Limits on rigid precession of planetary systems

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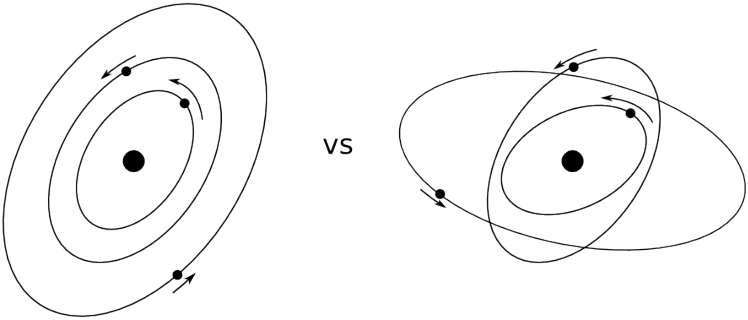
Advisors: Doug Lin & Man-Hoi Lee

6th August, 2014



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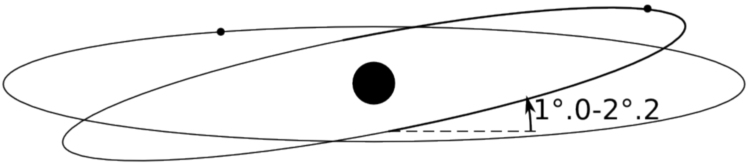

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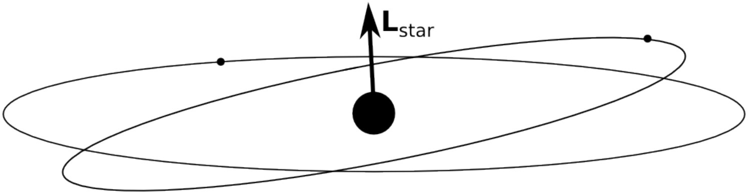

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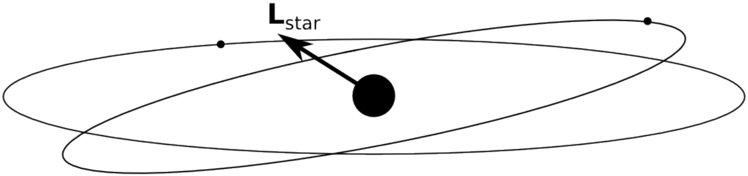

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- Kepler-25, Kepler-30, Kepler-50, Kepler-65, and KOI-94 are all obseved to be consistent with spin-orbit alignment (Albrecht et al. 2013 *ApJ*, 771:11).
- Two systems have definite, large misalignments between the stellar spin and the orbits: Kepler-56 (Huber et al. 2013 *Science*, 342:331H) and 55 Cnc (Bourrier & Hébard, arXiv:1406.6813).

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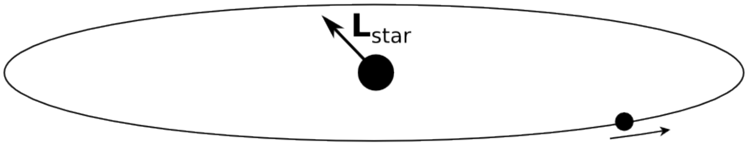

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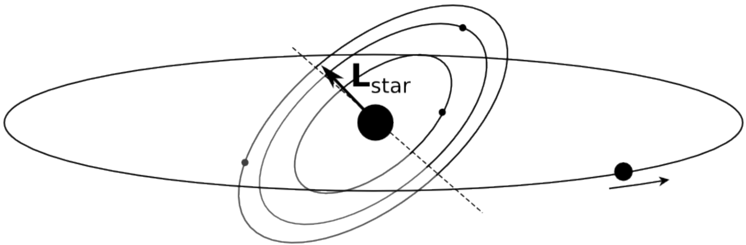

