Analytic Models of Globular Clusters with Potential Escapers

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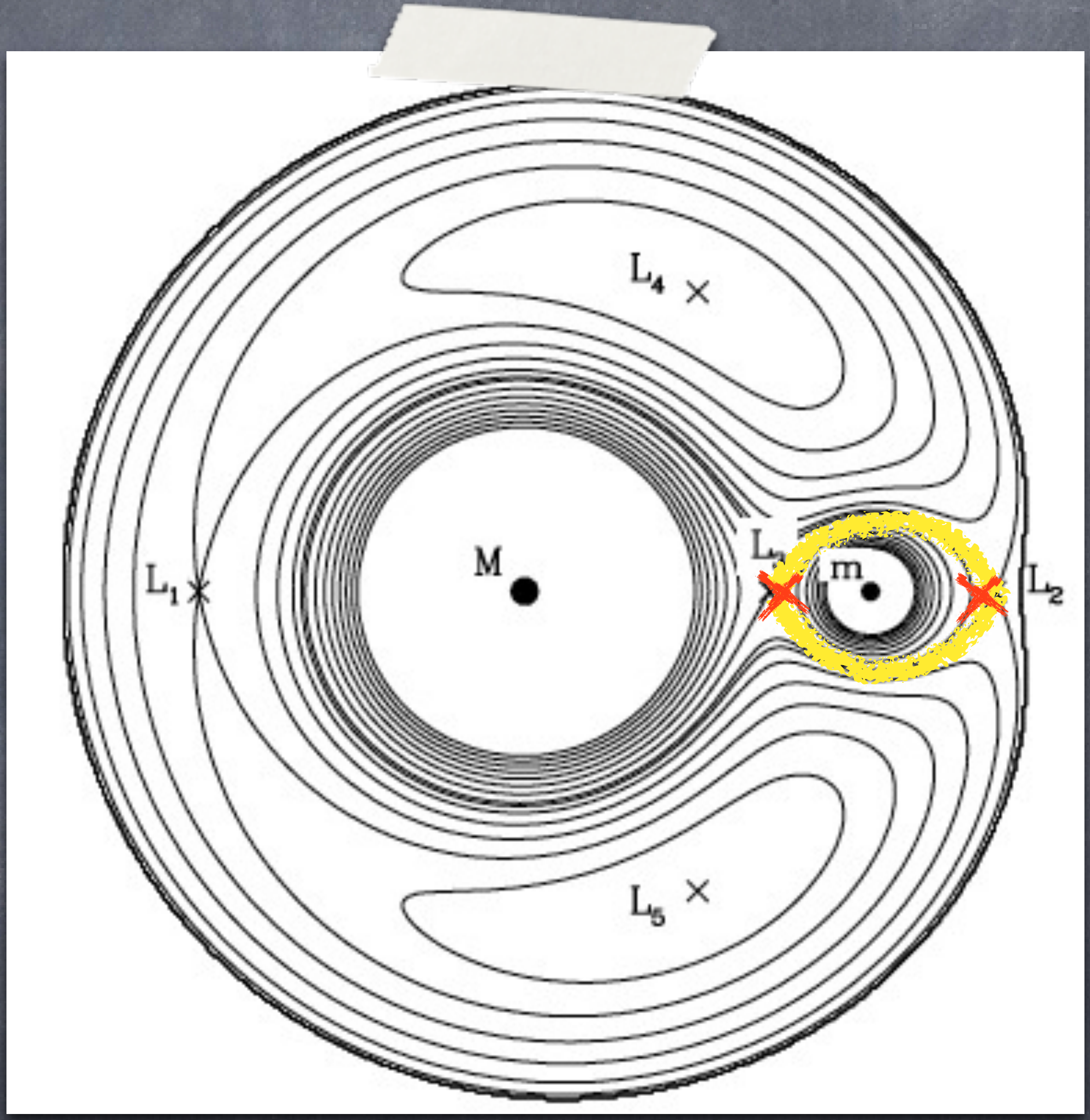
ISIMA

Potential in a tidal field

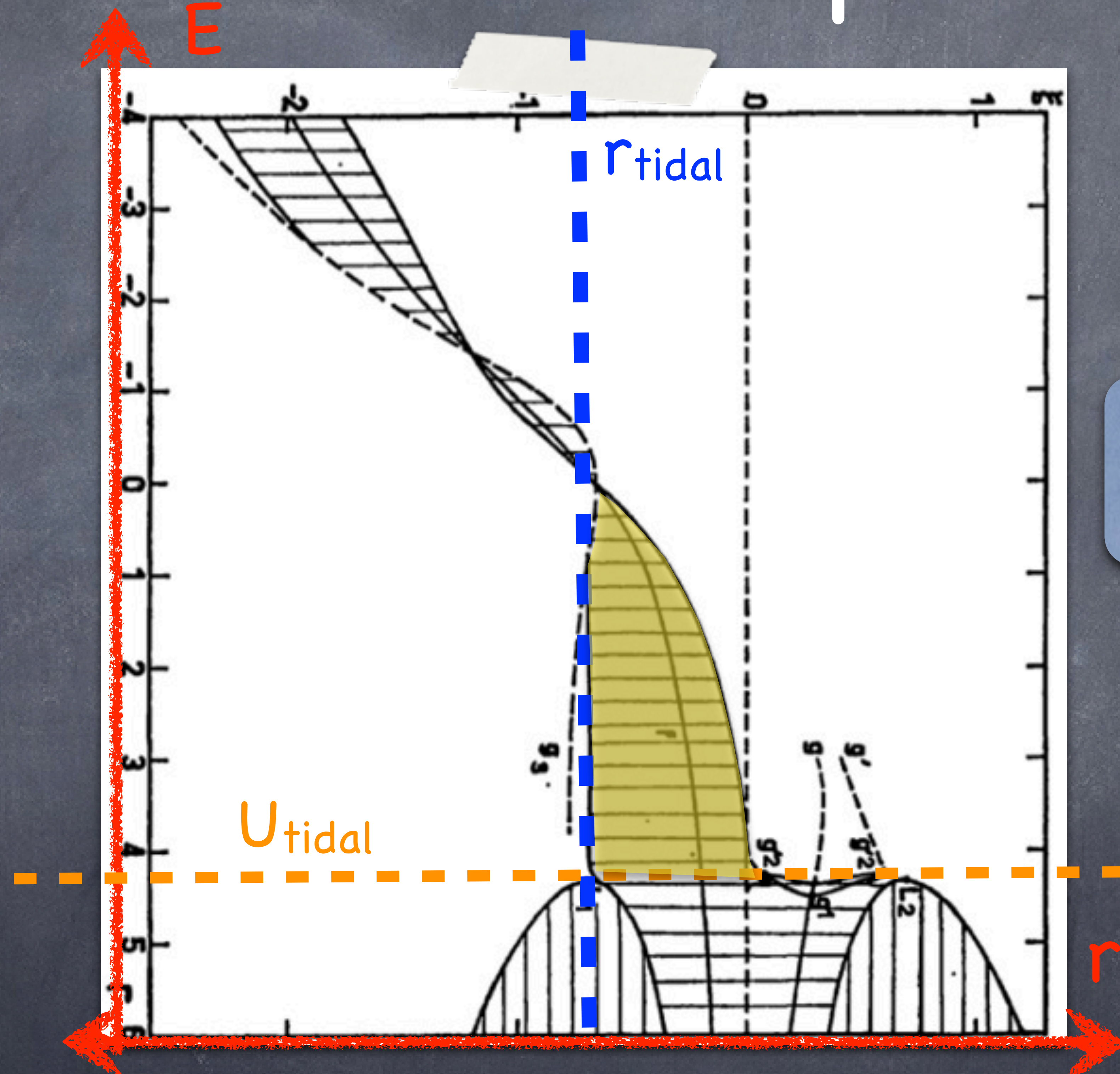
Potential in the rotating frame of stellar system in a tidal field.

There is an energy threshold (U_{tidal}) at the tidal radius (see Ben's talk)



Binney & Tremaine (2008)

Potential Escapers



E > U_{tidal}

r < r_{tidal}

Hénon (1970)

Snapshot Models

Aim to describe observed properties of a globular cluster (time-independent).

