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ISIMA report on "Kinematic signatures of tidal field in globular clusters"

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ABSTRACT

We present a large set of N-body simulations to study the evolution of tidally perturbed clusters. In our survey, we explored the effects of the tidal field on the dynamical evolution of star clusters by varying the number of stars, the orbital properties, the galactic potential and the initial concentration of the models.

Since the astrometric mission GAIA will soon allow us to explore the phase space of several Galactic globular clusters, we focused our analysis on the study of the kinematical properties of the models, especially in their outer parts. In particular, by studying the time evolution of the mean velocity vector, the velocity dispersion tensor, and the anisotropy parameter, we confirmed the existence of a significant degree of tangential anisotropy and of a moderate rotation in the outer parts of the clusters. We also started to explore the extent to which these properties could be explained in terms of the behaviour of the population of potential escapers and by other aspects of the dynamical evolution of the systems.

In addition, we found an unexpected instability in star clusters with a King profile $W_0 = 5.0$ and in Roche filling conditions. Such instability seems to be caused by a substantial and rapid expansion of the core of the cluster, associated with the escape of the neutron stars, as a result of the presence of an initial kick velocity of these stars.

Key words: methods: numerical, stellar dynamics – globular clusters: general

1 INTRODUCTION

It is well known that the tidal field associated with the host galaxy affects significantly the evolution of a globular clusters (GC). The primary goal of this study is to investigate the effects of such an external tidal field on the kinematics of globular clusters. First of all, the main effect of the tidal field on a GC (Heggie & Hut 2003) is the distortion of the shape of GC, which becomes a triaxial configuration, limited by a critical equipotential surface ("critical Hill's surface"). During this state the cluster may start to lose stars from the two lagrangian points L1 and L2. These points are at distance r_J from the center of the cluster, where

$$ {r_J}^3 = \frac{GM_{cl}}{\Omega^2 - \frac{d^2\phi}{dr^2}} \qquad \text{(King 1962)} \tag{1} $$

is the tidal radius (or Jacobi radius), which depends on the mass of the cluster M_{cl}, on the angular velocity Ω and on the potential of the galaxy ϕ where the cluster is moving.
A second effect is due to tidal heating (Heggie & Hut 2003), this occurs only in a non-static tidal field, such as for an eccentric orbit or an orbit which is not coplanar with the disk of the galaxy (called disk shocking effect). These effects may affect significantly the structural properties of a GC, especially its tidal radius. In the case of a cluster on an eccentric orbit, the tidal radius has its maximum value at the apocenter and its minimum value at the pericenter. The number of escaping stars increases when the tidal radius decreases. The magnitude of this effect strictly depends on the orbital properties and on initial conditions of the stellar system, however it can be very strong and speed up the dissolution of GCs.
The deviations from spherical symmetry induced by the tidal stretching can be, in principle, measured from the flattening of the isophotes of a globular cluster, especially in the outer parts. Recent measurements of the ellipticities of Galactic GCs performed by Chen & Chen (2010) seem to show a preferential alignment of the major axes of these systems with the direction of the Galactic Center. Such an evidence can be easily interpreted in terms of the flattening induced by the tidal field, but more accurate measurements are needed to confirm this result. In addition, from the kinematical point of view, it has been observed in globular cluster a flattening in the velocity dispersion by Drukier et al. (1998) for M15 and by Da Costa (2012) for $\omega\ Cen$; as shown in their papers, the external part of these globular clusters does not follow the keplerian trend. Moreover, van de Ven et al. (2006) found evidence of tangentiality in the outer part of $\omega\ Cen$, by studying its

three-dimensional velocity dispersion tensor (derived from a combination of proper motions and radial velocity data).

The ongoing astrometric mission GAIA will soon allow us to explore, for the first time and with unprecedented accuracy, many structural and kinematical properties of several Galactic GCs. In particular, GAIA will provide proper motions, temperature, mass, age, and elemental composition of stars between $5.7\ mag$ and $20\ mag$, and also the line of sight velocity for stars brighter than $17\ mag$ with precision between $1\ km/s$ (for $V = 11.5\ mag$) and $30\ km/s$ (for $V = 17.5\ mag$) (Jordan 2008). Together with the radial velocities from state-of-the-art spectroscopic surveys (e.g. the GAIA-ESO survey), the data from GAIA will "unlock" the 5-dimensional phase space of these systems.