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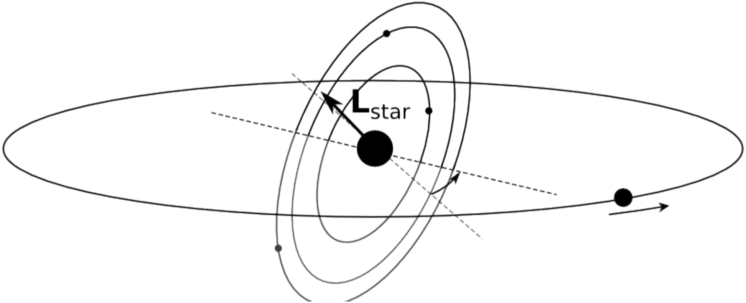

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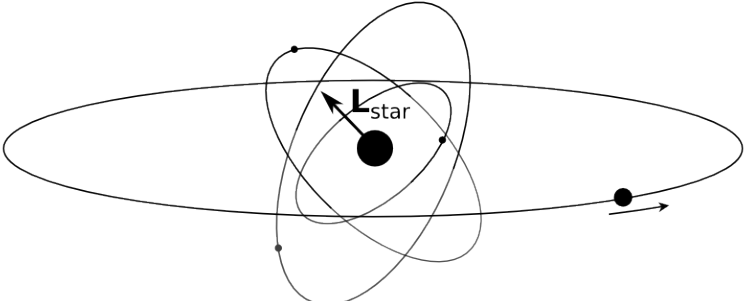

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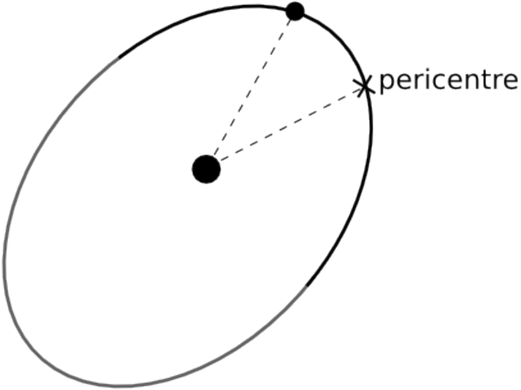
- Extend this to three or more planets.

Orbital Elements

- *a* - semi-major axis
- *e* - eccentricity



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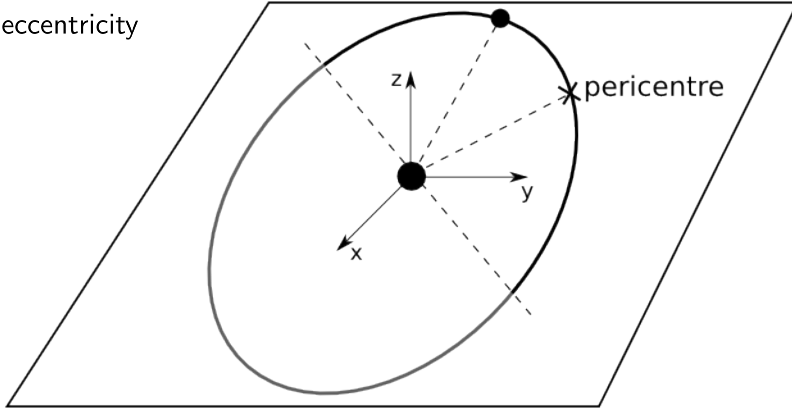

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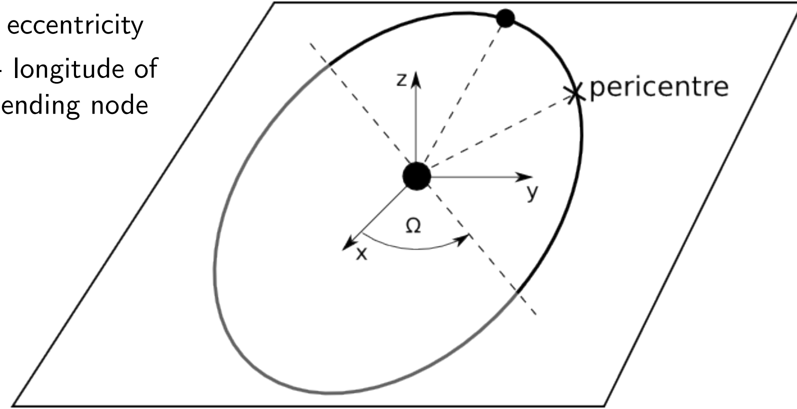

