Gravitational Instability in Planetesimal Disks

Bryce Bolin[1] (bolin@ifa.hawaii.edu), Yoram Lithwick[2], Margaret Pan[3], Hanno Rein[4], Yanqin Wu[5]

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[1]University of Hawaii, Institute for Astronomy, 2680 Woodlawn Dr, Honolulu, HI, 96822

[2]Deptartment of Physics and Astronomy, Northwestern University, 2145 Sheridan Rd., Evanston, IL 60208 & Center for Interdisciplinary Exploration and Research in Astrophysics (CIERA)

[3]Department of Astronomy and Astrophysics, University of Toronto, Toronto, ON M5S 3H4, Canada

[4]University of Toronto, Department of Environmental and Physical Sciences, Scarborough, 1265 Military Trail Toronto, Ontario Canada M1C 1A4

[5]NASA Goddard Space Flight Center, Exoplanets and Stellar Astrophysics Laboratory, Code 667, Greenbelt, MD 21230

ABSTRACT

Gravitational instability (GI) has been proposed as a method of forming giant gas planets enhanced by disk thermodynamics in a protoplanetary disk (Durisen *et al.* 2007; Boss 1997) and as a method of forming planetesimals through the focusing of boulders by the interaction between solids and gases in a turbulent circumstellar disk (Johansen *et al.* 2007; Youdin and Goodman 2005). GI is mediated through a gaseous circumstellar disk in each each of these scenarios. We explore the possibility of GI occurring in a planetesimal disk devoid of gas. In this regime, mutual collisions between planetesimals are required to dissipate their orbital shear and velocity dispersion enough for collapse to occur as described by the Toomre stability criterion (Goldreich and Lynden-Bell 1965; Toomre 1964). How frequent must collisions be between planetesimals in a gravitationally stable planetesimal disk for GI to occur? Are there collisional rates where GI is postponed indefinitely in an equilibrium state between gravitational stirring and collisional cooling? We present 3D shearing sheet simulations using the REBOUND N-body code with the symplectic epicyclic integrator (Rein and Liu (2012); **?**) in which the candidate collision rates are within a few orders of magnitude of the disk dynamical lifetime. Our simulations suggest that collisions rate directly controls disk cooling. The shape of the disk cooling curve is independent of the collision rate when scaled to the collision time.

Subject headings: Planetary formation; Planetesimal disks; Dynamics

Proposed Running Head:

Editorial correspondence to:

Bryce Bolin

Institute for Astronomy

University of Hawaii

2680 Woodlawn Drive

Honolulu, HI 96822

Phone: +1 808 294 6299

Fax: +1 808 988 2790

E-mail: bolin@ifa.hawaii.edu

1. Introduction

Observational evidence suggest that formation of terrestrial and gas-giants planets is a robust process (Petigura *et al.* 2013; Cassan *et al.* 2012). Gravitational instability (GI) (Boss 1997) and Core accretion (CA) (Pollack *et al.* 1996) have each been proposed as mechanism for planetary formation in protoplanetary disks. The later scenario relies on the presence of a sufficient amount of gas to be present in the protoplanetary disk as planetary embryos are grown from submicron-sized dust in order for the accretion of a gaseous envelope to occur. In the process of forming planetary embryos, the formation process must pass through the so-called $meter - size\ barrier$. As the dust coagulates into meter-sized bodies, the radial drag as a function of particle size approaches an inflection point where the particles have a tendency to collide with their host star within 100 orbits (Blum and Wurm 2008). Mutual collisions occur between meter-sized bodies at tens of meters per second due to different sized bodies having different radial velocities and also due to turbulent stirring (Dominik *et al.* 2007) which results in their destruction(Wurm *et al.* 2005).

The GI scenario in a protoplanetary disk also relies upon the presence of gas when considering the aerodynamic streaming instability as the source of gravitational collapse (Johansen *et al.* 2007; Youdin and Goodman 2005). Although the streaming instability forms kilometer-sized planetesimals from meter-sized objects rapidly enough so that they they do not drift into the central star and can withstand collisions, recent numerical experiments (Zsom *et al.* 2011; Bai and Stone 2010; Okuzumi *et al.* 2009) suggest that not enough meter-sized objects form for collapse to occur even under collisionless and laminar disk conditions. Shi and Chiang 2013 found that a disk with aerodynamically-coupled dust in the centimeter-size range requires up to four orders of magnitude higher density for collapse to occur than for a disk containing meter-sized objects. A physical example of a disk this dense with cm-size particles would require up to three times the mass of the *minimum mass solar nebula* (MMSN; Weidenschilling 1977; Hayashi 1981) and up to four times the solar metallicity. (Shariff and Cuzzi 2014) found that the collapse time is on the order of the dynamical time overcoming turbulent shear in that time.

