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In situe irregular satellite formation

Xiao-Chen Zheng[1,2], Hagai B. Perets[3], Alice Quillen[4]
[1] *Kavili institute for astronomy and Astrophysics, Peking University, Beijing, China*
[2] *Department of Astronomy, Peking University, Beijing, China*
[3] *Technion - Israel Institute of Technology, Haifa 32000, Israel*
[4] *Department of Physics and Astronomy, University of Rochester, Rochester, NY 14627, USA*

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ABSTRACT

We investigate formation of satellites through collisions in a planetesimal disk that is surrounding an oblique oblate planet. We find that Kozai resonance lifted particles cause large eccentricities and inclinations in the growing satellite population and this can lead to formation of a single large moon near the Laplace radius. It could be the formation mechanism for a single large moon. Besides, retrograde moon forms in some cases motivted us a possible fomation mechanism for Triton-like irregulars.

1 INTRODUCTION

Moons around the planets are grouped into two categories, regular and irregular. Regular moons have low inclination and eccentricity, with respect to the planet's spin axis. In contrast, irregular satellites can have high inclinations and eccentricities and even orbit the planet retrograde to the planet's orbit (for a review see Jewitt & Haghighipour 2007). Formation of regular moons is thought to take place in a gaseous circum-planetary disk (e.g., Canup & Ward 2006) whereas the a successful model accounting for irregular moons involves capture from heliocentric orbit (see discussion by Jewitt & Haghighipour 2007).

Models for terrestrial planet formation often start after the formation of a planetesimal disk (e.g., for a recent review see Raymond et al. 2014). At later stages a set of similar sized planets (oligarchs) can be formed, when runaway growth is limited by the depletion of low mass objects and the dispersion in the disk. Recently Perets & Payne 2014 showed that massive moons originally formed on coplanar orbits exterior to the region of regular moons in the solar system can scatter each other. Scattered moons, some of which have undergone Kozai-Lidov oscillations can give a population of high inclination and eccentricity irregular-like (but prograde) moons. Scattering can also eject moons from the planet's Hill sphere, and these if later recaptured, can temporarily have retrograde orbits.

In this paper we investigate formation of moons in near a planet with non-zero obliquity. A similar setting might be planet formation in a triple star system (Pelupessy & Portegies Zwart 2013). Studies for the inclinations of planetary systems affected by binary companion stars (Batygin et al. 2011; Batygin 2012; Terquem 2013; Xiang-Gruess & Papaloizou 2014) could be mentioned here.

1.1 Basic physics

A satellite in orbit about an oblate planet precesses about the planet's spin axis due to the planet's gravitational potential quadrupole moment. If the planet has a non-zero obliquity, then this axis is tilted with respect to the star's tidal field. The obliquity of the planet is the angle between the planet's spin axis and its orbital angular momentum axis. Inside the the Laplace radius, r_L, precession is predominantly due to planet's gravitational quadrupole moment and outside this radius, due to the stellar tide.

$$r_L \sim \left(\frac{2M_pJ_2R_p^2a_p^3}{M_*}\right)^{\frac{1}{5}} \tag{1}$$

where M_* is the stellar mass, M_p is the planet's mass, R_p is the planet's equatorial radius, a_p the planet's semi-major axis and J_2 the planet's quadrupole gravitational moment. The quadrupole moment can be modified to take into account the contribution from moons. The planet's orbital eccentricity e_p can be taken into account by multiplying the right hand side of equation 1 by $(1-e_p^2)^{\frac{3}{10}}$ (see equation 24 by Tremaine et al. 2009).

The Laplace surface has normal (as a function of radius from the planet) about which the satellite's orbit precesses due to the combined effects of stellar tide and the planet's quadrupole moment. The Laplace surface traces the predicted shape of a thin gas disk or dissipative particulate ring surrounding the planet (e.g., Tremaine et al. 2009). Most regular planetary satellites orbit within laplace radii and probably formed from a circumplanetary gas disk (e.g., Canup & Ward 2006).

Consider a particle in a circular orbit around the planet with semi-major axis a. We define an angle ϕ that is the angle between the angular momentum vector of the particle and the planet's orbital angular momentum vector. The Laplace surface ϕ could be writen as (using equation 7 by

Tremaine & Davis 2014)

$$\cot 2\phi = \frac{\cos 2\phi_p - a^5/2r_L^5}{\sin 2\phi_p}. \quad (2)$$

where ϕ_p is the azimuthal angles of the planet, the obliquity is $\phi_p - \phi_* = \phi_p - \frac{1}{2}\pi$

2 NUMERICAL SIMULATIONS

2.1 Initial settings

The code Mercury (Chambers 1999) was modified to take into account the oblateness of the planet by adding a quadrupole moment to the gravitational potential. When two particles collide they stick together forming a single body with total mass the sum of each impactor.