Driven by these motivations, in the present paper we explore the main kinematic properties of GCs in an external tidal field by studying the phase space properties of a series of N-body simulations of tidally perturbed stellar systems, in order to find specific signatures that may be recognized in the GAIA data.

2 DESCRIPTION OF THE RUNS

We simulate the evolution of the cluster as in Baumgardt & Makino (2003), but by using $NBODY6$ (Aarseth 2003; Nitadori & Aarseth 2012). We have performed a survey of simulations in a non-rotating frame, using a number of particles between $N = 8192$ and $N = 131072$, a Kroupa IMF, with the mass of the stars between $0.1\ M_{\odot}$ and $15\ M_{\odot}$, a theoretical mean mass $\langle m \rangle = 0.547\ M_{\odot}$, and metallicity $Z = 0.001$.

In our simulations, the cluster is in circular orbit or in elliptical orbit with eccentricity $\varepsilon = 0.5$, in a logarithmic potential $\phi = {V_c}^2 \ln(r)$, where ${V_c}^2$ is the square circular velocity; except for one case, which used a Keplerian Galactic potential. For the majority of our runs, we have used a King (1966) model with concentration of $W_0 = 5.0$ as initial condition. The clusters start at $(8.5, 0, 0)\ kpc$, with an initial velocity $V_{cl} = 220\ km/s$ (in the circular case); in the elliptical case these are the spatial coordinates of the apogalacticon. The initial conditions have been generated using McLuster (Küpper et al. 2011).

Not all simulations were carried out until complete cluster dissolution. The tidal radius of the cluster was determined iteratively by first assuming that all stars are still in the cluster and calculating the tidal radius. In a second step, we calculated the mass of all stars inside r_J and used it to obtain a new estimate for r_J. This method was repeated until convergence to a stable solution.

A few additional simulations have been performed by changing the initial concentration of the King profile ($W_0 = 7.0$), the potential of the galaxy (keplerian potential), and by introducing an initial rotation (pro-grade or retro-grade respect to the motion of the cluster around the galaxy). The properties of the simulations are presented in Table 1; where t_{end} is defined as the time when 95% of the mass was lost from a cluster; because some simulation were stopped before this point, we show the fraction of the bound mass and the time when the simulation stops.

Table 1. N-body simulation properties

$Model$	N	N_{sim}	W_0	Orbit	M_0 $[M_\odot]$	r_h $[pc]$	$r_J(t=0\ Myr)$ $[pc]$	t_{end} $[Myr]$	$\frac{M}{M_0}\Big\vert_{end}$	Kick velocity NS	Rotation	Potential
8k	8192	1	5.0	Circ	4507.9	4.62	24.74	2309	0.05	Yes	-	logarithmic
8k changing ICs	8192	1	5.0	Circ	4565.6	4.63	24.84	2356	0.05	Yes	-	logarithmic
16k	16384	1	5.0	Circ	9001.8	5.80	31.15	2665	0.05	Yes	-	logarithmic
16k changing ICs	16384	1	5.0	Circ	9047.6	5.80	31.20	2758	0.05	Yes	-	logarithmic
128k	131072	1	5.0	Circ	71787.3	11.63	62.24	5413	0.05	Yes	-	logarithmic
128k changing ICs	131072	1	5.0	Circ	70926.9	11.63	61.99	4206	0.25	Yes	-	logarithmic
8k NoKick	8192	2	5.0	Circ	4507.9	4.62	24.74	3921	0.05	No	-	logarithmic
8k NoKick changing ICs	8192	1	5.0	Circ	4565.6	4.63	24.84	4086	0.05	No	-	logarithmic
16k NoKick	16384	2	5.0	Circ	9001.8	5.80	31.15	6082	0.05	No	-	logarithmic
16k NoKick changing ICs	16384	1	5.0	Circ	9047.6	5.80	31.20	6094	0.05	No	-	logarithmic
32k NoKick	32768	1	5.0	Circ	18189.2	7.34	39.38	8837	0.07	No	-	logarithmic
64k NoKick	65536	1	5.0	Circ	36103.7	9.24	49.49	14411	0.05	No	-	logarithmic
128k NoKick	131072	1	5.0	Circ	71787.3	11.63	62.24	17622	0.13	No	-	logarithmic
128k EccA	131072	1	5.0	Ecc	71662.26	4.13	95.69	6361	0.19	Yes	-	logarithmic
128k EccP	131072	1	5.0	Ecc	71662.26	4.13	22.12	6284	0.19	Yes	-	logarithmic
128k NoKick Pro	131072	1	5.0	Circ	71787.3	10.66	62.24	1324	0.54	No	Prograde	logarithmic
128k NoKick Re	131072	1	5.0	Circ	71787.3	10.66	62.24	1423	0.59	No	Retrograde	logarithmic
128k W7	131072	1	7.0	Circ	71480.1	7.23	62.15	10308	0.20	Yes	-	logarithmic
128k NoKick Kepl	131072	1	5.0	Circ	71623.7	10.15	62.19	12496	0.10	No	-	keplerian