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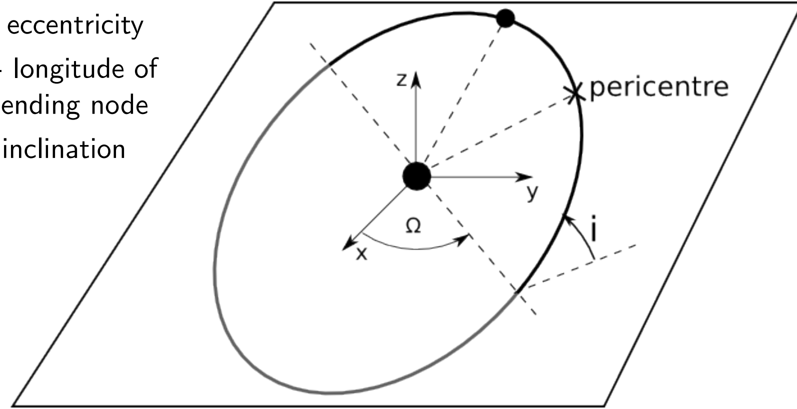

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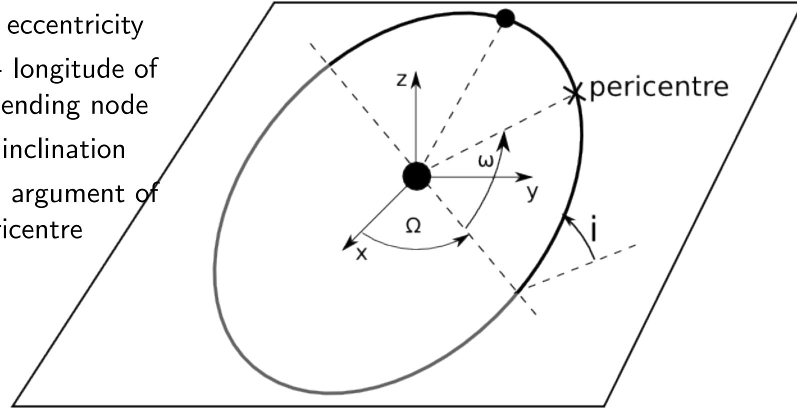

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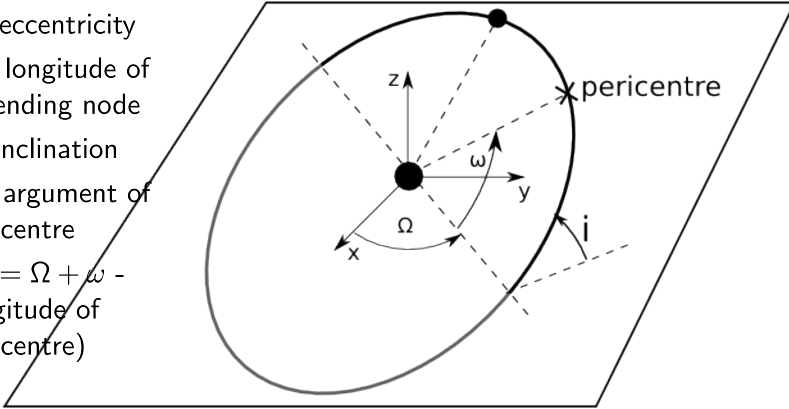

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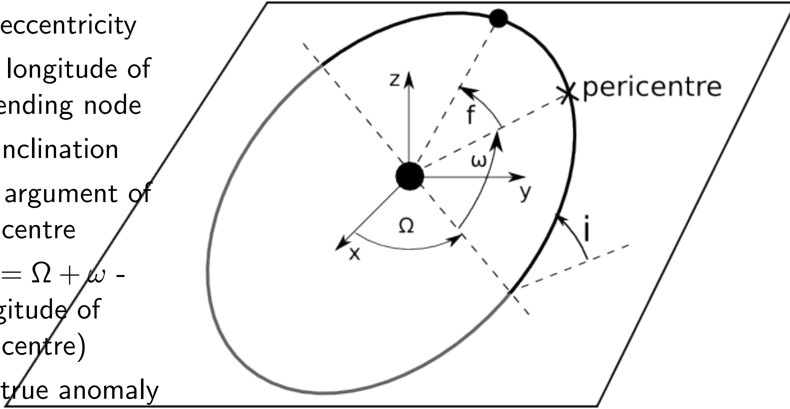