Simulations begin with a single planet in orbit about a star. The planet is initial surrounded by a disk of planetesimals in circular orbits that are in the planet/star orbital plane. Parameters for the simulations are listed in Table 1. The density assumed for planetesimals is that of the largest moon and is between 1-2 g cm^{-3}. Initial planetesimal separation was 7 r_H with the mutual Hill radius

$$R_H = (\frac{2\mu}{3})^{1/3}\frac{a_i + a_{i+1}}{2}, k = \frac{a_{i+1} - a_i}{R_H} \quad (3)$$

We ran three sets of simulations:

- Series A: started with planetesimals initial in the planet's orbital plane.
- Series B: with planetesimals started in the Laplace surface.
- Series C: with planetesimals started in a planet perpendicular to the planet's spin axis (the obliquity plane).

The A1 simulation uses parameters for Neptune.

We also run a couple of test simulations with the same parameters as these series but a zero obliquity.

2.2 Results and discussion

Figure 1 shows the result of the A1 simulations. Instead of the analyatic laplace surface derived by equation 2, the planetesimals are initially located at the reference plane, in another words, they are primarily considering to be born at the circumstellar disk. After 1 Myr evolving, there is a massive object formed within one r_L, caused by increase in collisions probability due to Kozai oscillations near r_L. Beyond the laplace radii, most plantesimals are smaller in mass, and even one or two dominated bodies formed, they are prefering to orbit around r_L rather than outer region. Most interesting, the largest object formed in the inner region is orbiting around the host planet in a retrograde manner, the matual inclination of this plantesimal orbit compare to the host planet obliquity is larger than 100 degree. It inspires us about the possible formation mechanism for Triton. Base on previous observation, Triton is at $0.237r_L$ to Neptune with a retrograde orbit, due to the large relative inclination to the equator plane of Neptune, Triton is classified into irregular moon group even though its mass and location are belong to regular moon features. It seems that the intensive collisons inspired by J2 of host plant which would result in the formation of a massive moon in retrograde orbit could be a prospective scenario for Triton-like satelliate formation without applying any other energy dissipation mechanism. However, the observed Triton is much closer to Neptune compare to the simulation and the inlination as well as the mass in our mimic result have nonignorable devergency with observed value. Actually, the latter difference could contribute to the model dependence and in the simulation we only consider the inelastic collision so the collison outcomes are resonable to be different to the real case, but how could Triton migrate into such a inner region is still a puzzle, additional migration scenario is neccessary to be considered, tidal dissipation could be the most luminous proposal.

In Fig.2, we study how moons growth with time in the top plot, while in the bottom pattern, their individual semimajor axis evolving are also presenting in detail. As same as the inferring by Fig.1, most moon mass prefer to concentrate in the inner region but there are large amount of small planetesimals bordely distribute in the few times of laplace radii to few tens percent of planet Hill radius with moderate excitation in orbit which could be the origin of irrgular group.

For comprison, planetsimals in Neptune system are also initially at the anylatic laplace plane and the equator plane, shown in Fig .3 and Fig.4, respectively. The final mass distribution in two plots show similar features, there is one massive object form at the reference plane within one laplace radii in both cases, while beyond the region, small planetesimals are randomly dispersed around the circum-stellar plane which is consistent to the A1 series. The absence of retrograde object in B1 and C1 series could contribute to the overlap of laplace surface with the primary location of plantesimals within laplace radii, therefore the inclination of most planetesimals in this region are impossible to be greatly excietd by quadropole torque of host planet.

To have a basic idea that how could the planet's obliquity take effect on moon formation, in Fig.5, we consider the planetesimals evolving around one zero-obliquity planet. As what we expected, collisions of planetesimals in such a non-J2 system are not as effeciency as A1, B1 and C1 series resulting in the absence of massive moon after 1 Myr evolution. Therefore, we could rougly conclude that the planet's obliquity has significant influnece on the observed configuration of surrounding moons.

Besides, the moon formation around Saturn system is also studied in Fig.6, Fig.7 and Fig.8. The planetsimals are initially located at reference plane, laplace surface and equator plane, respectively. In both cases, after 1 Myr evolution, there is no massive moon formed within one laplace radii, it is different from the Neptune cases. However, slightly outer laplace radii, one or two dominated moons have formed around the laplace surface, while in the outside regime where stellar tide is dominated, only small planetesimals surviving due to the lack of orbital crossing.

