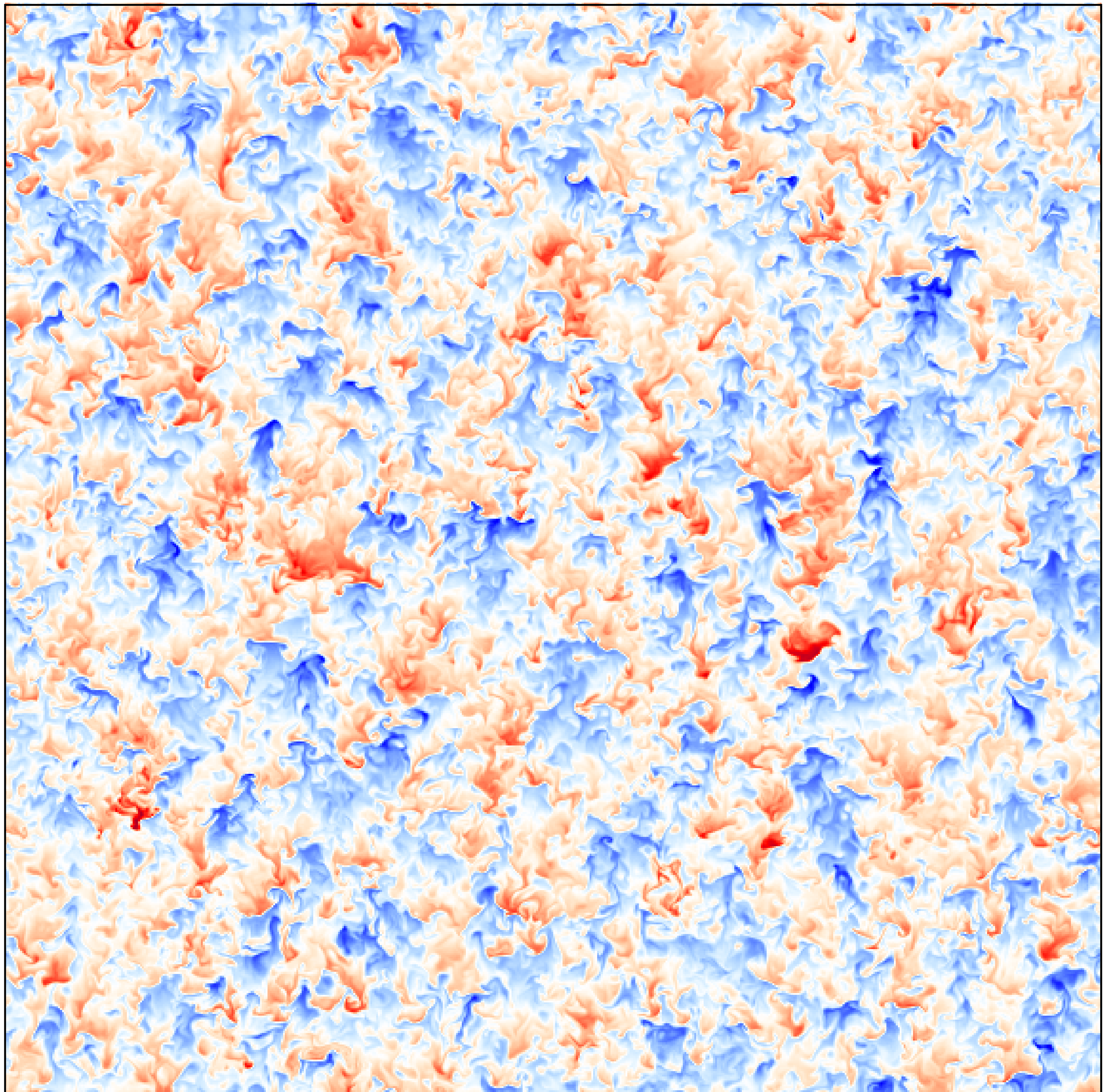
$\mathrm{Pr} = 0.1, \quad \tau = 0.03,$

$R_0 = 3.0$ (Note: parameters not very "stellar" yet)



Brown, Garaud & Stellmach (2013)

Concentration field

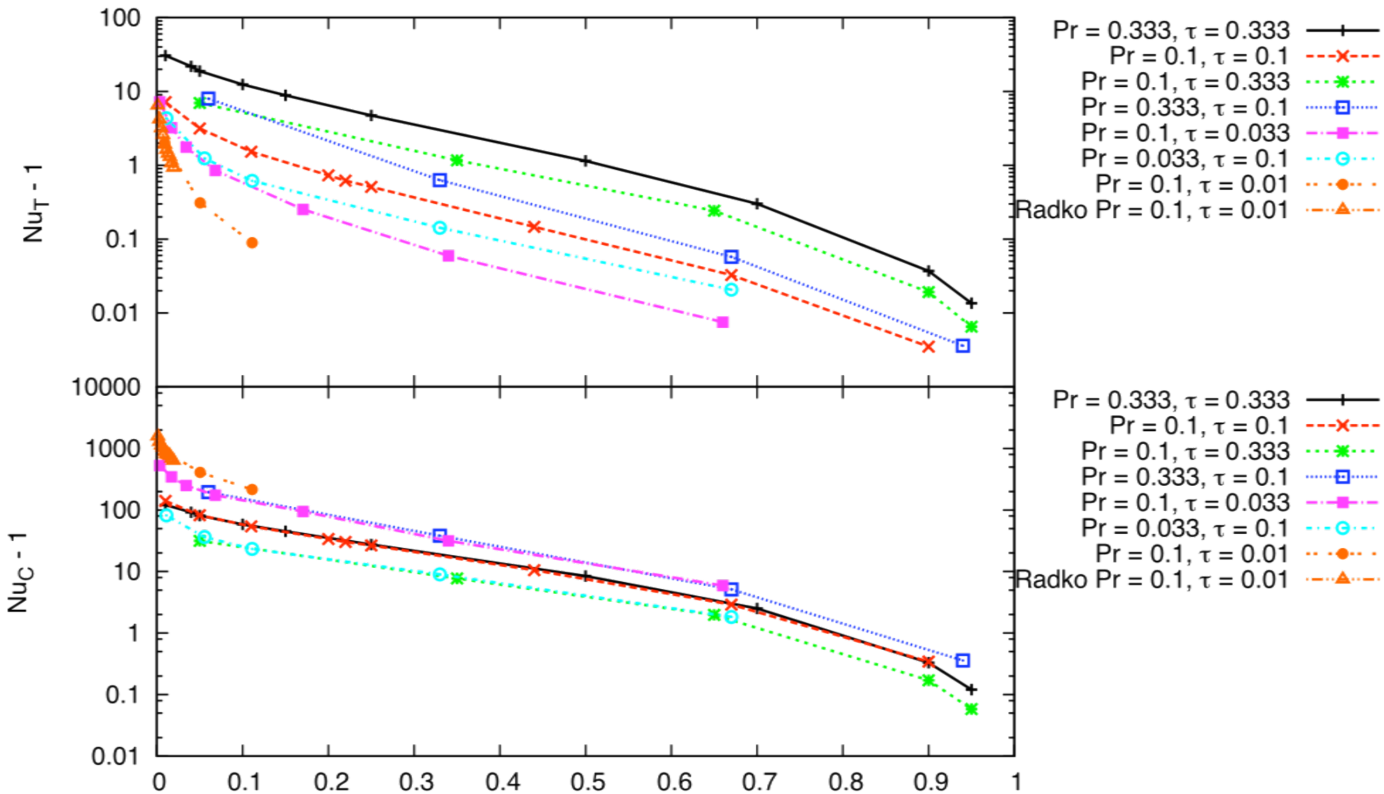


From: Garaud & Brummell, in prep.

Numerical modeling

$$Nu - 1 = \frac{D_{\text{turb}}}{\kappa_{\text{micro}}}$$

Experimental results:



$$r = \frac{R_0 - 1}{\tau^{-1} - 1}$$

Can we predict what the fluxes may be for lower Pr, τ ?