Construct a distribution function (f), that will produce:

- density distribution (ρ) with core radius and truncation at r_{tidal}
- velocity dispersion (σ)

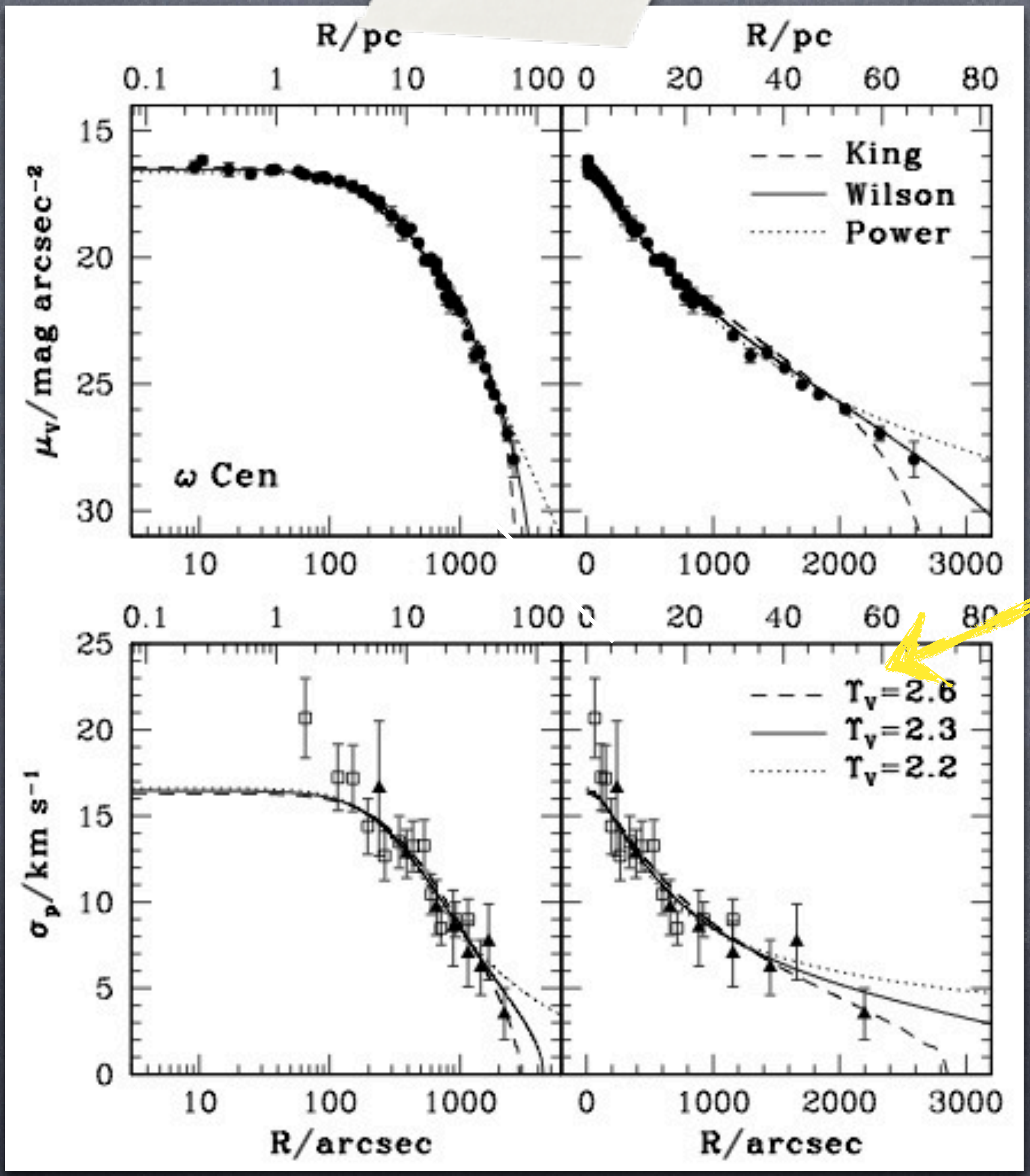
Several rather successful models use f=f(E) with energy truncation at $E \geq E_{crit}$ [eg. Woolley & Dickens (1961), King (1966), Wilson (1975)]

Snapshot Models

ω Centauri

Good 0^{th} order description!

(see Filippo's talk for details about flattening of velocity dispersion)



M/L

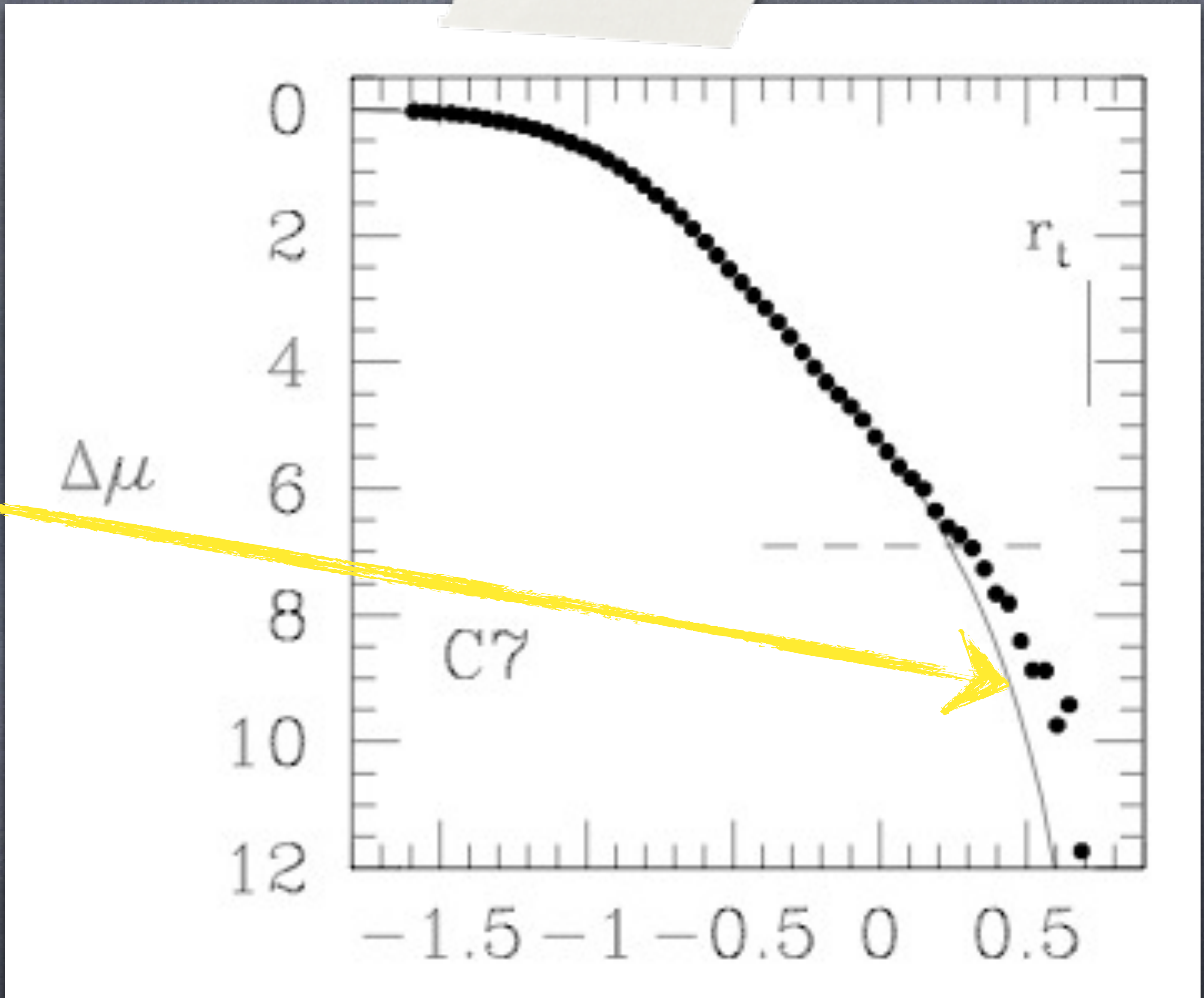
McLaughlin & van der Marel (2005)

...but this is not enough!

"Extra-tidal lights"

Fit to King model

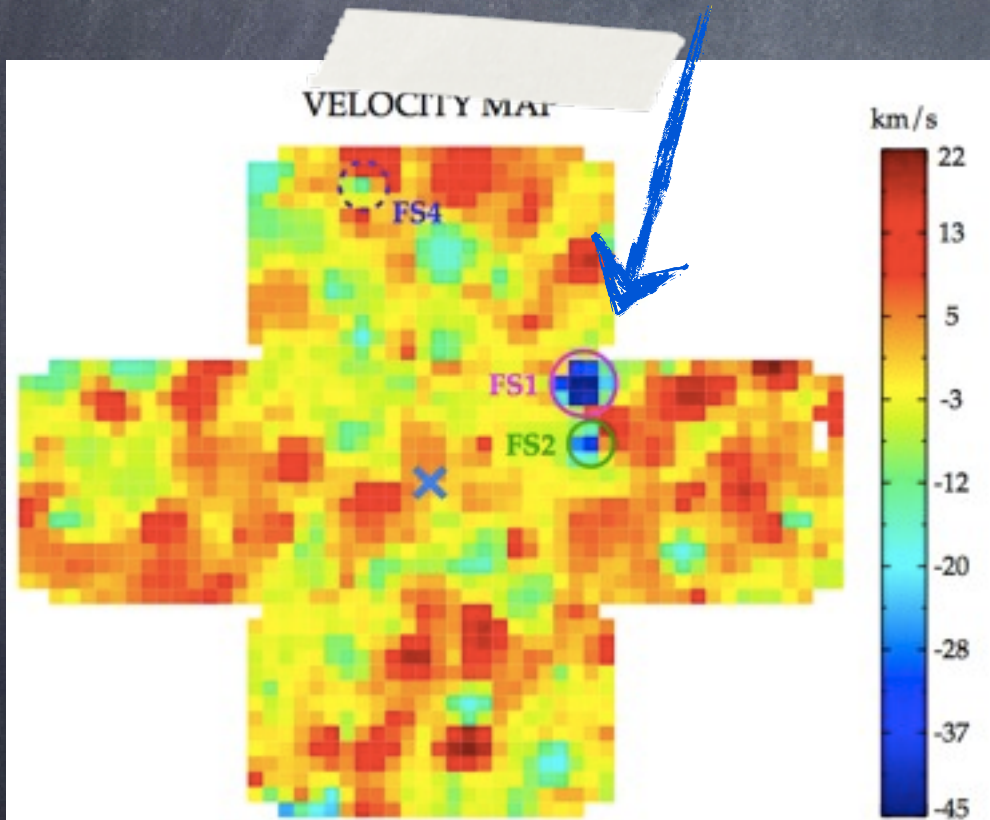
Wilson models seems to work better the the other two.