Laplace-Lagrange Theory

- Laplace-Lagrange low order secular theory describes the interactions between a pair of planets with small eccentricities and with small inclinations relative to the reference plane (physically, they must have low *mutual* inclinations).

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- Work with variables

$$\begin{aligned} h_j &= e_j \sin \varpi_j \qquad k_j = e_j \cos \varpi_j \\ p_j &= I_j \sin \Omega_j \qquad q_j = I_j \cos \Omega_j \end{aligned} \tag{1}$$

- The non-Keplarian part of the Hamiltonian becomes

$$\begin{aligned} H_{\rm int} =& \frac{1}{2} n_1 {a_1}^2 A_{11} \left({h_1}^2 + {k_1}^2\right) + \frac{1}{2} n_2 {a_2}^2 A_{22} \left({h_2}^2 + {k_2}^2\right) \\ &+ (n_1 {a_1}^2 A_{12} + n_2 {a_2}^2 A_{21}) \left(h_1 h_2 + k_1 k_2\right) + \frac{1}{2} n_1 {a_1}^2 B_{11} \left({p_1}^2 + {q_1}^2\right) \\ &+ \frac{1}{2} n_2 {a_2}^2 B_{22} ({p_2}^2 + {q_2}^2) + \left(n_1 {a_1}^2 B_{12} + n_2 {a_2}^2 B_{21}\right) \left(p_1 p_2 + q_1 q_2\right) \end{aligned} \tag{2}$$

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- The (e_j, ϖ_j) and (I_j, Ω_j) problems are decoupled from one another. Will focus on the latter from now on.
- The equations of motion are

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} p_1 \\ p_2 \end{pmatrix} = \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix}\begin{pmatrix} q_1 \\ q_2 \end{pmatrix}, \quad \frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} q_1 \\ q_2 \end{pmatrix} = -\begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix}\begin{pmatrix} p_1 \\ p_2 \end{pmatrix}$$

- The non-zero eigenvalue of this matrix is the precession frequency of the two planet system.

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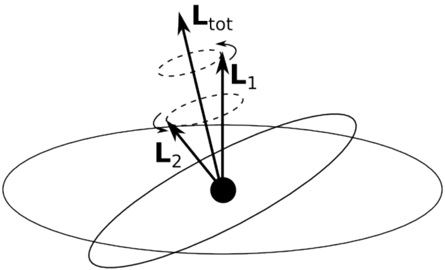

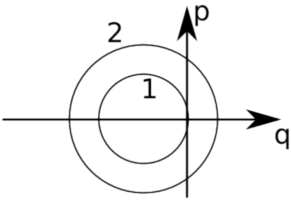

- The angular momentum vectors precess at matching rates around the (constant) total angular momentum vector.
- In q, p space the paths traced out are concentric circles.
- Precession frequency depends only on the masses and semi-major axes.

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- The fact that the perturber is very distant allows us to discard higher order terms in its multipole expansion, and retain only the quadrupole part of its potential. Averaging over the mean anomalies of both bodies gives an interaction energy of

$$H_{\mathrm{int}} = \frac{m_1 m_2 {a_1}^2}{8 {b_2}^3} \left(2 + 3{e_1}^2 - \left(3 + 12{e_1}^2 - 15{e_1}^2 \cos^2 \omega\right) \sin^2 I_{\mathrm{mut}}\right)$$

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- If $I \gtrsim 39^\circ.23$, Lidov-Kozai oscillations result: both ${e_1}^2$ and $I_{\rm mut}$ become large periodically, with $\sqrt{1-{e_1}^2}\cos I_{\rm mut}$ remaining constant.