For the eccentric orbit $\varepsilon = 0.5$; $t = t_{end}$ when $\frac{M}{M_0}\Big|_{end} = 0.05$, if $\frac{M}{M_0}\Big|_{end} > 0.05$ then $t = t_{end}$ is when our simulation stopped; when $N_{sim} > 1$ we are evolving the same initial conditions (ICs).

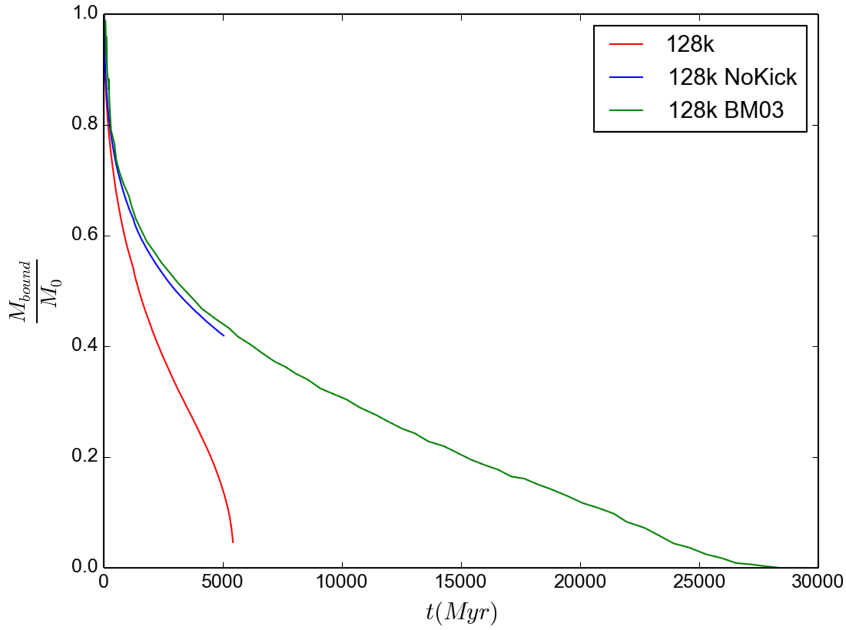

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Figure 1. The plot shows a comparison between the evolution of a star cluster in the presence (red line; model $128k$) and in the absence (blue line; model $128k$ $NoKick$) of an initial kick velocity of the NS. For reference, the corresponding model studied by Baumgardt & Makino (2003) is also presented (green line).

3 INSTABILITY

The rapid escape of neutron stars is a well-known effect determined by their initial kick velocity but, surprisingly, we found that, in the context of our simulations, it creates a fatal instability of the clusters. For instance, our model with $N = 131072$ and an initial concentration of $W_0 = 5.0$ in Roche filling conditions is completely dissolved in about 5 Gyr, while, from their corresponding simulation, Baumgardt & Makino (2003) found a dissolution time of about 25 Gyr, in the absence of initial kick velocity for the neutron stars (NS). In their case, more than 95% of the NS remain inside the cluster, while in our simulation only 5% remain bound. Fig. 1 shows a comparison between the evolution of the bound mass of the cluster in our simulations and the corresponding one in Baumgardt & Makino (2003); it is clear that, when we include a kick velocity for NS, there is a very significant decrease of the dissolution time.