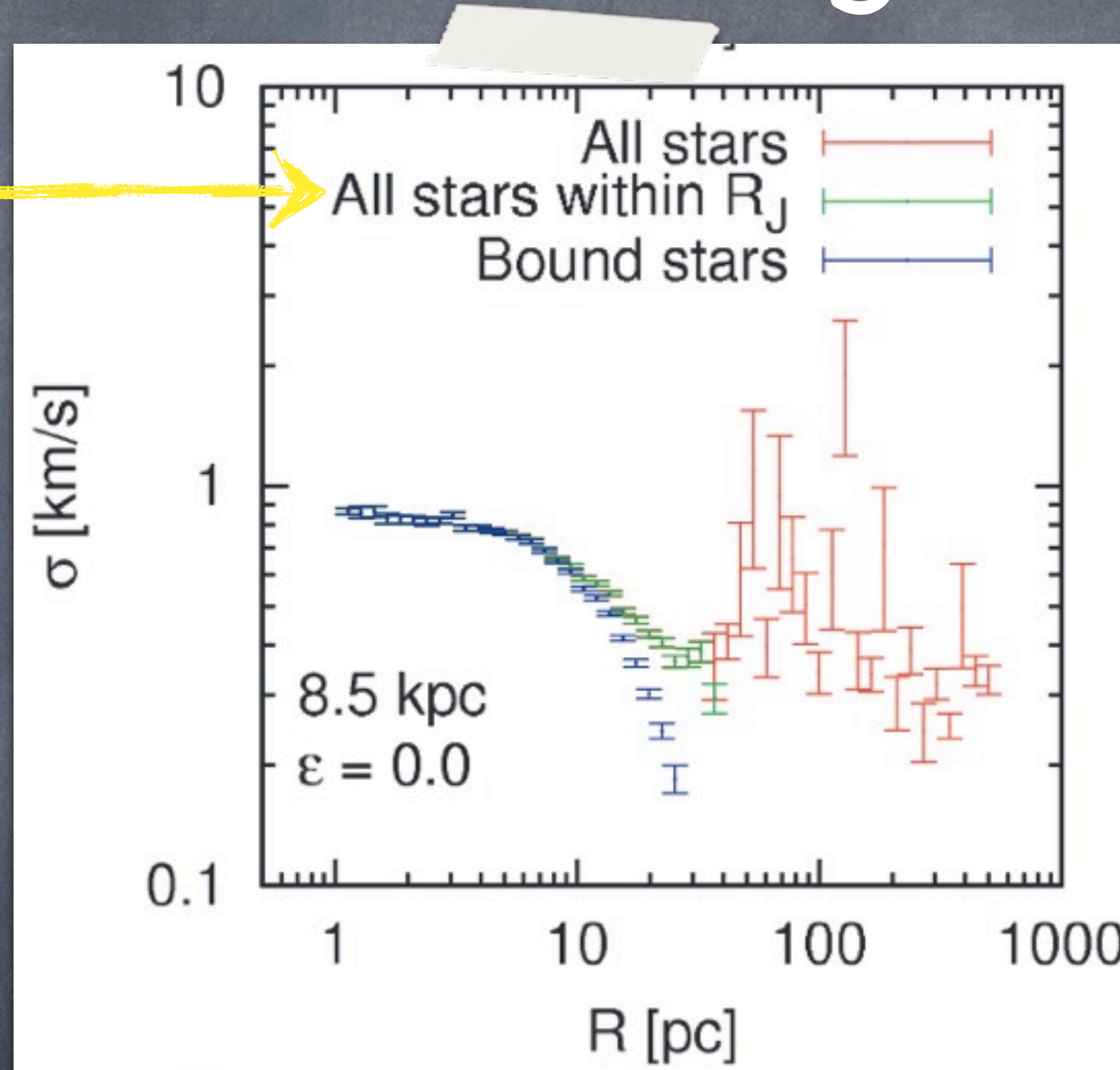
Harris et al. (2002)

...but this is not enough!

Enhanced values for the velocity dispersion at large radii



NGC 2028
Lützgendorf et al. (2012)



N-body
Küpper et al. (2010)

How to build a "Snapshot" Model?

- Compose a physically motivated distribution function, as function of integrals of motion
- Find associated density
- Use Poisson Eqn to find self-consistent potential
- Find structural and kinematical properties from moments in velocity space

$$\cancel{\frac{\partial f}{\partial t}} + \mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{x}} - \frac{\partial \Phi}{\partial \mathbf{x}} \cdot \frac{\partial f}{\partial \mathbf{v}} = 0$$

$$\rho(\mathbf{x}, t) = \int f(\mathbf{x}, \mathbf{v}, t) d^3\mathbf{v}$$

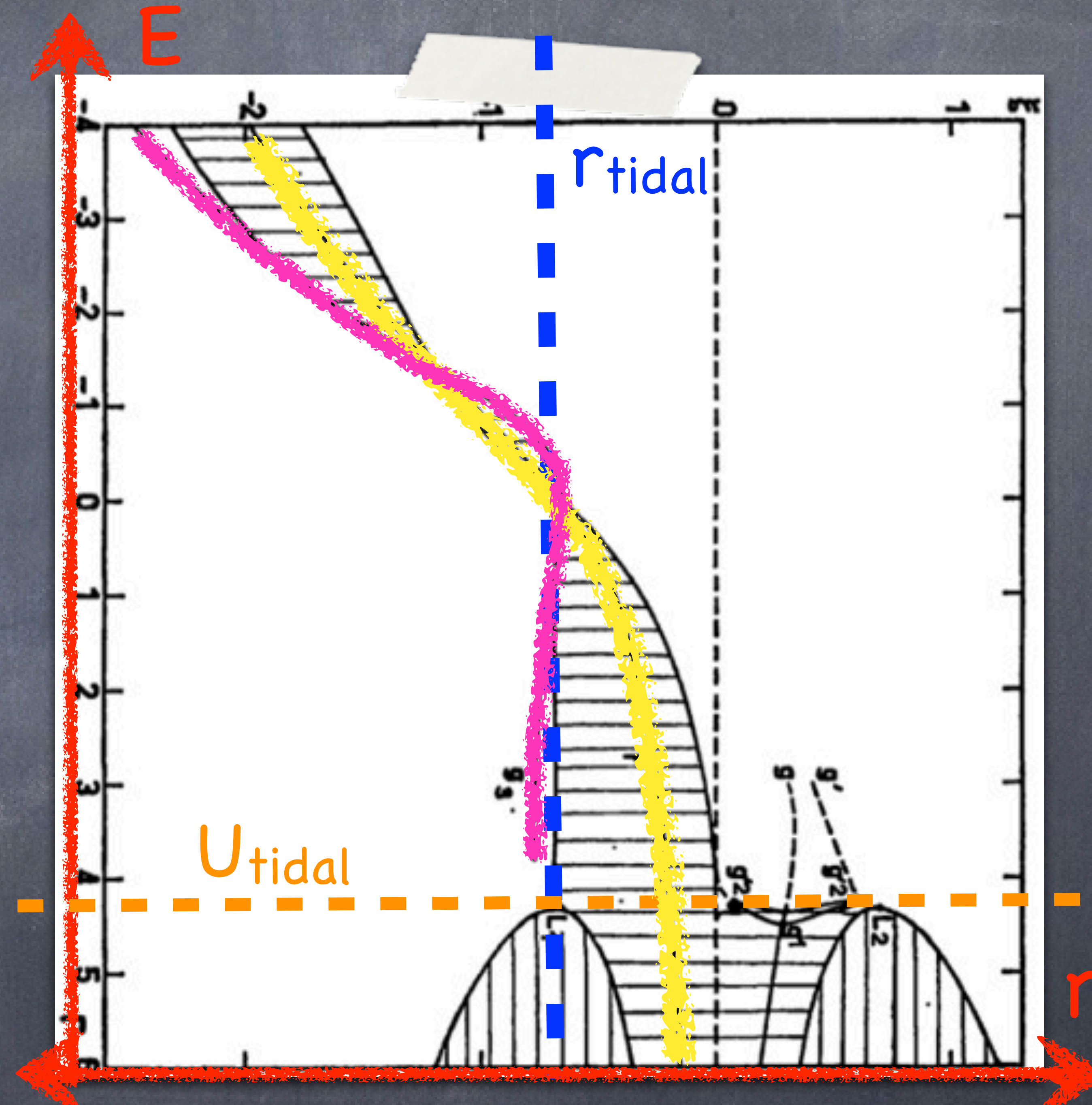
$$\nabla^2 \Phi = 4\pi G \rho(\mathbf{x}, t)$$

$$\langle \mathbf{v_i} \rangle = \frac{1}{\rho(\mathbf{x}, t)} \langle \int \mathbf{v}_i f(\mathbf{x}, \mathbf{v}, t)\, d^3\mathbf{v} \rangle$$

$$\sigma_i^2(\mathbf{x}, t) = \frac{1}{\rho(\mathbf{x}, t)} \int \mathbf{v_i}^2 f(\mathbf{x}, \mathbf{v}, t)\, d^3\mathbf{v} - \langle \mathbf{v_i} \rangle^2$$