Similar planetary formation processes such as GI could occur in disks devoid of gas such as planetesimal disks. The median age at which stars cease to accrete gas is 3 Myrs (Calvet *et al.* 2000). It is reasonable to assume that in some fraction of cases a protoplanetary disk could lose its gas envelope in $<$1 Myr period after the formation of the planetary nebula where the formation of gas giants is favorable to GI (Boss 1998) not forming any major gas planets due to the lack of gas. Studies by Cuzzi *et al.* 2010 and Morbidelli *et al.* 2009 found that the formation of 100 km-sized planetesimals occurred several million years after the formation of the solar nebular which precluded the formation of Jupiter in the case of the solar system (Sco). This suggest that that planetesimal formation can be incomplete well into the median accretion life time of circumstellar gas.

Gravitational instability in a planetesimal disk

How would the onset of collapse occur in a planetesimal disk devoid of gas? One advantage of a planetesimal disk over a gaseous disk is that there is do radial drag and cooling is provided through the inelastic collisions between particles assuming the particles are able to survive the collisions. Collapse should occur if $\Omega t_{col} \leq 1$ similar to a completely gaseous disk (Gammie 2001) where Ω is the average orbital frequency of the particles and t_{col} is the mean collisional time between particles. t_{col} is defined as

$$t_{col} = \frac{1}{3}\frac{\rho\,r}{\Sigma\,\Omega} \tag{1}$$

where ρ is the average particle density, r is the average particle radius, Σ is the local surface density of the disk. We use a value of 20 g cm^{-2} for Σ comparable to the mass of solids at 1 au in the MMSN and a value for Ω comparable to the value for particles at 1 au. ρ and r depend on the number of particles in our simulations which we define in section 2.1.

We define the stability criterion in therms of the dimensionless quantity

$$Q = \frac{\sigma_{vel}\,\Omega}{\pi\,G\,\Sigma} \tag{2}$$

where σ_{vel} is the velocity dispersion of particles in the disk, Ω is the average angular frequency of the particles, G is the gravitational constant and Σ is the local surface density of the disk (Toomre 1964; Goldreich and Lynden-Bell 1965). We use a value of 20 g cm^{-2} for Σ comparable to the mass of solids at 1 au in the MMSN. σ_{vel} is defined as the sum of the x, y and z coordinate velocity dispersion components

$$\sigma_{vel} = \sqrt{\sigma_{vx}^2 + \sigma_{vy}^2 + \sigma_{vz}^2} \tag{3}$$

The velocity dispersion of the k$^{\rm th}$ component, $\sigma_{v{\rm k}}$, is determined by an incremental algorithm for calculating variance (Knuth 1981)

$$\sigma_{v{\rm k}}^2 \;=\; \sum_{i=1}^{N} \frac{\sigma_{v{\rm k}_{i-1}}^2 \;+\; \left(v{\rm k}_i \;-\; \bar{v{\rm k}}_{i-1}\right)\;\left(v{\rm k}_i \;-\; \bar{v{\rm k}}_i\right)}{N} \qquad (4)$$

where vk is is the velocity component of the k$^{\rm th}$ coordinate and N is the total number of particles. The sample mean of the k$^{\rm th}$ coordinate's velocity component, $\bar{v{\rm k}}$, for all particles i is

$$\bar{v{\rm k}}_i \;=\; \bar{v{\rm k}}_{i-1} \;+\; \frac{\bar{v{\rm k}}_i \;-\; \bar{v{\rm k}}_{i-1}}{i} \qquad (5)$$

An exception to eqns. 4 and 5 is made for the y component in order to account for shear (**?**)

$$\sigma_{v{\rm y}}^2 \;=\; \sum_{i=1}^{N} \frac{\sigma_{v{\rm y}_{i-1}}^2 \;+\; \left(v{\rm y}_i \;+\frac{3}{2}\,\Omega\,{\rm x}_i \;-\; \bar{v{\rm y}}_{i-1}\right)\;\left(v{\rm y}_i \;+\frac{3}{2}\,\Omega\,{\rm x}_i \;-\; \bar{v{\rm y}}_i\right)}{N} \qquad (6)$$

$$\bar{v{\rm y}}_i \;=\; \bar{v{\rm y}}_{i-1} \;+\; \frac{\bar{v{\rm y}}_i \;+\frac{3}{2}\,\Omega\,{\rm x}_i -\; \bar{v{\rm y}}_{i-1}}{i} \qquad (7)$$

Values of Q $\gtrsim$ 1 represent cases where the random velocities and rotational shear of the particles in the disk overcome their de-stabilizing self-gravity (Binney and Tremaine 2008). Our hypothesis is if the collisional rate isn't high enough for $\Omega {\rm t}_{col} \;\leq\; 1$, the disk should cool to $Q \sim 1$ with transiently collapsing clumps re-exciting one another.