Fig. 2 shows a similar comparison plot, but for simulations with $N = 16k$. In order to evaluate the magnitude of the stochastic effects associated with the generation of the initial conditions and with the direct N-body calculations, we performed several additional simulations by changing the seed used in the sampling of the initial conditions or by evolving the same initial conditions in

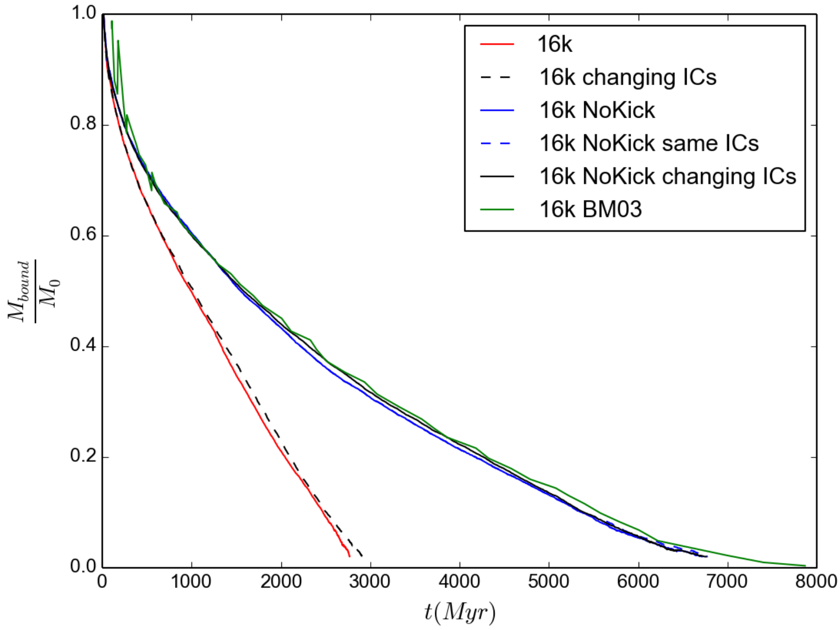


Figure 2. As in Fig. 1, but for simulations with $N = 16k$. Models with initial kick velocity are denoted by solid red (model $16k$) and dashed black (model $16k$ *changing ICs*) lines; models without kick velocity are denoted by solid black (model $16k$ $NoKick$ $changing$ ICs), solid blue (model $16k$ $NoKick$), and dashed blue (model $16k$ $NoKick$) lines. As in Fig. 1, the green line represent the corresponding models from Baumgardt & Makino (2003). We also performed some additional simulations with different realizations of the same initial conditions or by evolving the same realization, in order to assess the magnitude of the stochastic effects of the N-body calculations.

independent runs.

4 KINEMATICS

Despite the interesting result presented in the previous Section, the main goal of this project is to study the kinematics of the stars in the clusters to be able to compare our data with GAIA.
First of all, we need to define some quantities to describe the kinematic properties of each cluster. We will use the anisotropy parameters:

$$\tilde{\beta} = 1 - \frac{\langle {v_t}^2 \rangle}{2 \langle {v_r}^2 \rangle} \qquad \text{(as in Baumgardt \& Makino 2003)} \tag{2}$$

and

$$\beta = 1 - \frac{{\sigma_t}^2}{2{\sigma_r}^2} \tag{3}$$

where $\langle {v_t}^2 \rangle$ (${\sigma_t}^2$) and $\langle {v_r}^2 \rangle$ (${\sigma_r}^2$) denote the mean square tangential and radial velocity components, and the tangential and radial diagonal components of the velocity dispersion tensor, respectively. To be noted that ${\sigma_t}^2 = {\sigma_\theta}^2 + {\sigma_\phi}^2$, and that ${\sigma_\phi}^2 = \langle {v_\phi}^2 \rangle - \langle v_\phi \rangle^2$, ${\sigma_\theta}^2 = \langle {v_\theta}^2 \rangle - \langle v_\theta \rangle^2$, and