Brown, Garaud & Stellmach (2013)

Mixing by fingering

Idea: (cf. Radko & Smith 2012)

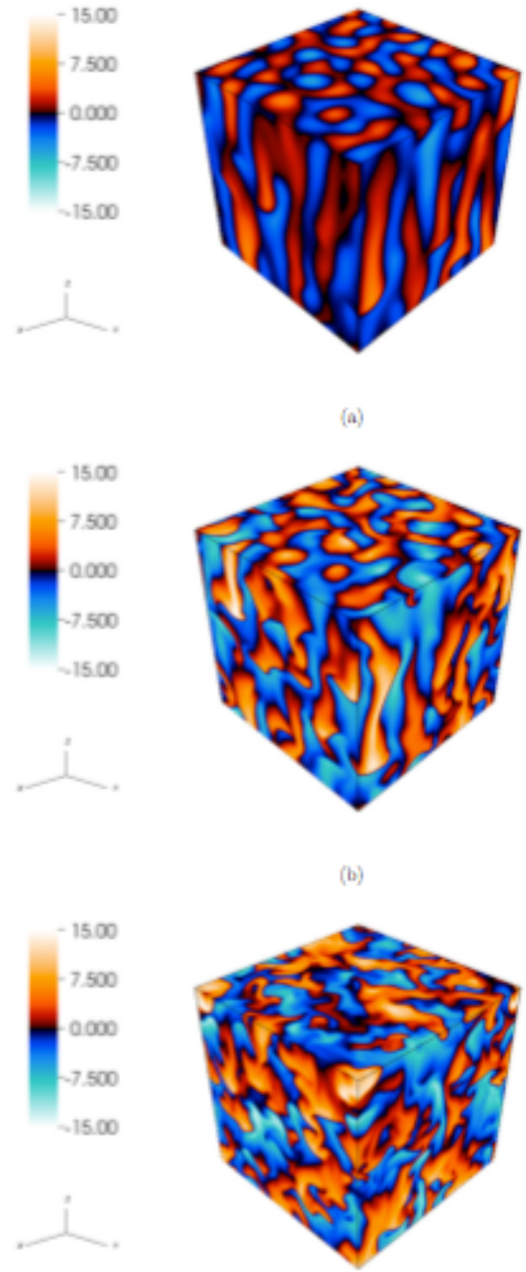
- Saturation occurs when the growth rate σ of the shearing instability associated with the fluid motion within the fingers is of the order of the growth rate of the fingering instability λ :

$$\sigma = C\lambda$$

- Applying this idea yields

$$\frac{D_{BGS}}{\kappa_\mu} = \frac{1}{\tau} \frac{C^2\lambda^2}{\lambda m^2 + \tau m^4}$$

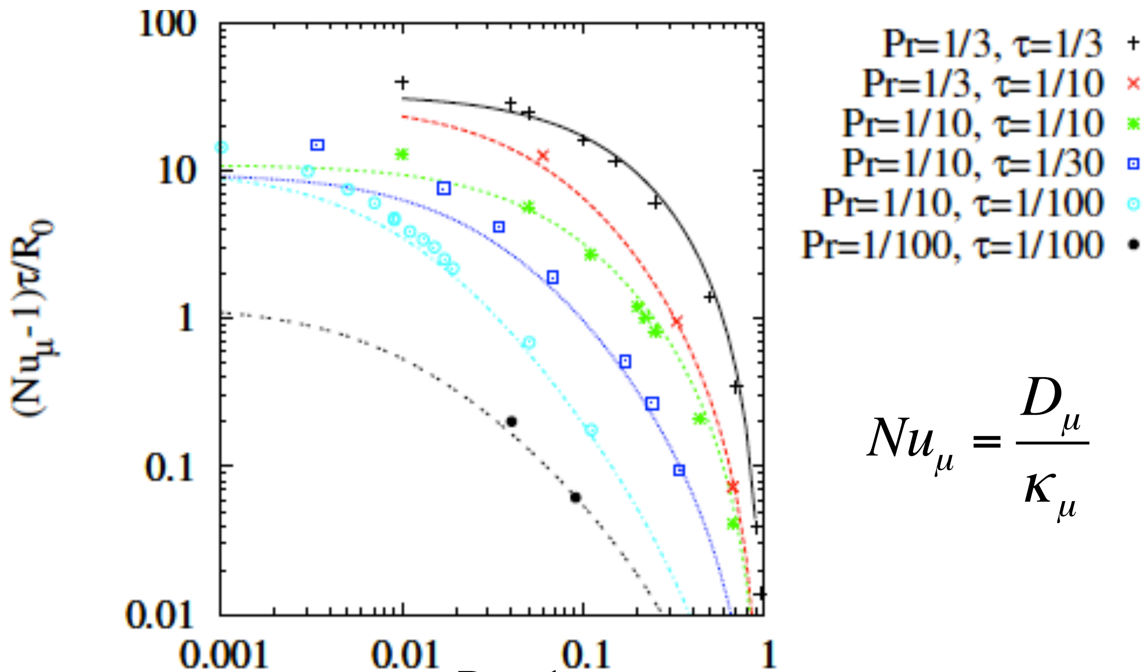
where m is wavenumber of fastest-growing mode, C constant to be fitted



Brown, Garaud & Stellmach (2013)

Mixing by fingering

C=7 gives an excellent fit to ALL data for all (astrophysically-relevant) cases with Pr > τ



$$Nu_\mu = \frac{D_\mu}{\kappa_\mu}$$

$$r = \frac{R_0 - 1}{\tau^{-1} - 1}$$

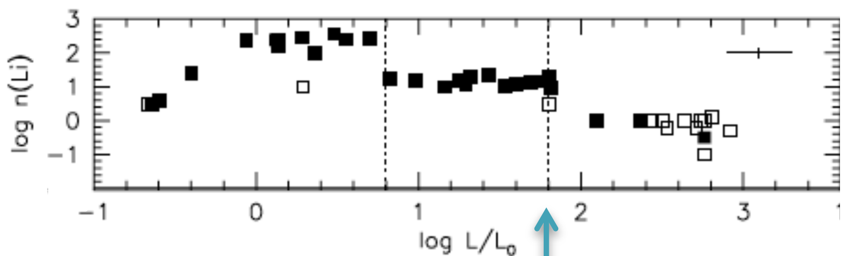
We now have a way of estimating transport by fingering convection at astrophysical parameters!

Brown, Garaud & Stellmach (2013)

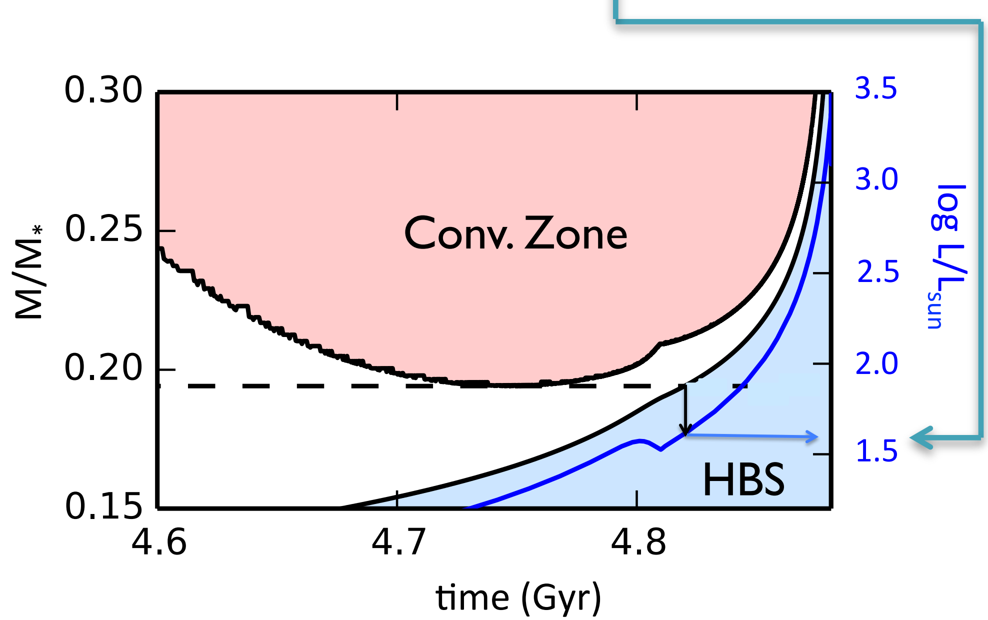
Outline

- The success of stellar evolution models
- Missing mixing in RGB stars
- How to model missing mixing
- Fingering (thermohaline) convection
  - What is fingering convection
  - Mathematical & numerical models
- Implications for RGB observations
- Fingering convection and the effect of planetary infall.