Identify Potential Escapers

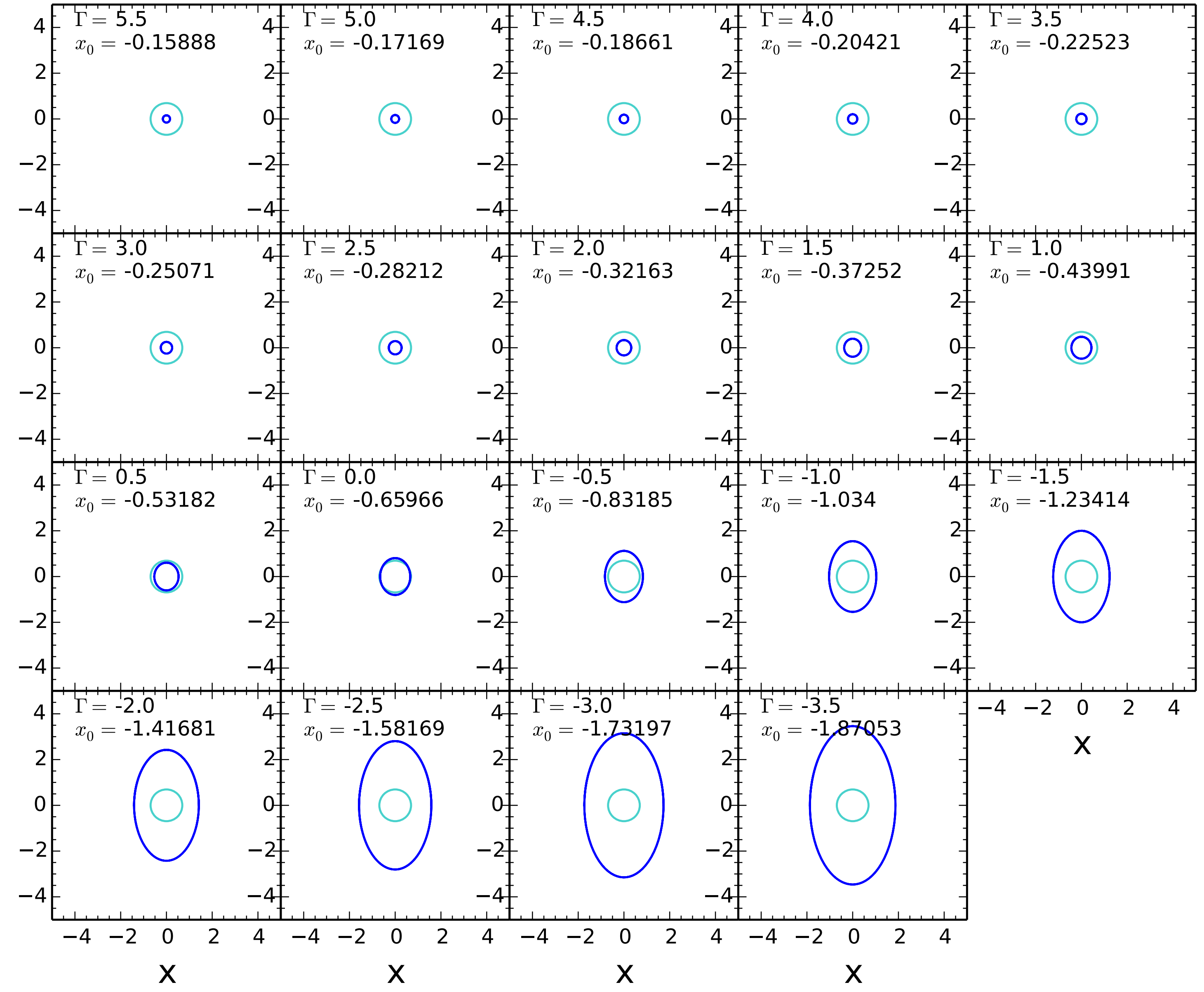
- Hénon identified family “f” and “g3” in his exploration of solutions of the restricted three body problem, in the Hill's limit.
- Stable and marginally stable orbits with $E > U_{tidal}$ and $r < r_{tidal}$
- The Jacobi Energy, Γ, is an integral of motion related to the energy ($E=U_{tidal} \Rightarrow \Gamma=3^{4/3}$)



$$\Gamma = 3x^2 + \frac{2}{\sqrt{x^2+y^2+z^2}} - z^2 - \dot{x}^2 - \dot{y}^2 - \dot{z}^2$$

Orbit Exploration

- Use initial conditions given by Hénon (1969) as a guide
- Begin by finding "f" orbits
- Stable
- From low to high values of Γ, orbits appear as epicycles to orbits around the galactic center with guiding center at x=0 to circular Keplerian orbits



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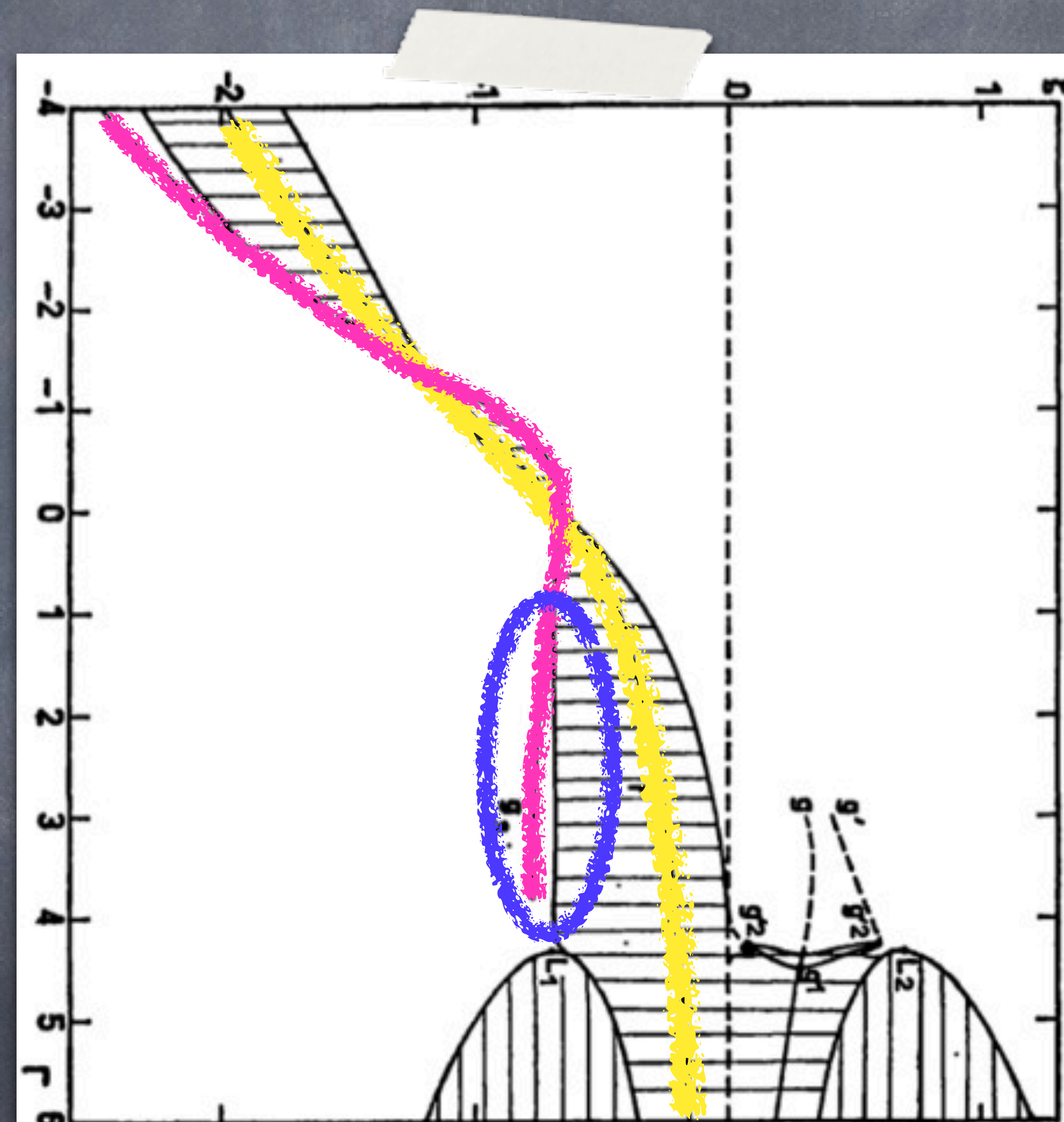