${\sigma_r}^2 = \langle {v_r}^2 \rangle - \langle v_r \rangle^2$. The difference between $\tilde{\beta}$ and β is given by the contribution of the components of the mean velocity vector. The β and $\tilde{\beta}$ parameters allow us to quantify the degree of anisotropy in the velocity space of a stellar systems. If all orbits are circular, $\sigma_r = 0$ and $\beta = -\infty$; if the system is isotropic in the velocity space, $\beta = 0$; if all orbits are perfectly radial, $\sigma_\theta = \sigma_\phi = 0$ and $\beta = 1$ (Binney & Tremaine 1987). As a result, the systems with $\beta > 0$ are "radially biased", while those with $\beta < 0$ are "tangentially biased". In our case, in the outer part of our cluster, we have a radially biased cluster during the first hundred Myr, whereas after this we have a tangentially biased cluster, see Fig. 3(a) and Fig. 3(b).

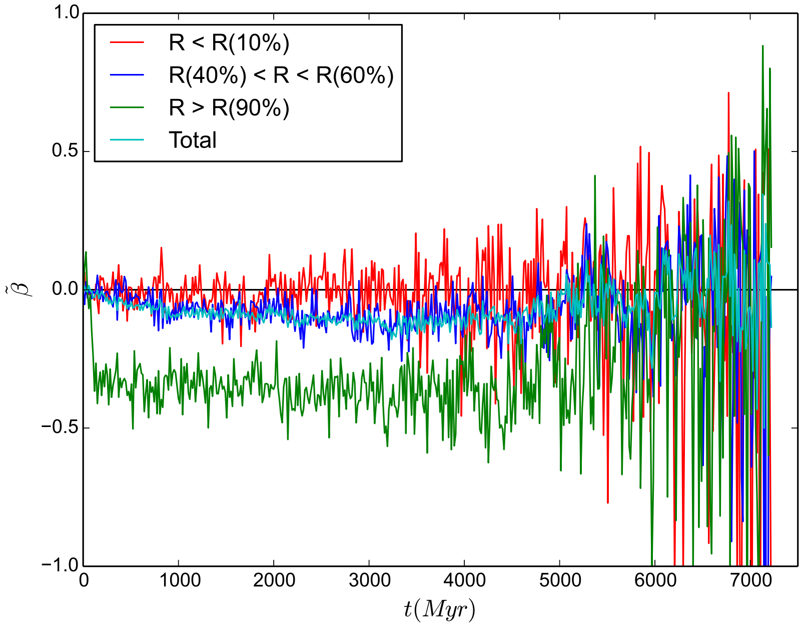
About the rotation of the cluster, we know that the cluster is orbiting around the galaxy on the x-y plane, therefore we expect to see a sign of rotation inside the cluster in the v_ϕ component, see Fig. 4(a), while as expected the v_θ component is fluctuating around zero, see Fig. 4(b).

The preliminary analysis of some of the simulations performed in this study confirms the existence of significant tangential anisotropy and of a weak rotation in the outer parts of the clusters, in agreement with the analysis performed by Baumgardt & Makino (2003). At the present stage of the project, the physical origin of these properties is only partially understood. Baumgardt & Makino (2003) state that if a cluster fills its Roche lobe and starts losing mass, there is a preferential loss of stars on radial orbits induced by the external tidal field at large radii, where tangential anisotropy in velocity space is thus established (see also Takahashi & Lee 2000). In addition, they motivate the existence of a weak rotation as a result of the fact that, in the outer parts of the cluster, most of the stars are counter-rotating, since retro-grade orbits are more stable against escape than pro-grade orbits (see Keenan & Innanen 1975). In the near future we would like to test these interpretations with our simulations, with particular attention to the time scale of these processes and to the role played by the frame of reference.