Mixing by fingering on RGB



Recall: we want to explain the "second-dip" in abundances.



Courtesy: Corentin Cadiou

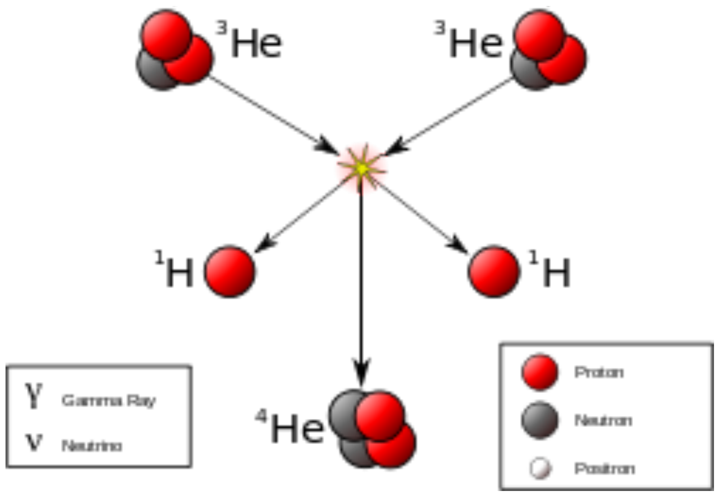
Note how second dip in RGB surface abundances corresponds to luminosity bump in star.

The luminosity bump also happens when the hydrogen-burning shell moves into the region previously mixed by dredge-up.

This is not a coincidence (Eggleton et al., 2006; Charbonnel & Zahn 2007).

Clues to the origin of missing mixing

- Near the outer edge of the hydrogen burning shell, the dominant reaction is second part of PP chain.



(Source: Wikipedia)

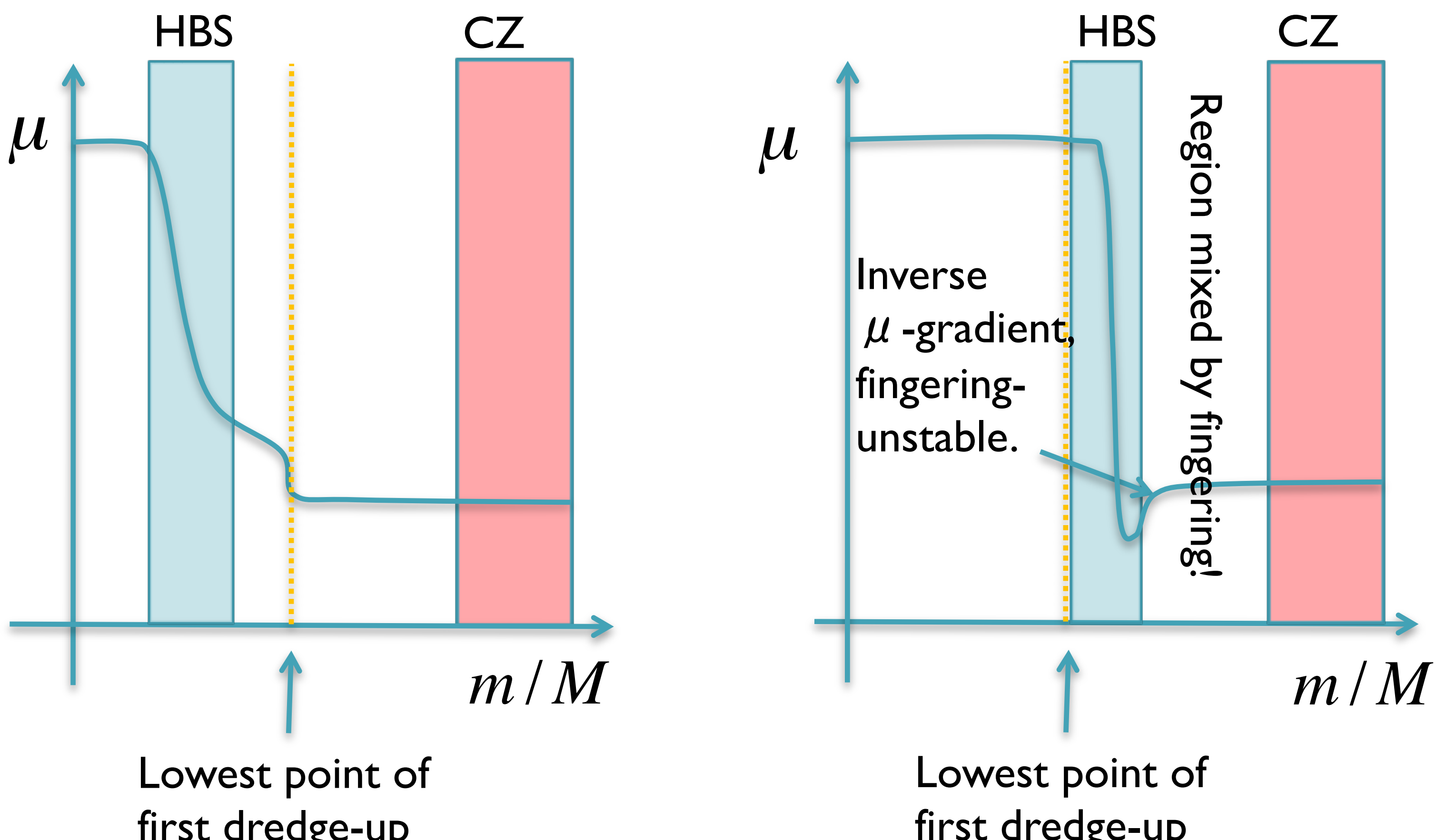
2 particles of total mass 6

3 particles of total mass 6

This reaction locally destroys ^{3}He and decreases the mean molecular weight.