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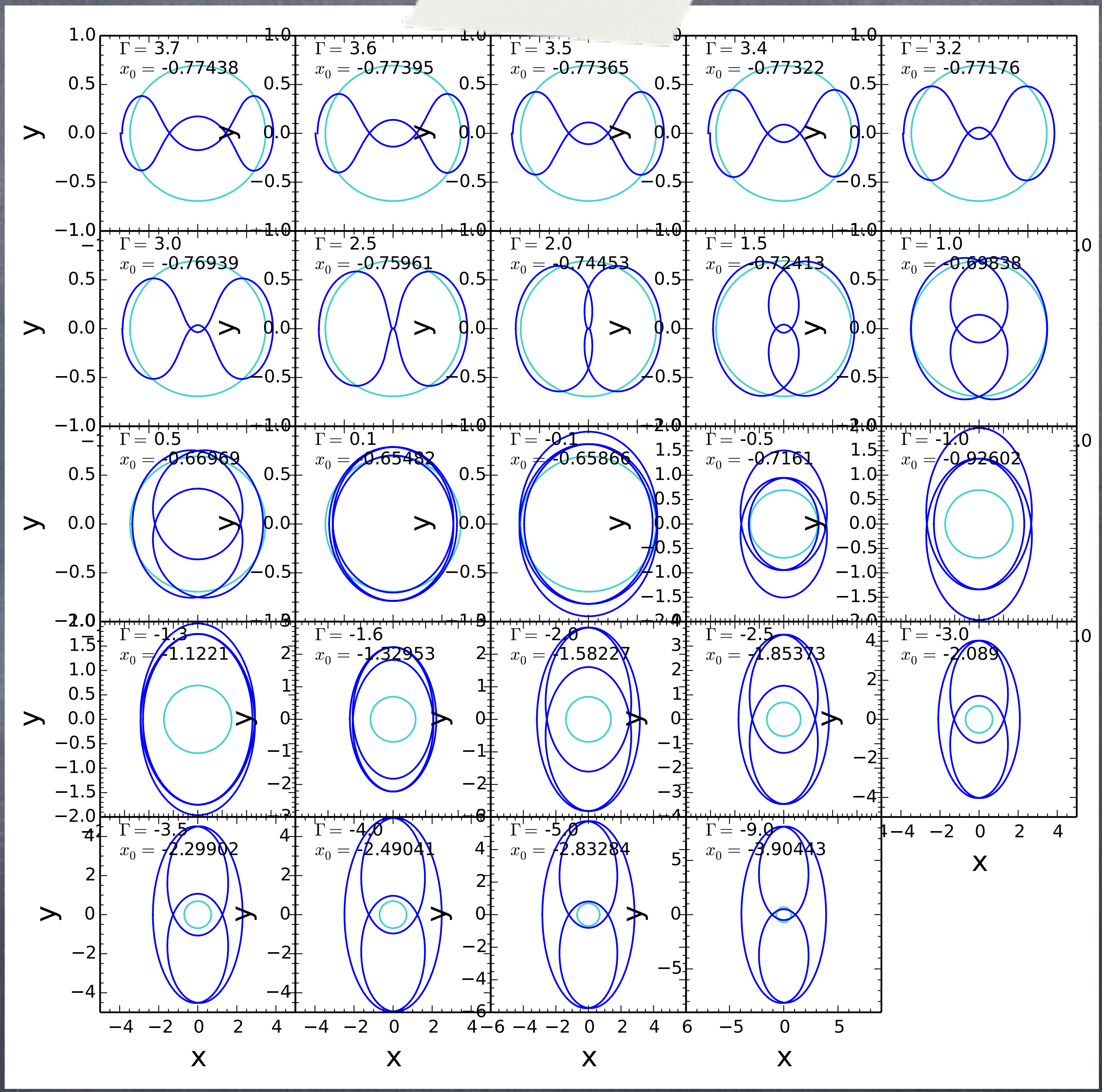
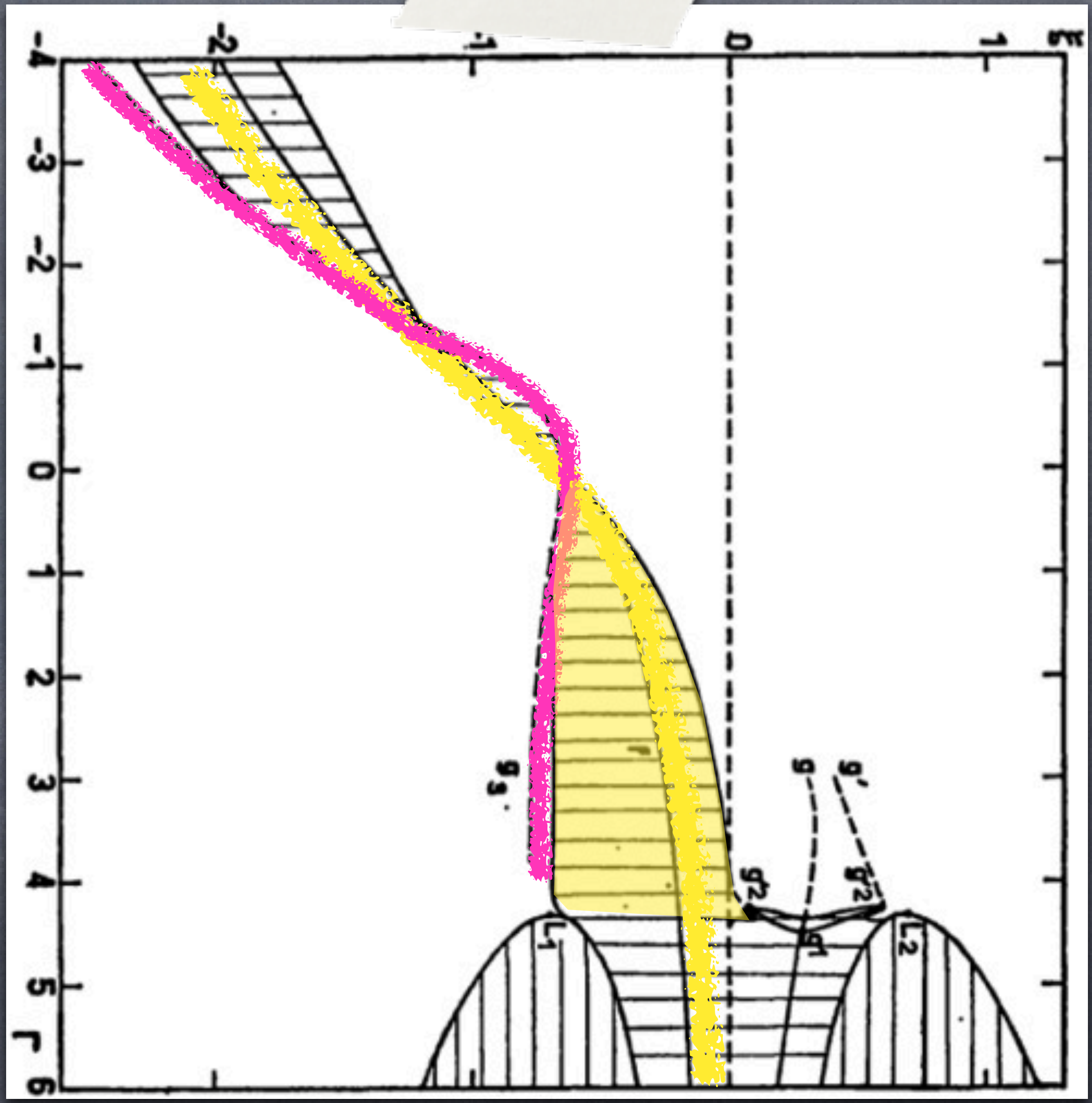
Orbit Exploration

- Found "g3" orbits given by Hénon (1970)
- Marginally stable and Unstable
- 3:1 resonant orbit
- Low Γ appear as having a rocking guiding center



Stability

- Need to find quasi-periodic orbits in order to understand the parameters that limit stability
- For now, just look at orbits with $r < r_{tidal}$ and $\Gamma > 0$
- Explore the 6D phase space within these bounds



Stability

Empirical stability analysis!