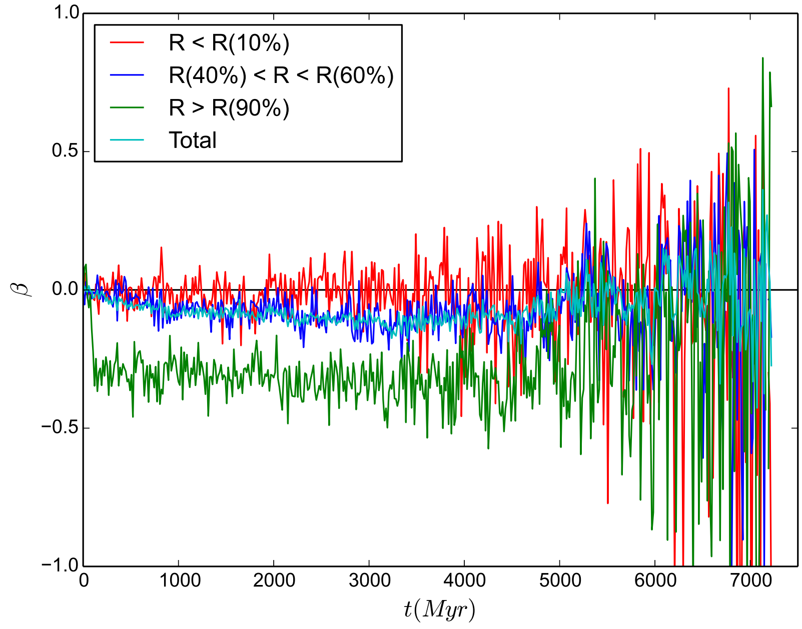
Finally, we studied the time evolution of the trace of the velocity dispersion tensor:

$$\sigma = \sqrt{\frac{1}{3}\left({\sigma_r}^2 + {\sigma_\phi}^2 + {\sigma_\theta}^2\right)} \tag{4}$$

as shown in Fig. 5. As we can see, the lines that represent the outer part of the cluster (purple, green and yellow) they go closer to each other and then they merge at about $5.6\ Gyr$, this mean that if we plot the velocity dispersion as a function of radius we will see a flattening (because their are reaching the same value). To see better this effect we can plot the ratio between the velocity dispersion in each lagrangian shell and the total velocity dispersion, see Fig. 6.