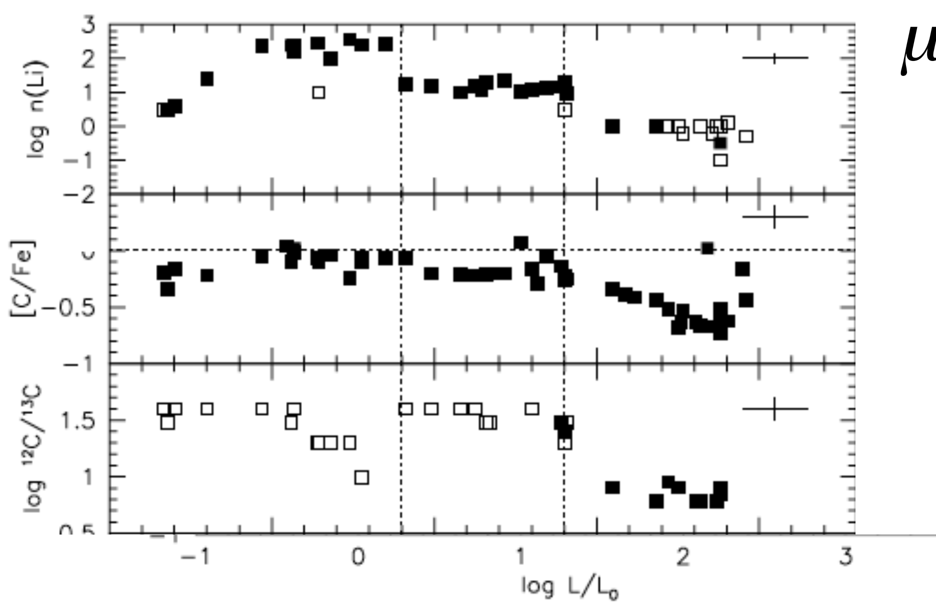
Clues to the origin of missing mixing

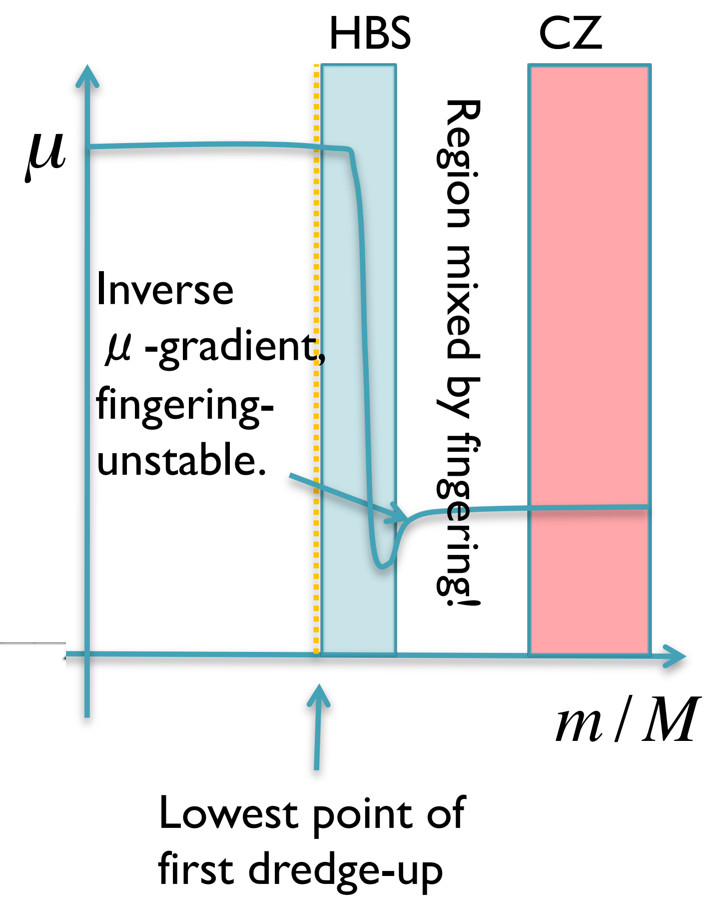
- As a result, an inverse μ –gradient can form after luminosity bump, but not before…



Clues to the origin of missing mixing

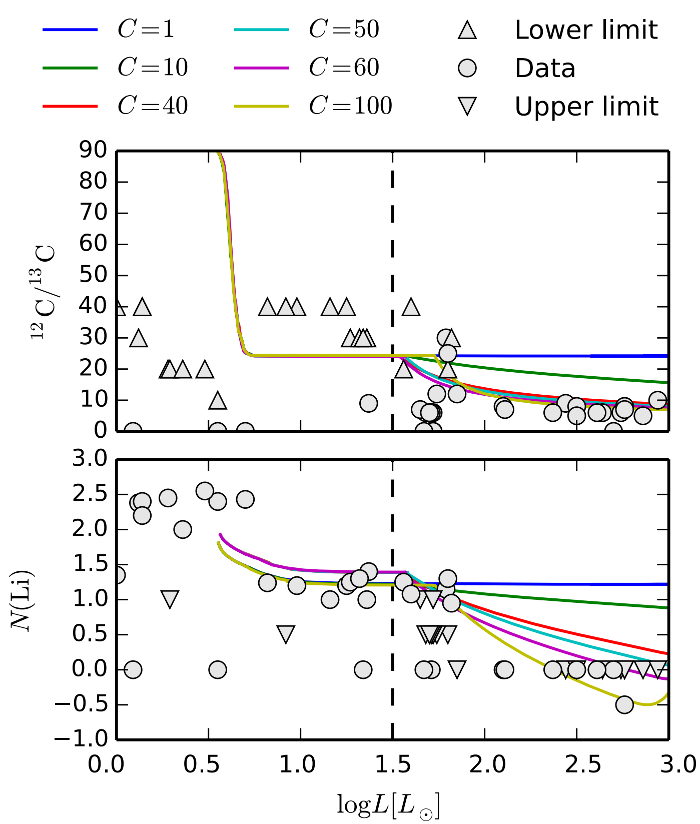
- As a result, an inverse μ –gradient can form after luminosity bump, but not before…





Mixing by fingering on the RGB

- However, mixing by fingering convection turns out to be quite weak, and probably cannot explain RGB abundances



From: Cadiou et al, in prep.

Surface abundances in RGB stars computed with:

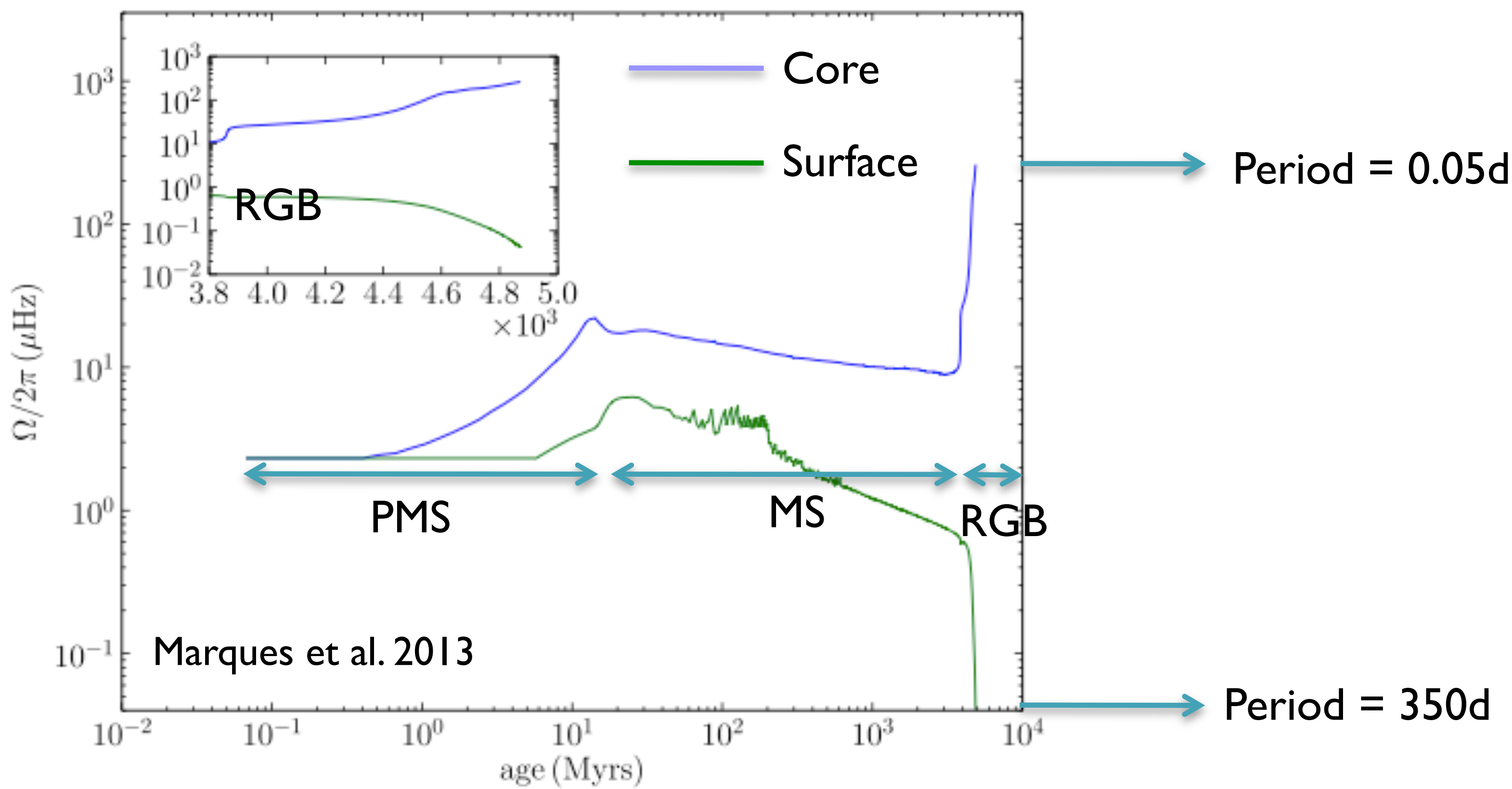
$$D_{fingering} = CD_{BGS}$$

✓ This correctly accounts for the correlation between the luminosity bump and the second abundance dip

✗ Basic fingering convection alone appears to be 50-100 times too weak to explain second dip in RGB abundances!