Lidov-Kozai Oscillations

- The timescale for Lidov-Kozai oscillations due to distant, massive body 2 on body 1 is

$$\tau_{\mathrm{Koz}} = \frac{2}{3\pi} \frac{M_* + m_1 + m_2}{m_2} \frac{{P_2}^2}{P_1} \left(1 - {e_2}^2\right)^{3/2}$$

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- We shall now test this claim.

2-Planet Numerical Experiments

- We perform a series of numerical experiments using the Mercury symplectic integrator code. Our reference system is

Planet	$M/M_\odot$	a/au	e	ϖ	I	Ω
Central star	1					
b	$3 \times 10^{-5}\,(= 10M_\oplus)$	0.1	0	0	0	0
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- We'll diagnose systems as 'aligned' or 'misaligned' by measuring for what fraction of the simulation time the two planets have a mutual inclination of less than 5°.

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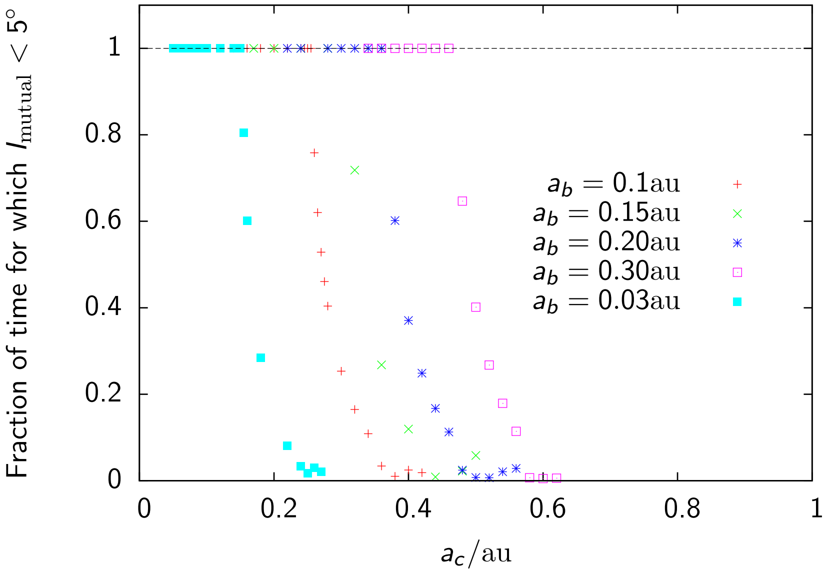

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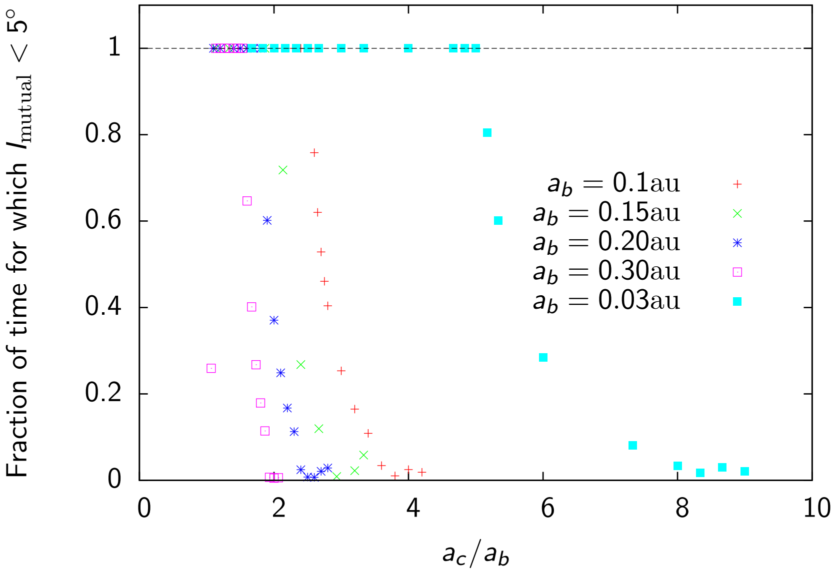

The timescale criterion

- Having established that changing a_b changes significantly the value of a_c (and of a_c/a_b) at which the transition from coplanar to non-coplanar behaviour occurs, let's go back and look at Takeda et al's criterion: $\tau_{\mathrm{Koz},c} \gtrsim C\tau_{\Omega bc}$.