2. Methods

2.1. Code

We use the REBOUND code, a modular N-body code (Rein and Liu 2012) with a symplectic epicyclic integrator (Rein and Tremaine 2011) and tree codes for gravitational and collisional calculations (Barnes and Hut 1986).

3. Results

Our results are summarized in Fig. 1 and Fig. 2. We show that the Q vs t_{col} cooling curve is invariant with t_{col} for which there is collapse occurring as the curves approach $Q \sim 1$. As the collision time expands to $\gtrsim 30$ orbits, it becomes less apparent that collapse is occurring as Q approaches 1. For disks with longer collision times > 50 orbits, the two-body interaction between the particles dominates the dynamics within the disk and causes there to be a secular heating trend. 10,000 particles were used in these experiments which gives a relaxation time on the order of days. Increasing the number of particles used would alleviate the trend as this would increase the relaxation time Binney and Tremaine (2008).

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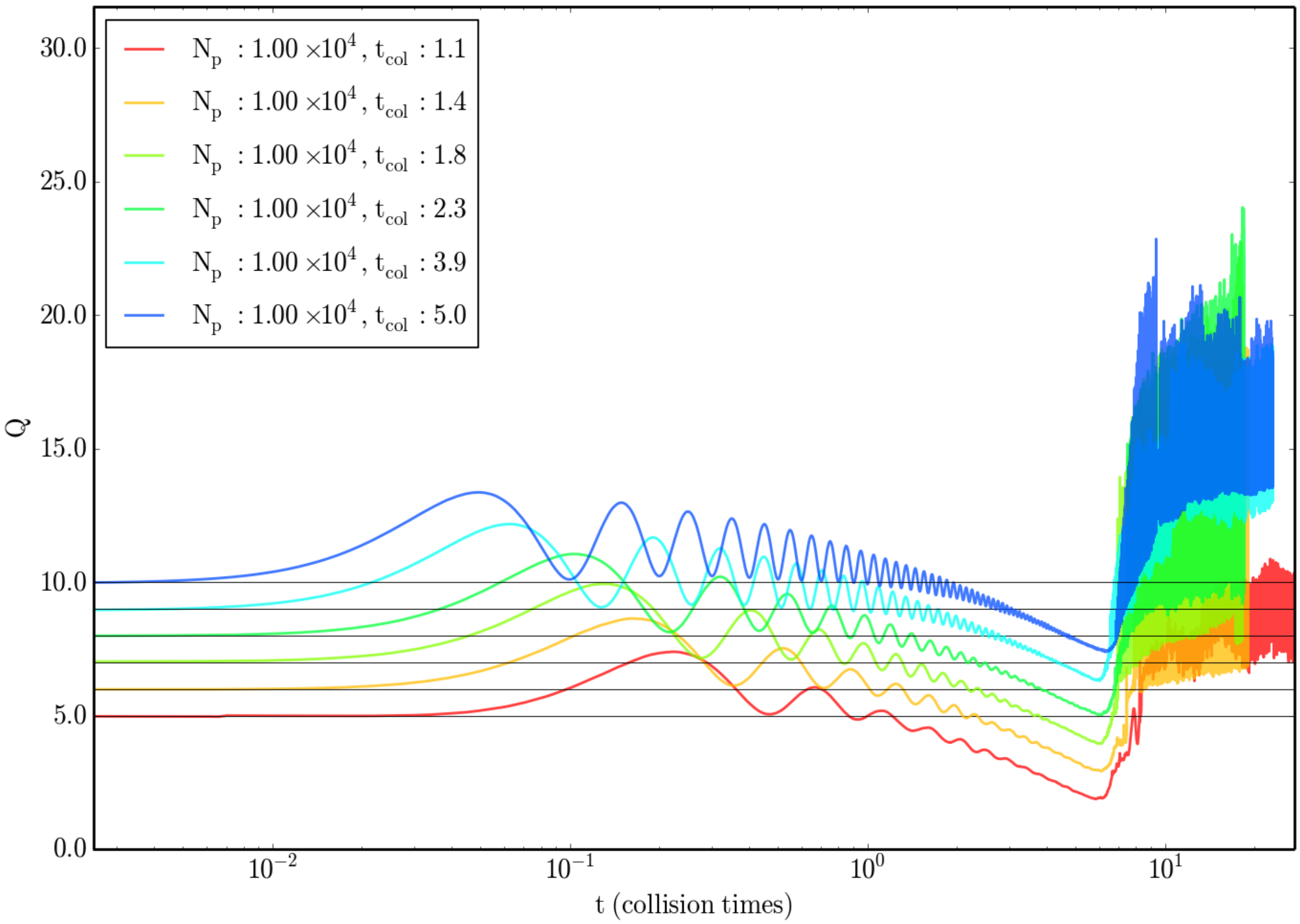

Fig. 1.— Q vs t_{col} profile for collision time < 5 orbits.