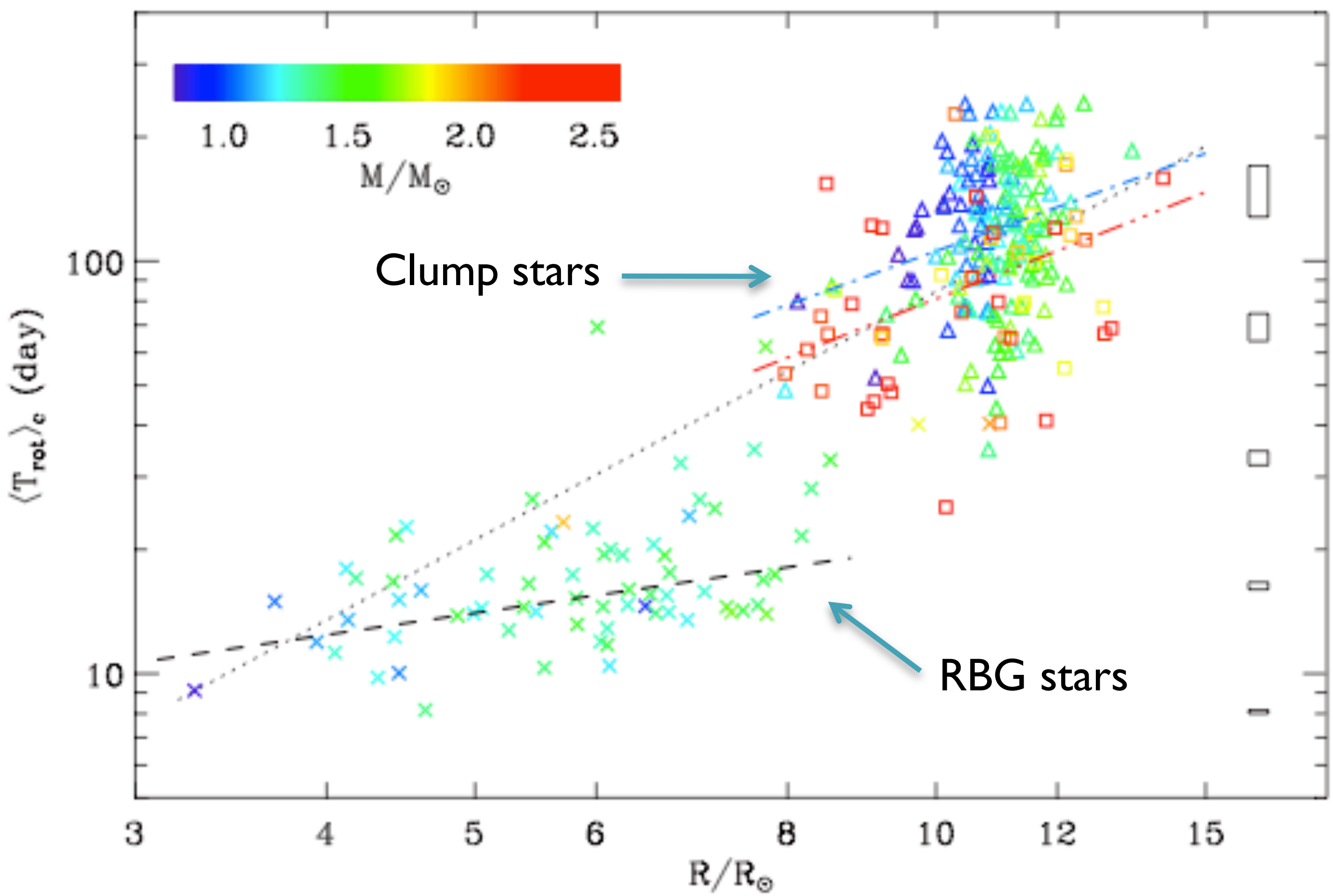
Missing rotational mixing in RGB stars

"Missing mixing" on the RGB doesn't seem to just affect abundances: Standard models predictions for the evolution of the rotation profile without extra angular momentum mixing predict huge difference between core and surface rotation rates.



Missing rotational mixing in RGB stars

This is at odds with observations, however.

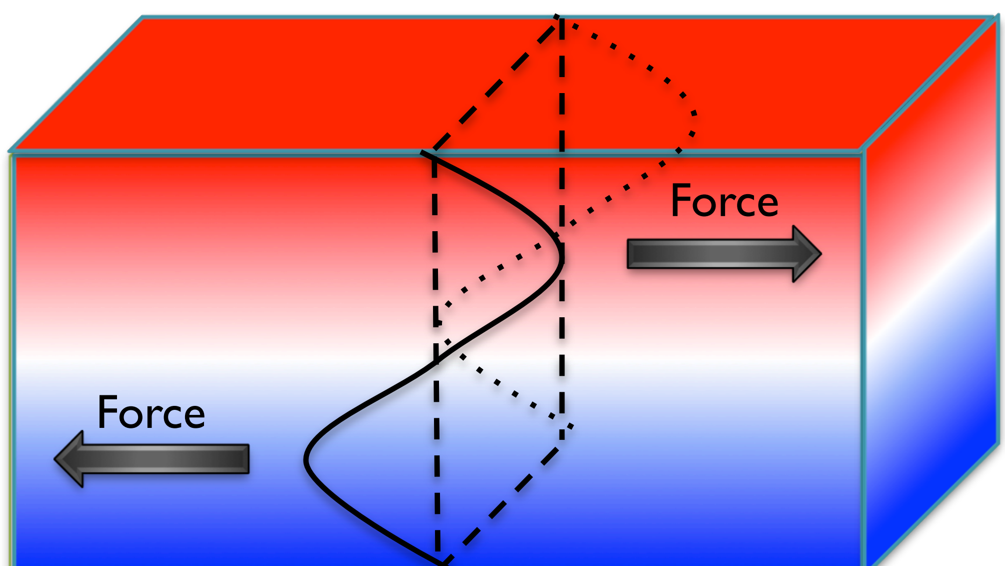


Mosser et al. 2012

Missing rotational mixing in RGB stars

Could the same process be responsible for both "missing mixing" problems?