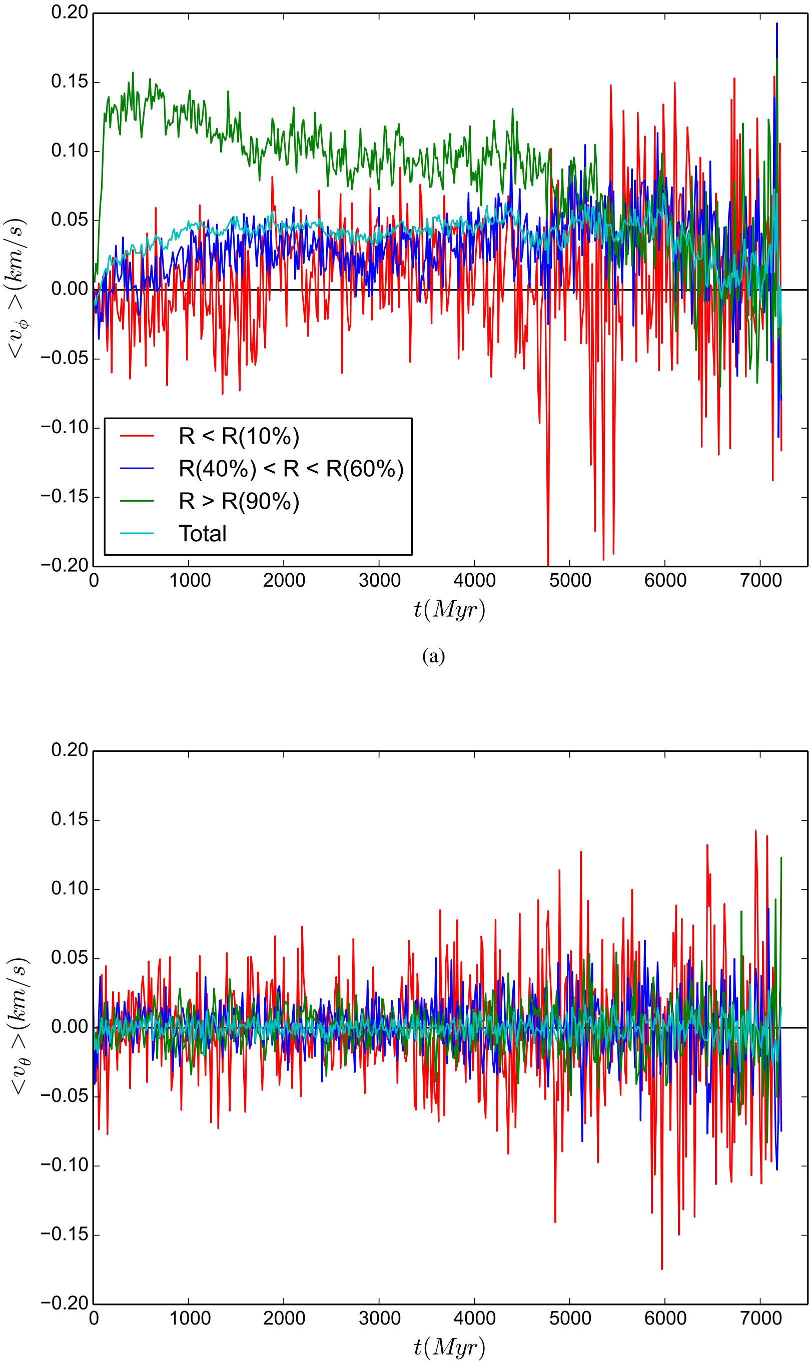


(a)



(b)

Figure 3. Anisotropy parameters of model $16k\ NoKick$ in selected lagrangian shells, containing 10%, 40%-60%, 90%-100% and 100% of the mass inside the tidal radius; (a) The $\tilde{\beta}$ parameter shows a clear evidence of tangentiality in the outer parts of the model (green line); (b) The β parameter confirms a tangentially biased cluster in the outer parts (green line).



(a)

(b)

Figure 4. Polar and azimuthal components of the mean velocty vector of model $16k\ NoKick$ in selected lagrangian shells, containing 10%, 40%-60%, 90%-100% and 100% of the mass inside the tidal radius; (a) The rotation curve ($\langle v_\phi \rangle$) shows a clear evidence of moderate rotation in the outer parts of the model; (b) The time evolution of $\langle v_\theta \rangle$ confirms the absence of any average motion in this velocity component.

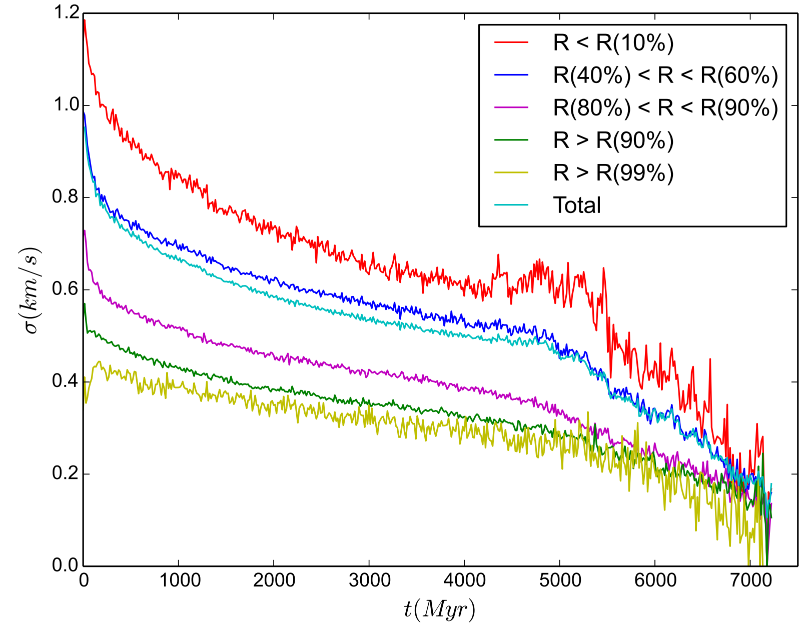


Figure 5. Trace of the velocity dispersion tensor of model $16k\ NoKick$, calculated in selected lagrangian shells, containing 10%, 40%-60%, 80%-90%, 90%-100%, 99%-100% and 100% of the mass inside the tidal radius