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- Using expressions for the Laplace-Lagrange precession of the ascending node, and the Kozai timescale for the interaction of the distant perturber with body b, we can rewrite this as

$$6\pi C \frac{m_B}{{a_B}^3\left(1-{e_B}^2\right)^{3/2}} \lesssim \frac{1}{{a_b}^3} b_{3/2}^{(1)}\left(\frac{a_b}{a_c}\right)\left(m_c\left(\frac{a_b}{a_c}\right)^{7/2}+m_b\left(\frac{a_b}{a_c}\right)^4\right)$$

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- So, for a given value of a_b, can estimate $a_{c,\mathrm{crit}}$ from the above expression.

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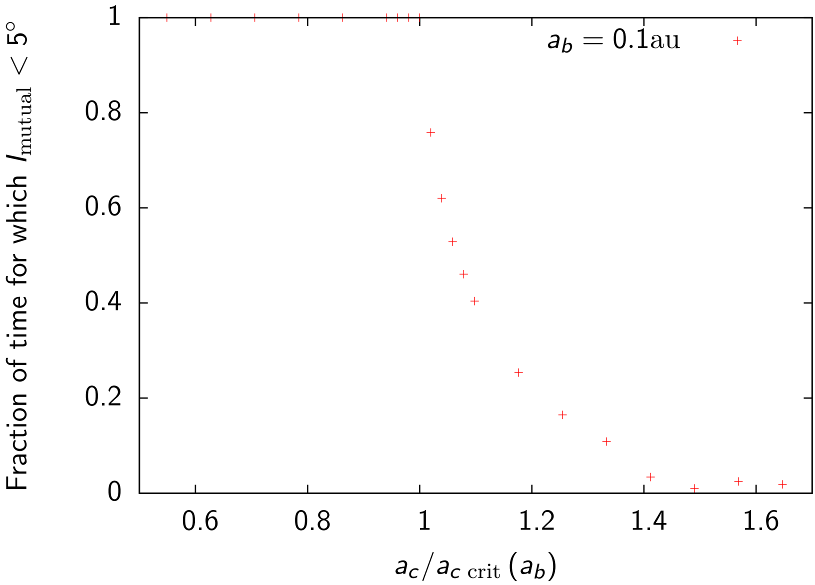

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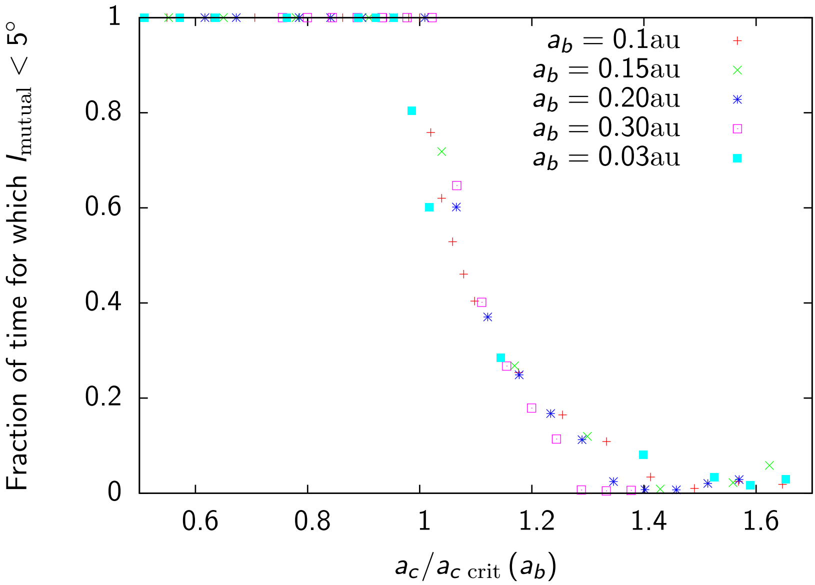

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- What else can we vary about our nominal system?