- The development of instabilities related to the strong shear between the core and the envelope of RGB stars could be responsible for smoothing out the shear, and for mixing chemical species.
- Process currently under investigation using numerical simluations

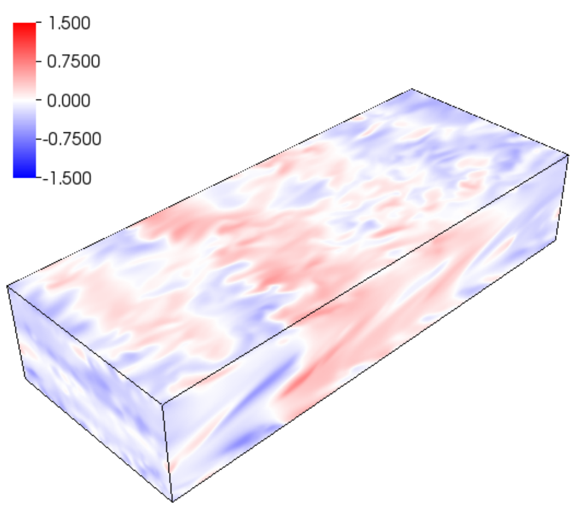


Missing rotational mixing in RGB stars

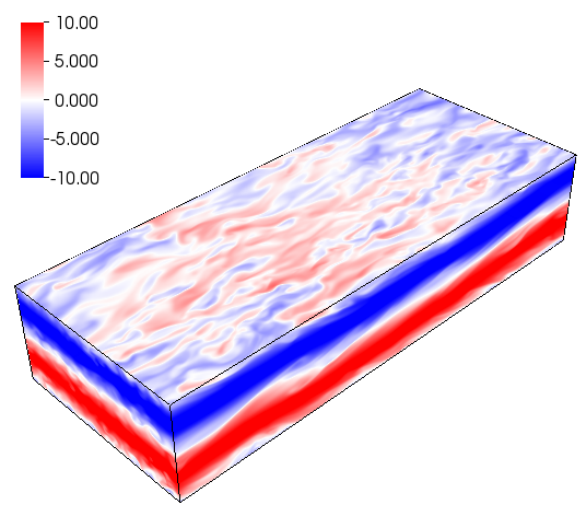
Could the same process be responsible for both "missing mixing" problems?

- The development of instabilities related to the strong shear between the core and the envelope of RGB stars could be responsible for smoothing out the shear, and for mixing chemical species.

T perturbations



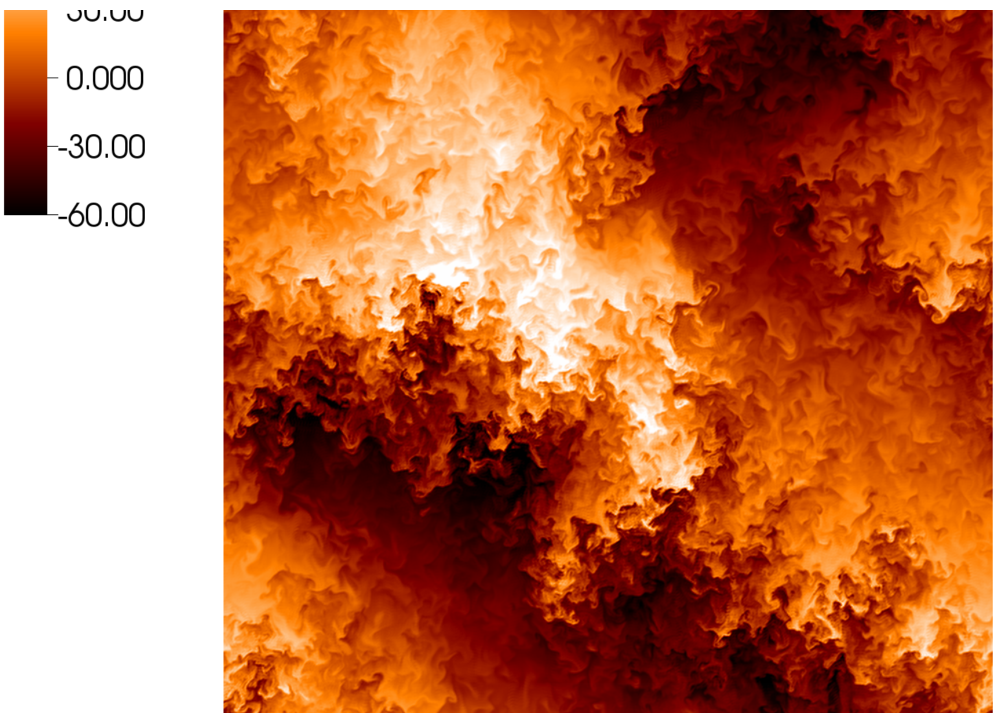
u_x perturbations



Missing rotational mixing in RGB stars

Could the same process be responsible for both "missing mixing" problems?

- Other fingering-related instabilities could also just about save the day.. Also currently under investigation.

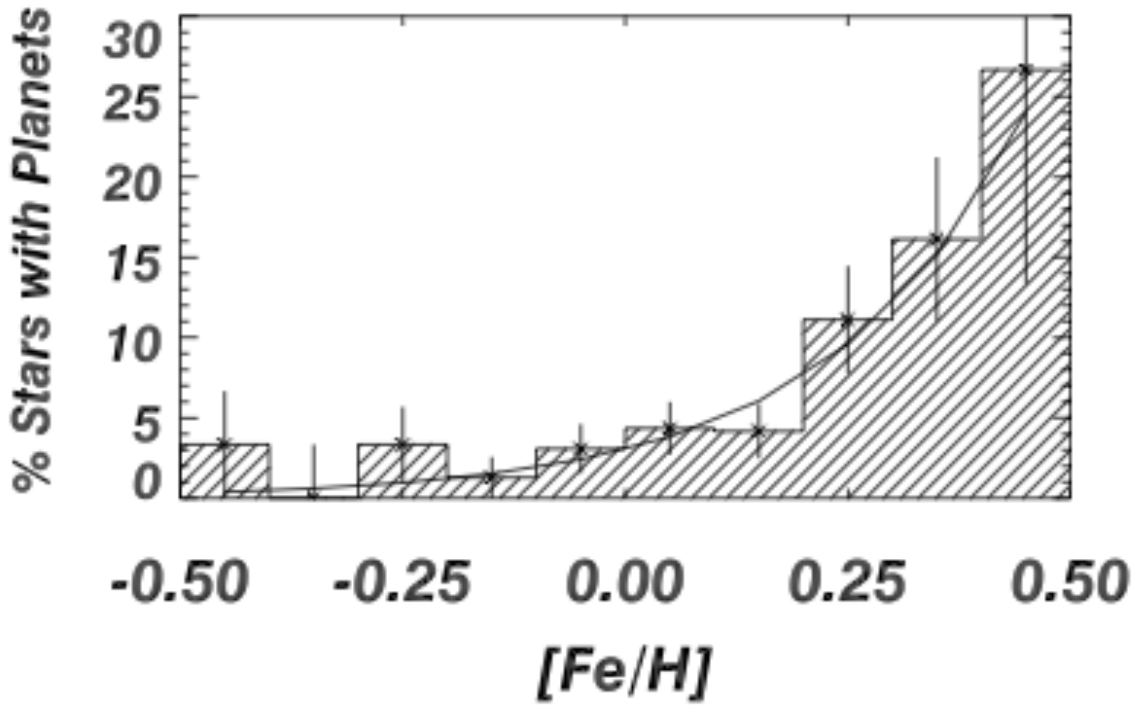


Outline

- The success of stellar evolution models
- Missing mixing in RGB stars
- How to model missing mixing
- Fingering (thermohaline) convection
  - What is fingering convection
  - Mathematical & numerical models
- Implications for RGB observations
- Fingering convection and the effect of planetary infall.

Mixing by fingering on early MS

- Giant-planet / stellar metallicity connection:



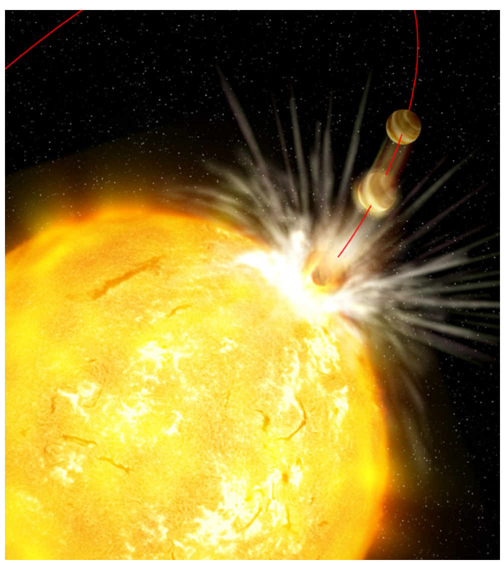
Primordial effect or caused by planetary infall?