Integrate orbits over several orbital periods

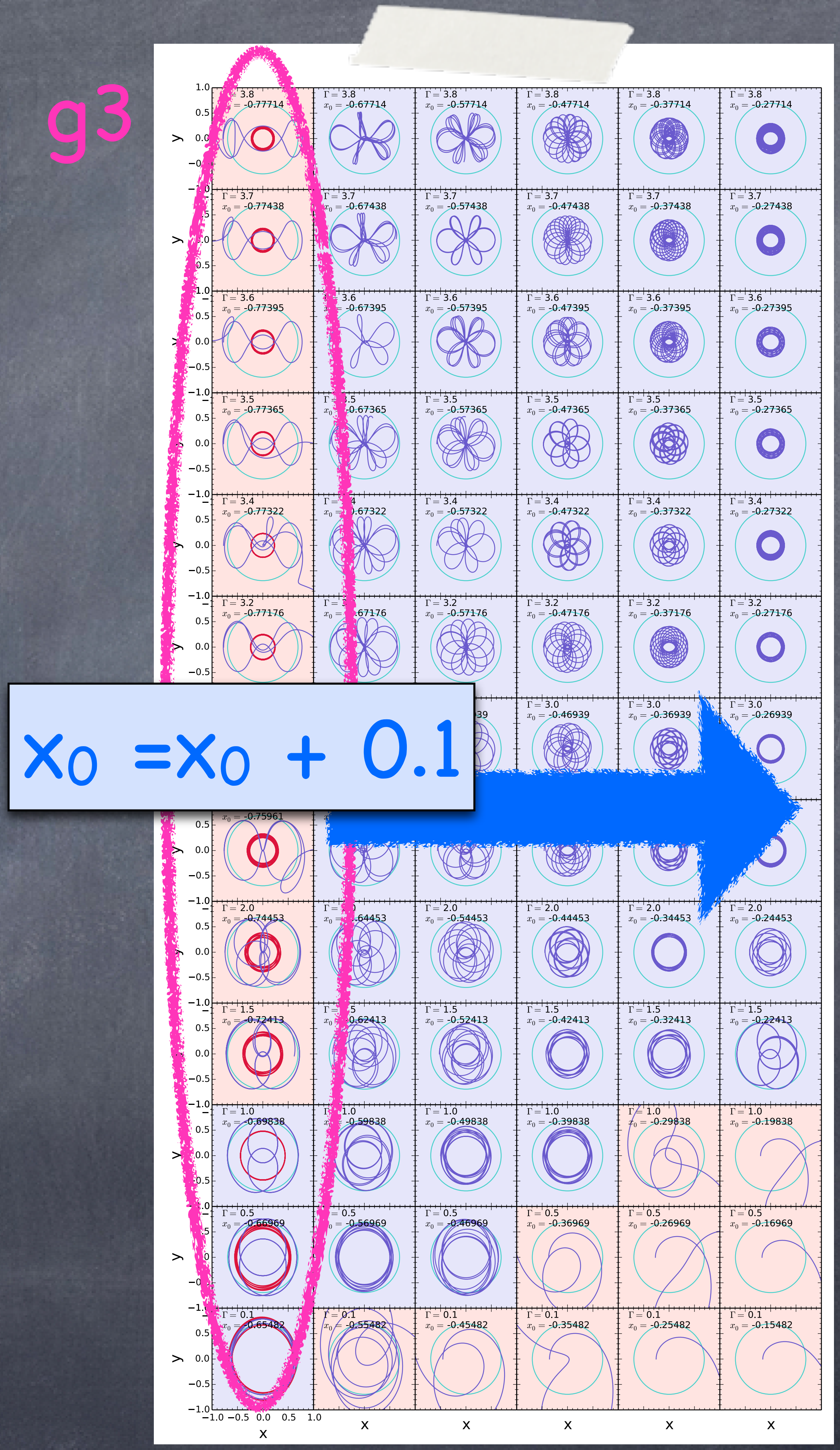
Unstable orbits have $r > r_{tidal}$ at some time during integration

Dusty rose = unstable

Misty Lavender = stable

Associate with integrals of motion

Know Γ, can calculate L_z & $|L|$



Stability f

Exploring the sensitivity of the stability to the 3rd dimension

Dusty rose = unstable
Misty Lavender = stable
Ivory = unclear

Associate with integrals of motion

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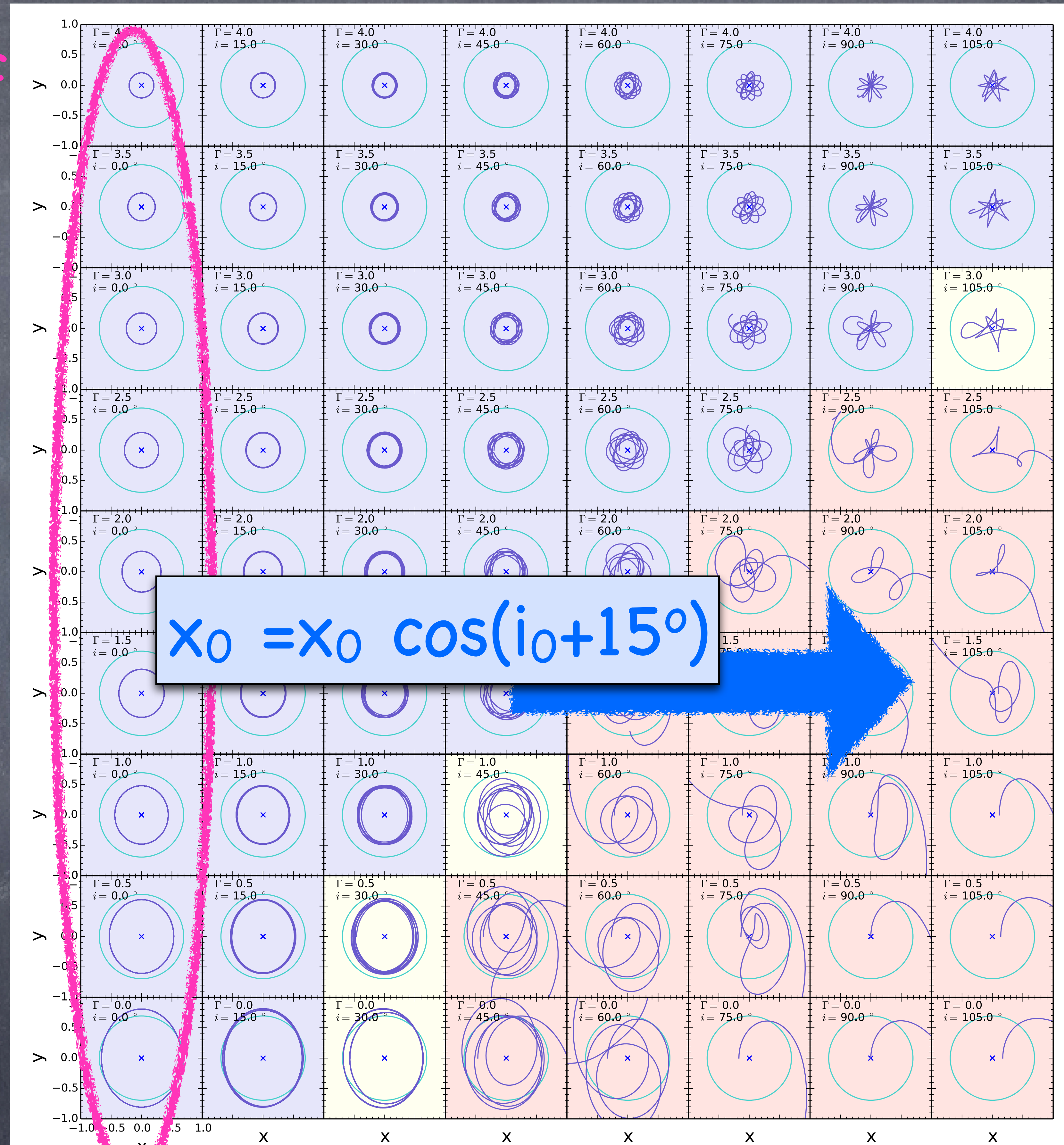
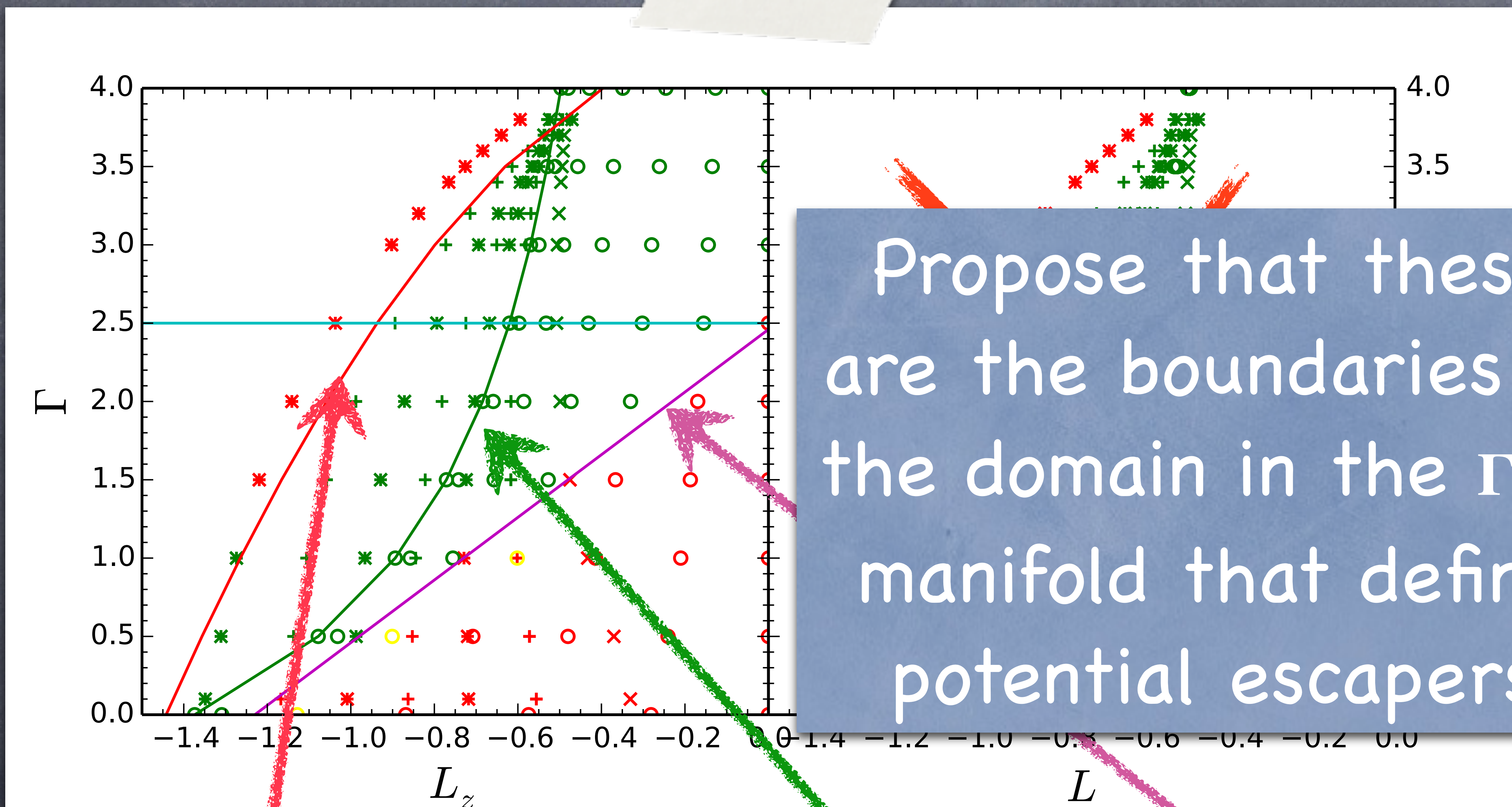