Actually, the moon configuration in Saturn and Neptune systems should be similar because the host planets both have a intermediate obliquity, by contrast, Uranus should be an exception as its euqtor plane is nearly perpendicular to the orbital plane. We also study the inclination evolving of planetesimals around Uranus in Fig.9 and Fig.10. For planetesimals initially in equator plane, the final distribution

Table 1. Initial integration parameters

Series	M_p (M_*)	m (M_*)	a_p (AU)	I (°)	J_2	R_p (AU)	r_H (AU)	r_L (AU)	$a_{min} - a_{max}$ (AU)
1-Neptune	5×10^{-5}	2×10^{-10}	30	30°	24×10^{-3}	1.65×10^{-4}	0.77	0.01	0.0005 - 0.119
2-Uranus	4.3×10^{-5}	1×10^{-10}	19	98°	18.7×10^{-3}	1.75×10^{-4}	0.46	0.011	0.0005 - 0.048
3-Saturn	28.5×10^{-5}	15×10^{-10}	9.58	26.7°	70.56×10^{-3}	4.0×10^{-4}	0.44	0.0195	0.0005 - 0.12
4-Jupiter	1.0×10^{-3}	3.93×10^{-9}	5.2	3°	45×10^{-3}	4.8×10^{-4}	0.36	0.017	0.0005 - 0.119

M_p is the mass of the planet divided by the mass of the star, m is the initial mass of the planetesimals (divided by the mass of the star). R_p is the planet's radius. I is the planet's obliquity and a_p the semi-major axis of the planet. Simulations are started with 50 planetesimals spaced by 10 mutual Hill radii and ranging from a_{min} to a_{max} (in orbit about the planet). The Hill radius is r_H and the Laplace radius r_L (equation 1). We adopt a density for the planetesimals of 1 g cm^{-3}. Simulation was run for 1 Myr.

after 1 Myr is kind of plain, only one small planetesimal is still surviving in the system. Comparably, planetesimals are trending to form massive moons if they are primarily setted at the reference plane. Those results motivited us a suppose to constriant the time when Uranus trends to be obliquid base on previous moon configuration, if Uranus gets its high obliquity at an early age, it is hard to form large moon, but if Uranus gets its high obliquity at later age, it is possible for moons' formation in inclined orbits.

3 TIDAL EVOLUTION

If spin is important then satellites move outward if they are outside spin-synchronous orbit with the planet. If spin is unimportant then satellites move inward due to tidal circularization. For drift due to spin of planet and tide raised on planet from the satellite

$$\tau_a^{-1} = \frac{\dot{a}}{a} \sim n \sin(\Omega_p - n) \frac{3k_2}{Q_p} \frac{M_s}{M_p} \left(\frac{R_p}{a}\right)^5 \quad (4)$$

The sign would be positive for the region we are in as Ω_p is large compared to n for us (we are outside spin-synchronous orbit). For tidal circularization and a rigid body (and the satellites probably have rigidity like ice) an eccentricity damping timescale is

$$\tau_e \sim \frac{4}{63} \frac{M_s}{M_p} \left(\frac{a}{R_s}\right)^5 \frac{\mu_s Q_s}{n} \quad (5)$$

where μ_s is the rigidity of the satellite. Through angular momentum conservation

$$\tau_a \sim \tau_e \quad (6)$$

and the satellite moves inward. Usually satellite eccentricity is damped and this follows through angular momentum with the orbit, (though the Moon is a counter example with increasing eccentricity and semi-major axis because the Earth's tidal dissipation and spin are more important than that of the moon).