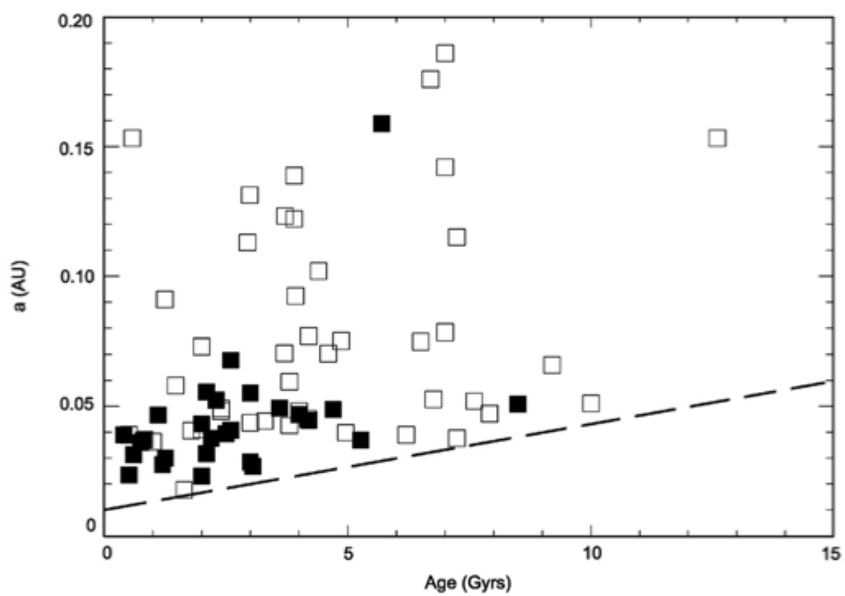
Fisher and Valenti 2005

Mixing by fingering on early MS

Interactions between planets and their host disk, or planets with one another, are thought to lead to very close orbits (3-day period planets)

Tidal interactions with the host stars then causes further orbital decay on Gyr timescale

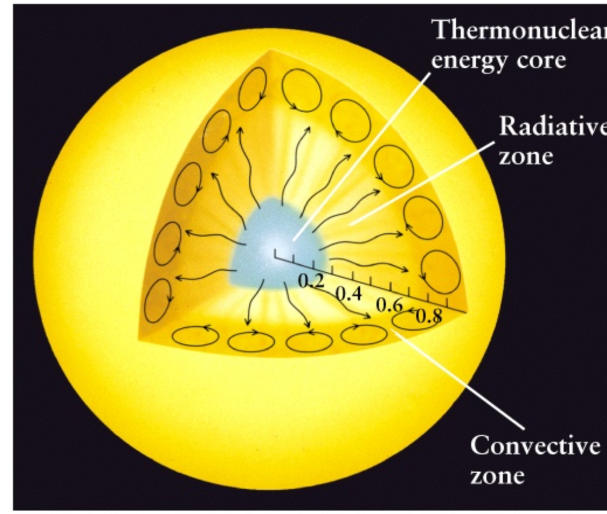




Jackson et al 2009

Mixing by fingering on early MS

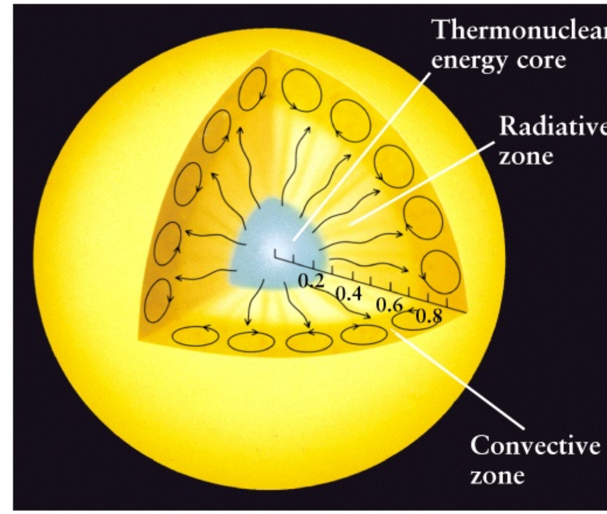
- Solar-type stars have outer convective zones.
- If infalling planetary material is mixed only within the outer convection zone, we may expect to see (Gonzalez, 1997):



- Higher metallicity (dispersion) in planet-host stars
- Even higher metallicity (dispersion) in planet-host stars with shallower convection zones.

Mixing by fingering on early MS

- Solar-type stars have outer convective zones.
- If infalling planetary material is mixed only within the outer convection zone, we may expect to see (Gonzalez, 1997):

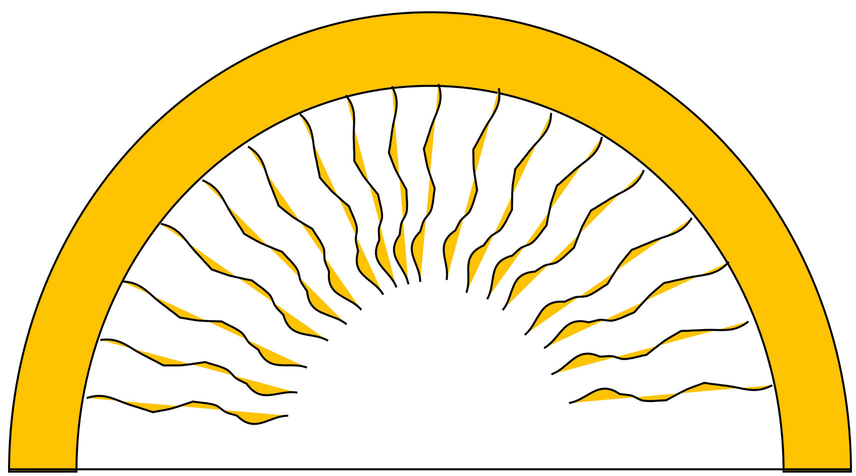


- Higher metallicity (dispersion) in planet-host stars
- Even higher metallicity (dispersion) in planet-host stars with shallower convection zone.

Not observed!

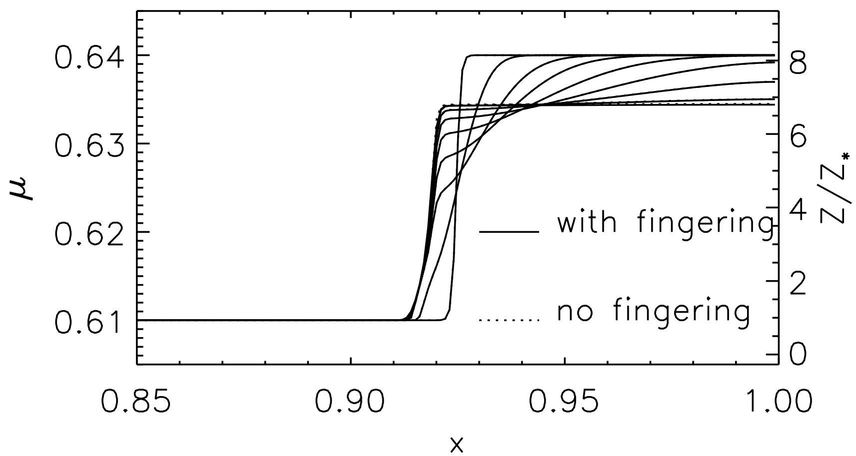
Mixing by fingering on early MS

- Possible resolution of the problem (Vauclair 2004):
  - Planetary infall adds a significant amount of high-μ material into the convection zone.
  - This creates an inverse μ–gradient at the top of the radiation zone, which becomes unstable to fingering convection.
  - The added mixing drains the excess metallicity into the radiative interior



Mixing by fingering on early MS

- Example of a $1M_J$ planet falling onto a $1.4M_{sun}$ star.