Define Parameter Space

Integrals of the motion in a Lindblad diagram



Propose that these are the boundaries of the domain in the Γ-L_z manifold that define potential escapers

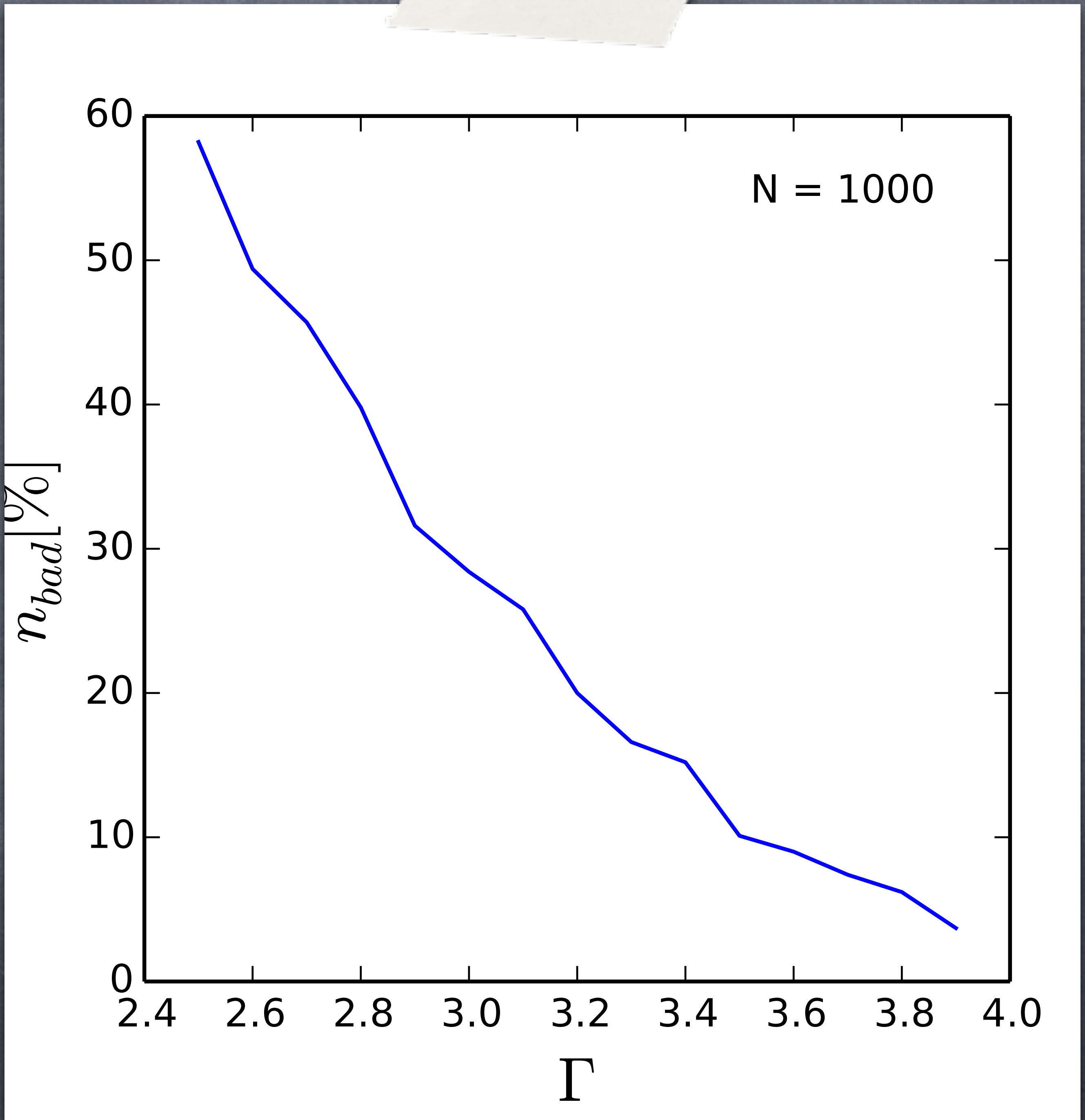
$L_z = r_{tidal}(3r_{tidal}^2 + /|r_{tidal}| - \Gamma)$

f orbits

"best fit"

Monte Carlo Exploration

- Monte Carlo simulation to check our characterization of potential escapers by extensive random sampling
- Initial conditions:
  - |r| < r_{tidal}
  - |v| defined from Γ
  - randomly selected angles
  - select only orbits that have L_z within the bounds defined by our manifold
  - orbits where |r|>r_{tidal} at any time during integration are flagged