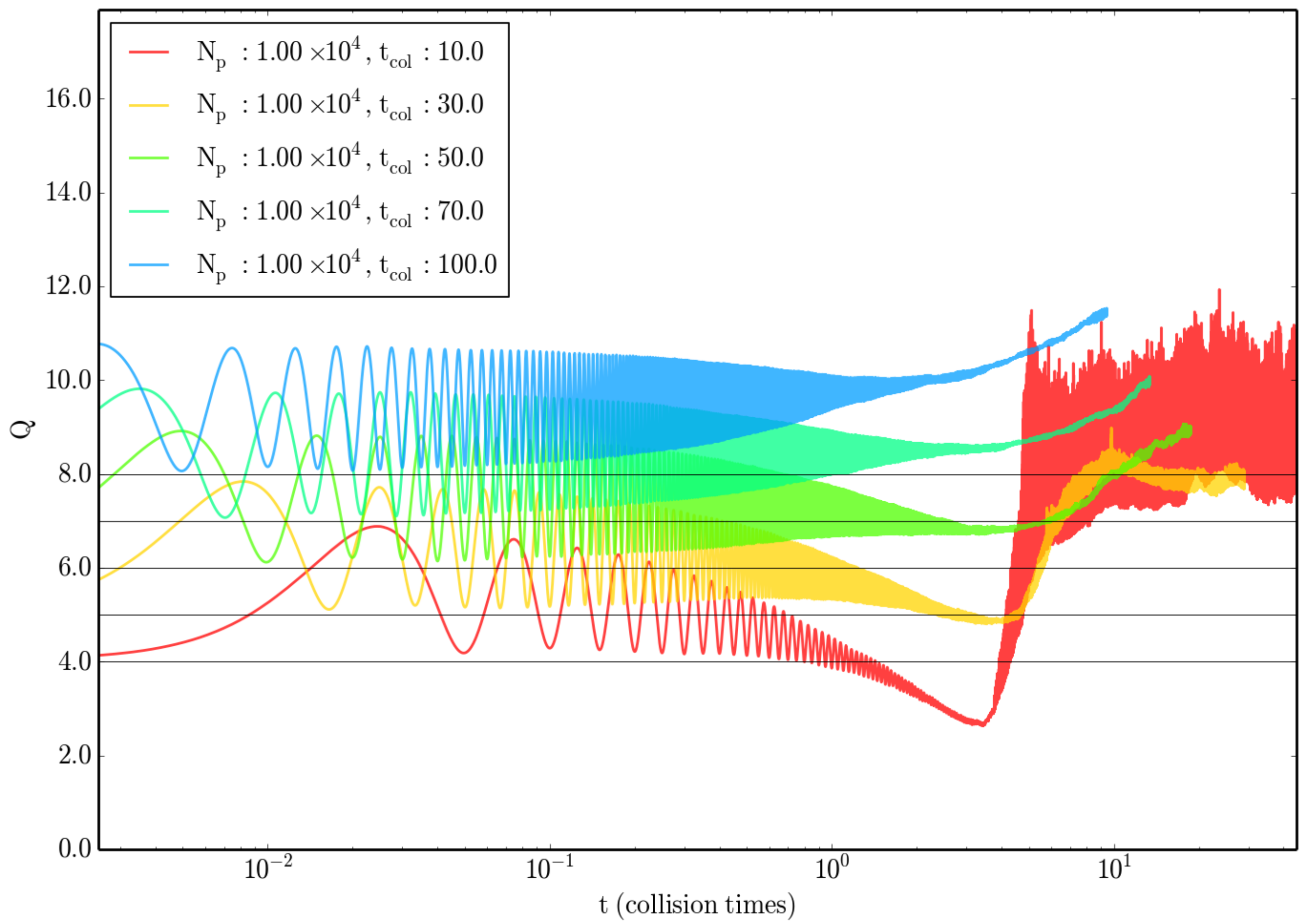

Fig. 2.— Q vs t_{col} profile for collision time < 100 orbits.