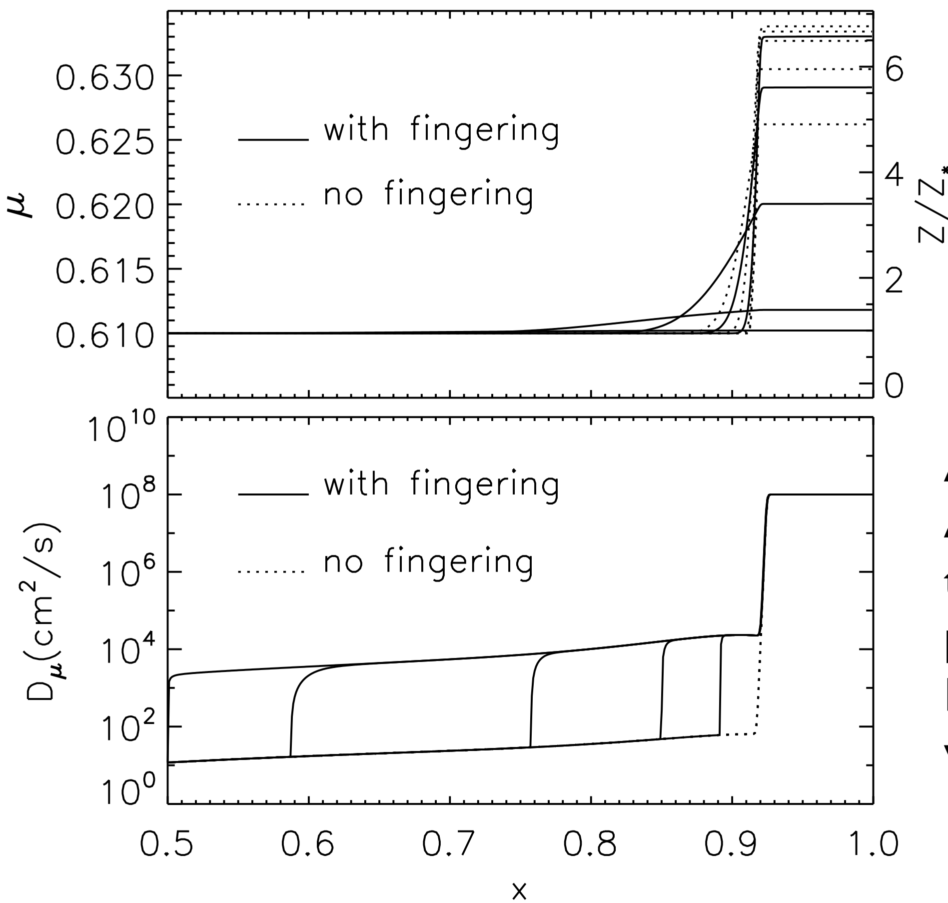


First 10,000 yrs after impact.: the added material renders the base of the convection zone Ledoux-unstable, and the convection zone deepens accordingly.

Garaud 2011

Mixing by fingering on early MS

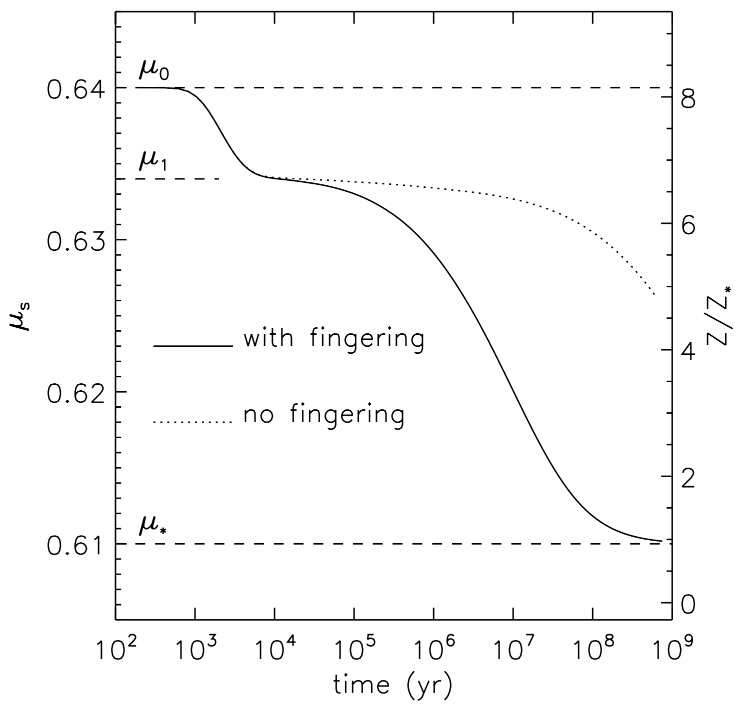
- Example of a $1M_J$ planet falling onto a $1.4M_{sun}$ star.



After 10^5, 10^6, 10^7, 10^8 and 10^9 years: A fingering region develops under the base of the convection zone, and progressively deepens with time. Most excess metallicity disappears within 10^8 years

Garaud 2011

Mixing by fingering on early MS

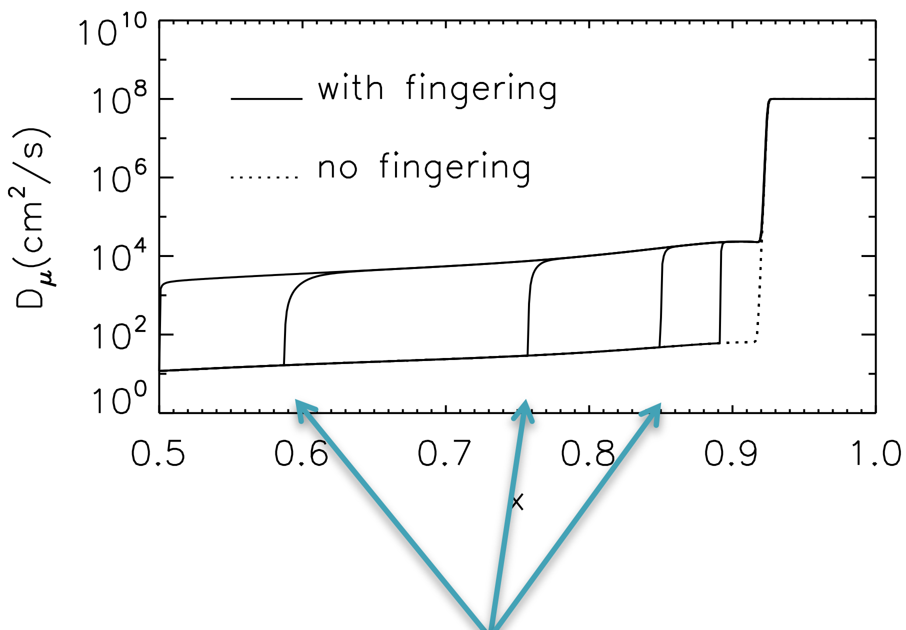


- Fingering convection strongly enhances dilution of metals into stellar interior.
- After about 100Myr, all evidence for planetary infall has disappeared.

Observed giant planet-stellar metallicity trend must be primordial

Garaud 2011

Mixing by fingering on early MS



Extent of mixed layer increases with time, eventually overlaps with Li-destruction region

- In addition, the fingering region extends all the way to the Li-burning region

Could explain observed (but controversial) claim of higher Li-depletion in planet-bearing stars.

Could explain Li dispersion in solar-type stars

Garaud 2011

Conclusion

Numerical experiments in astrophysical fluid dynamics *with proper follow-up analysis to extrapolate results to actual stellar conditions* can provide strong constraints on stellar evolution models.

CREDITS

Adrienne Traxler
Kevin Moore
Nic Brummell
Timour Radko
Stephan Stellmach
Justin Brown
Michael Medrano
Corentin Cadiou