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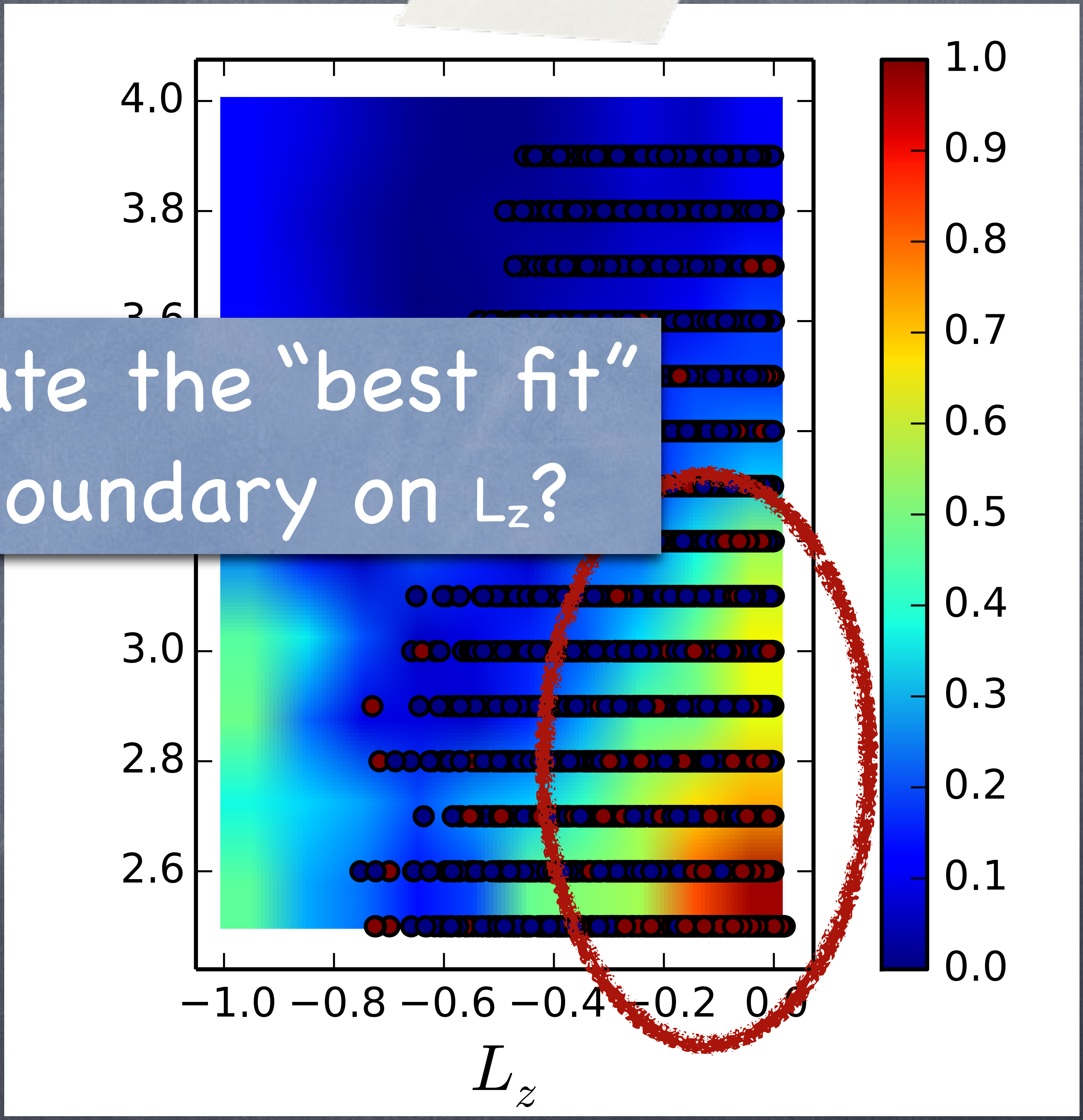
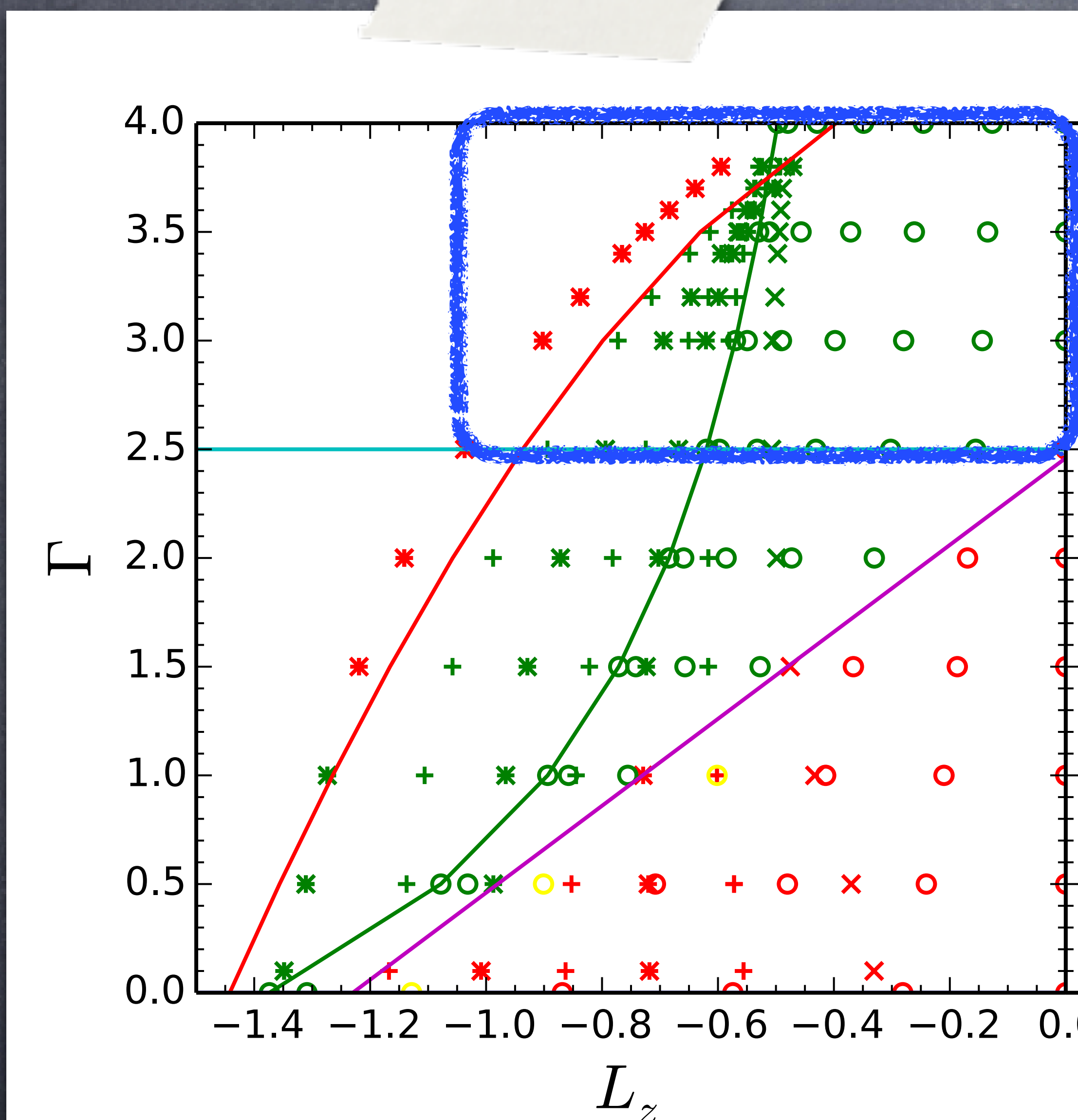
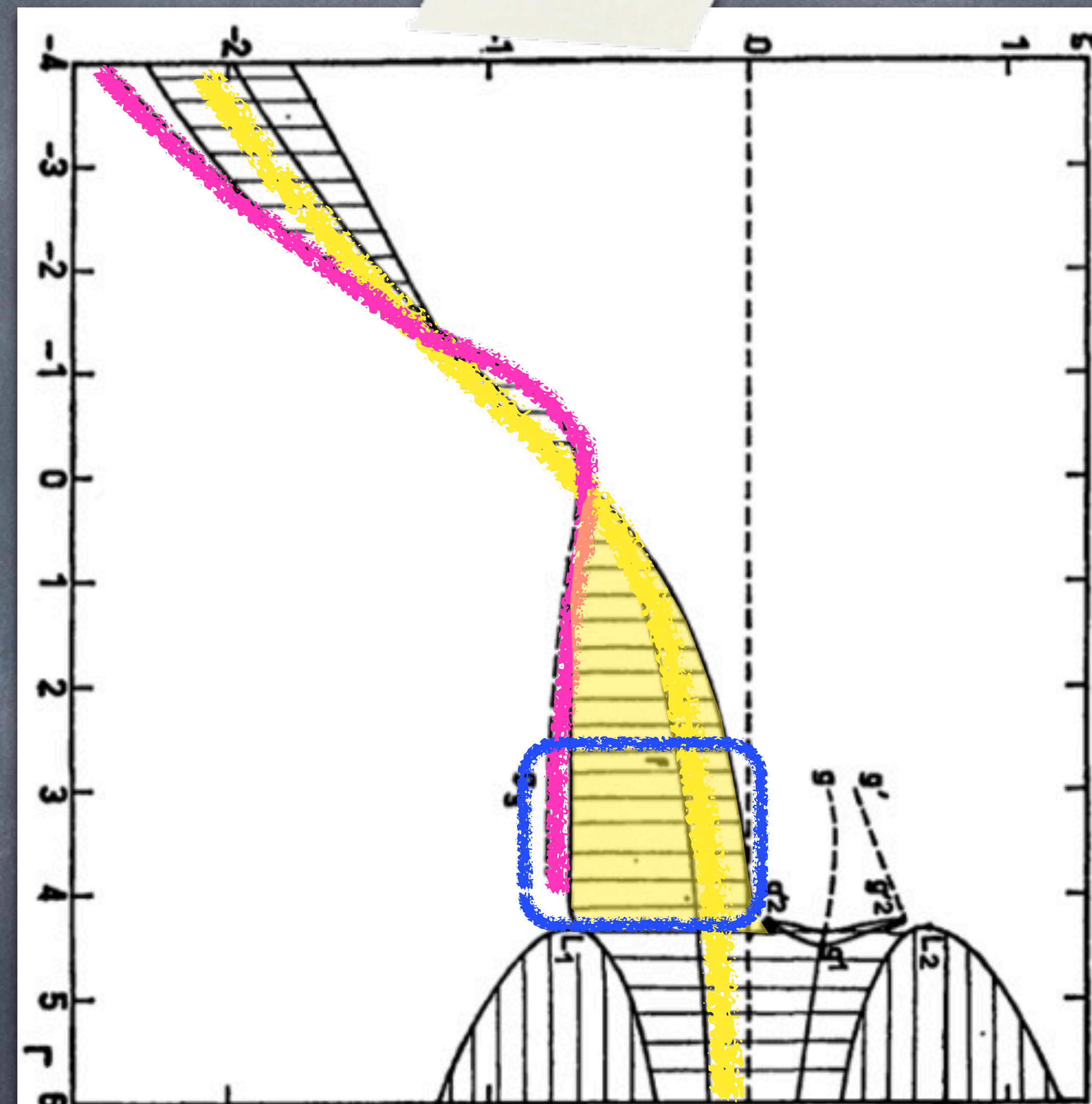


Re-evaluate the "best fit" upper boundary on L_z?

Select Sub-region of Interest





First attempt to find density

Use 2-Integral (Γ-L_z) distribution function based model with f=const as first step

$$\rho(\mathbf{x}, t) = \int f(\mathbf{x}, \mathbf{v}, t) d^3\mathbf{v}$$

$$\Gamma = 3x^2 + \frac{2}{\sqrt{x^2 + y^2 + z^2}} - z^2 - \dot{x}^2 - \dot{y}^2 - \dot{z}^2$$

$$\rho(\mathbf{x}) = \frac{\pi}{R} \int_{\Gamma_{L1}}^{2.5} d\Gamma \int_{x_{L1}(3x_{L1}^2 + \frac{2}{|x_{L1}|} - \Gamma)^{1/2}}^{0} dL_z\, f$$

Lacking a condition on the potential

Future Work

- Improve the distribution function:
  - Include a boundary that includes the potential
  - Refine using the resulting moments as a guide
  - Convert into appropriate coordinates for Poisson solver
- Use Poisson solver to find a fully self-consistent model, with characterization of the phase-space properties
- Apply to the interpretation of N-body models and observational data of globular clusters

Summary

- Used the work of Hénon to begin an exploration of periodic and quasi-periodic orbits of potential escapers
- Identified and explored the important domain in the Γ-L_z manifold for potential escapers with stable orbits
- Wrote an initial 0^{th} order moment of the distribution function as a first step toward finding a self-consistent prescription for potential escapers