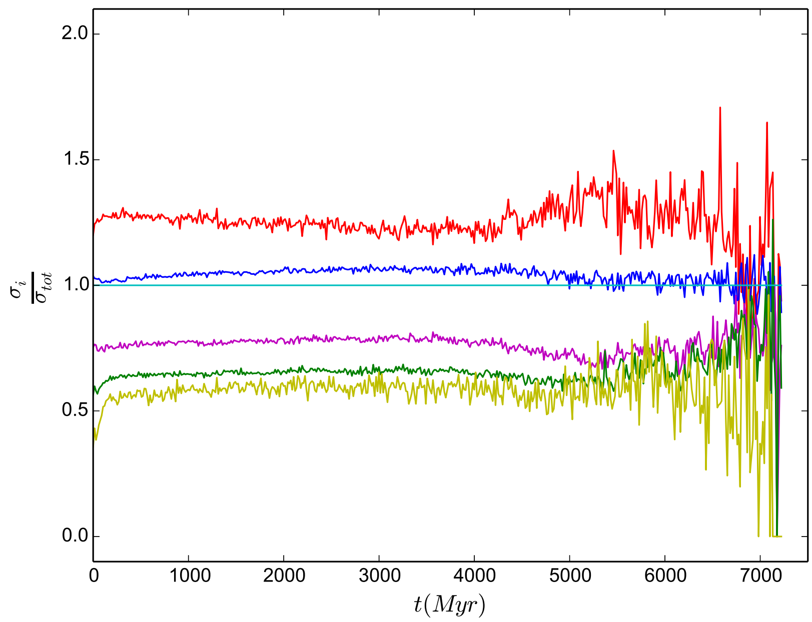


Figure 6. Ratio between the velocity dispersion in each lagrangian shell, containing 10%, 40%-60%, 80%-90%, 90%-100%, 99%-100% and 100% of the mass inside the tidal radius, and the total velocity dispersion of the model $16k\ NoKick$.

5 CONCLUSION AND FUTURE WORK

The first result of the current state of the project, as discussed in Section 3, is the very surprising evidence that, as a result of the presence of an initial kick velocity of the NS, we cannot consider any more a globular cluster with a King profile $W_0 = 5.0$ and in Roche filling a stable system. It is clear that, in particular situations, this parameter could be very important even if the fraction of neutron stars and the fraction in mass (of NS) are very small quantities, 0.5% and 1.8%, respectively. At the moment $NBODY6$ is using a gaussian distribution where the peak is 190 km/s and this determines that about 5% of NS are still inside the cluster after the initial expansion because they have a kick velocity less than the escape velocity of a generic star in the cluster. However, it is not clear whether this assumption is reasonable or not.

The second result concerns the kinematical properties of tidally-perturbed star clusters. We have confirmed the existence of tangentiality in the outer parts of these systems, as originally noted by Baumgardt & Makino (2003). We further explored this property by evaluating the existence and the contribution of a genuine rotational component (mean azimuthal velocity) by means of the inspection of the time evolution of the mean velocity vector and of the parameter β (instead of using exclusively $\tilde{\beta}$, as in Baumgardt & Makino 2003). Finally, we studied the properties of the velocity dispersion profile in the outer parts of the cluster, to evaluate the possible presence of a flattening at large radii.

The next step of our analysis will be focussed on the study of the role played by the population of potential escapers in the cluster (Fukushige & Heggie 2000; Baumgardt 2001; Küpper et al. 2010a,b). After an assessment of the size of the population of potential escapers in our simulations, we will inspect their contribution in the time evolution of the fundamental kinematic observables explored so far (mean velocity vector, anisotropy parameters, and velocity dispersion tensor). In addition, we plan to perform a detailed study of the orbits of selected pro-grade and retro-grade potential escapers, to analyze their behaviour and their stability, with particular reference to the previous investigations by Keenan & Innanen (1975). The subsequent part of our future analysis will be devoted to the study of the entire radial profile of the fundamental kinematical observables, ideally in projection, in order to facilitate the comparison with the corresponding observational quantities. Finally, we will also perform a full morphological characterization of our main models, by studying the radial profile of the surface brightness, the two-dimensional isophotal maps, and the associated ellipticity profile. We will perform our kinematical and structural study also on simulations with a different galactic potential, to evaluate its role in the dynamical evolution of the cluster.

As a separate line of investigation, we will continue our study of the effects of the kick velocity of the NS on the global evolution of the cluster. In particular, we will carefully analyse the simulation starting from a King (1966) with concentration $W_0 = 7.0$, to understand the non-trivial interplay between the expansion of the core associated with the rapid escape of NS and the initial concentration of the system.

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