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- We'll look at varying m_b, m_c, and I_B.

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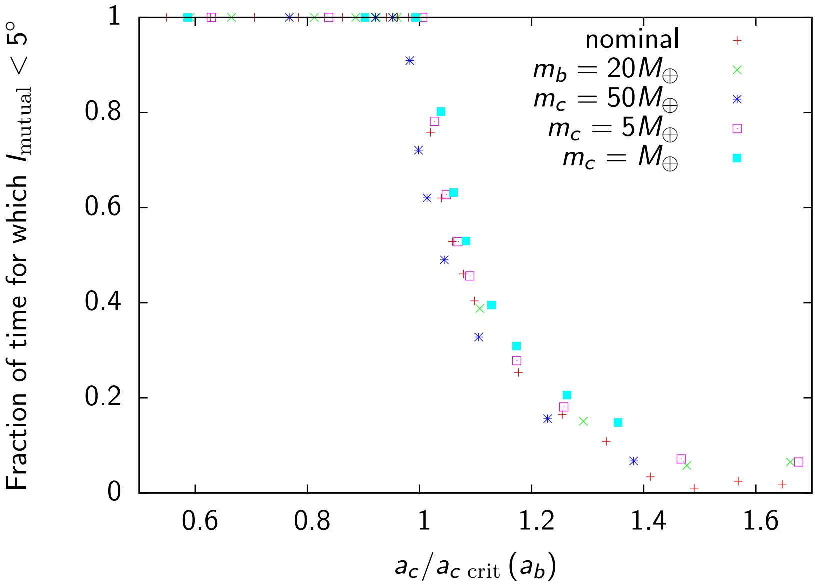

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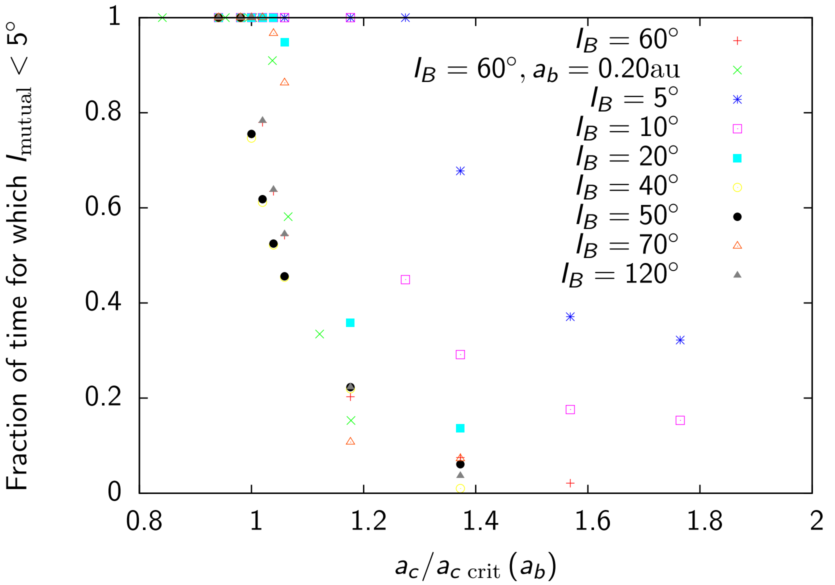

Three planet investigations

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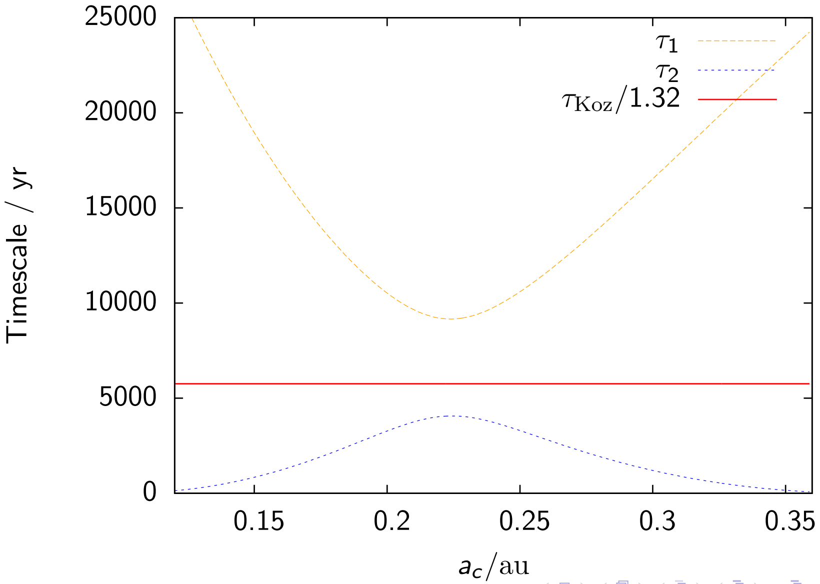
- Do I need $\tau_{\rm Koz} > \tau_1 \geq \tau_2$? Or is $\tau_1 > \tau_{\rm Koz} > \tau_2$ sufficient?

Three planet investigations

- The case of three inner planets plus a perturber is more complex. The fastest Kozai timescale is still that of the outer planet. However, there are now two timescales from the Laplace-Lagrange secular theory. Call them τ_1 and τ_2, with $\tau_1 \geq \tau_2$.
- Do I need $\tau_{\mathrm{Koz}} > \tau_1 \geq \tau_2$? Or is $\tau_1 > \tau_{\mathrm{Koz}} > \tau_2$ sufficient?
- Certainly there are cases where the second planet ‘couples’ the inner and outer planet more strongly.

Three planet investigations

$a_b = 0.1, a_c = 0.38$



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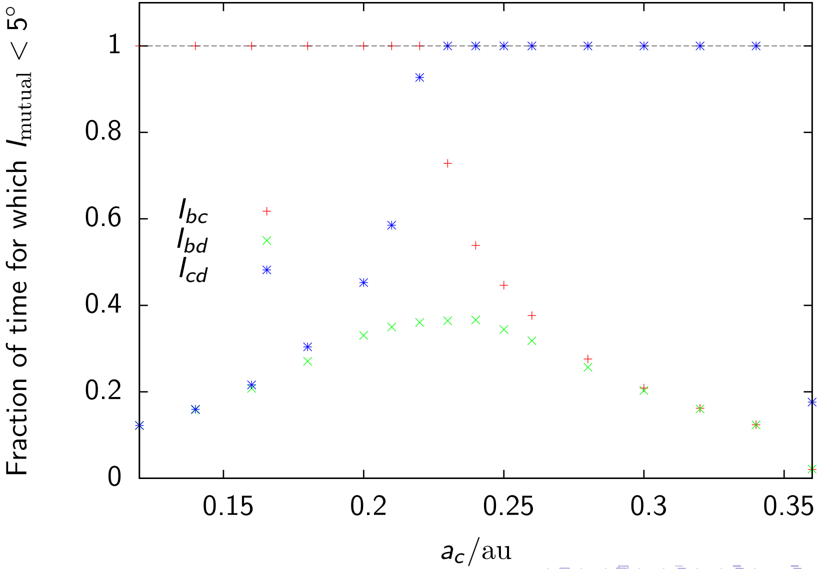

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- Mutual inclination may not be the best variable to work with here. Maximum inclination of a planet from the invariable plane may be better.
- Haven't yet looked at a case where $\tau_{\rm Koz} > \tau_1 \geq \tau_2$. Will there be a significant change in behaviour, or does the multiple Laplace-Lagrange timescales 'soften' the transition compared to the two planet case?

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- The three-planet case is more complicated, and is still a work in progress.
- It looks like there's certainly a change in behaviour when the Kozai timescale passes the shorter Laplace-Lagrange timescale.

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- If a system is observed to be coplanar, but does not fit the criteria shown here, can we infer the existence of an additional planet 'coupling' the system together.