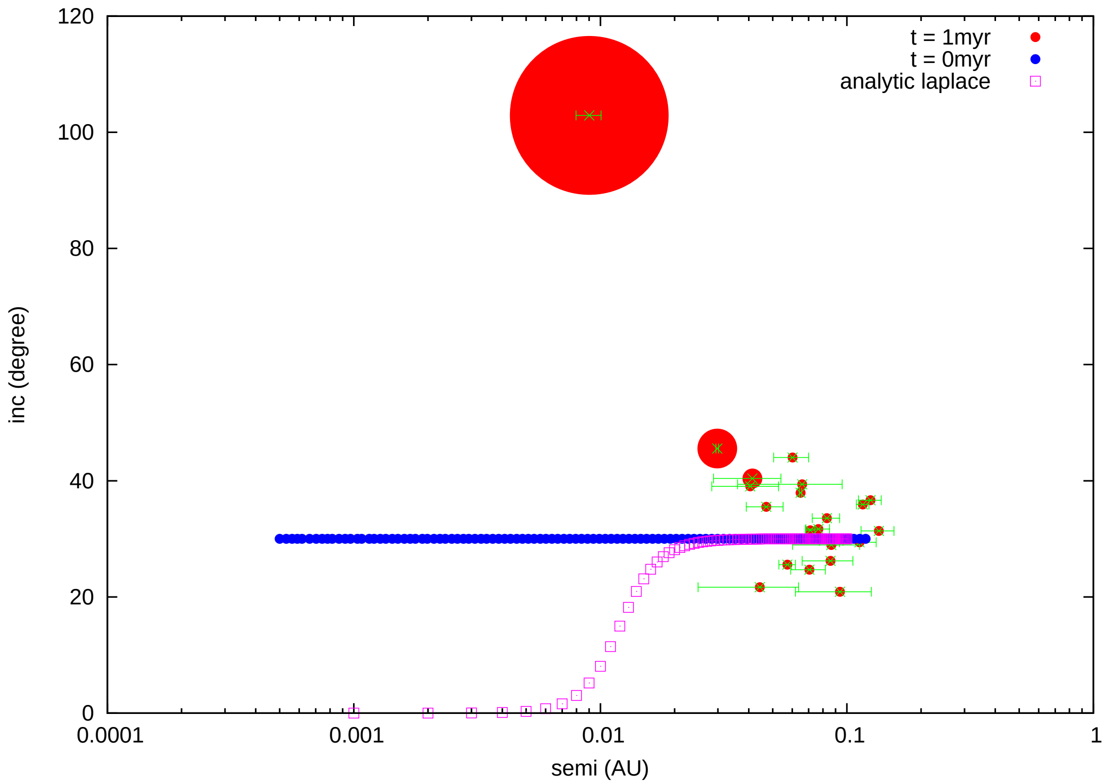

Figure 1. Final masses moon inclinations and semi-major axes in the A1 series (see Table 1). As a function of inclination with respect to the planet's orbital plane, we plot semi-major axes of planetesimals in the A1 series. The error bars give peri-center and apocenter radii and have length related to particle eccentricity. The size of the dots is related to moon mass. Colors labels each system, and Blue circles show the analytical Laplace surface (calculated using equation 2).

Possibly the above is consistent with

$$\tau_a^{-1} = \frac{\dot{a}}{a} = -n\frac{21}{64}\left(\frac{R_s}{a}\right)^5 \frac{k_{2s}}{Q_s}\frac{1}{(1-e)^6}\frac{M_p}{M_s} \quad (7)$$

Which is equation 1 by Cuk & Gladman (2005). Matija averaged over Kozai cycles to predict drift of the satellite.

$$(n\tau_a)^{-1} \sim 10^{-15}\left(\frac{M_s}{M_{Triton}}\right)^{-1}\left(\frac{M_p}{M_{Neptune}}\right) \left(\frac{k_{2s}}{0.1}\right)\left(\frac{Q_s}{100}\right)^{-1}\left(\frac{a}{0.01\mathrm{AU}}\right)^{-5} \left(\frac{R_s}{R_{Triton}}\right)^5\left(\frac{1}{1-e}\right)^6 \quad (8)$$

Here for semi-major axis I have used Neptune's current Laplace radius and k_{2s} and Q_s used by Cuk & Gladman (2005) for Triton. The timescale is too long unless there is a high eccentricity.

Tidal circularization does bring a satellite inward, so if a single large body is made at r_L at eccentricity it would be brought inward by tidal circularization. It is straightforward to predict Δa from the eccentricity after formation using angular momentum conservation, but it is only sensible to do this if there is no Kozai oscillation and if the timescale for tidal circularization is short.

Fig.11 shows how the eccetricity and tidal circularison of each planetsimals evolving with time. In the eccentricity regime, there are two goups, one is bound planetesimals, another is escapers. Base on equation8, the timescale of tidal circularison is also present at the bottom plot. For most planetesimals, the tidal effect could be ignore due to the long circularison timescale, only for few particles, the tidal circularison timescale are comparable to the evolving timscale which could be the candidates for the following inward migrations. Therefore, those plantesimals which are effecient for tidal circularison could be our research majors in next step.

3.1 J_2 and r_L with time

A planet that is oblate because of rotation

$$J_2 \equiv k_{2p}\frac{q_{rot}}{3} \quad (9)$$

with a centrifugal parameter

$$q_{rot} \equiv \frac{\bar{R}_p^3\Omega_p^2}{GM_p} \quad (10)$$

and Ω_p the spin of the planet and $\bar{R}_p$ the mean radius (not equatorial radius). Angular momentum of the planet

$$L = M_p\alpha_p\bar{R}_p^2\Omega_p \quad (11)$$

With angular momentum conservation at two different times

$$J_{2,new} = J_{2,old}\frac{k_{2p,new}}{k_{2p,old}}\left(\frac{\alpha_{p,new}}{\alpha_{p,old}}\right)^{-2}\left(\frac{\bar{R}_{p,new}}{\bar{R}_{p,old}}\right)^{-1} \quad (12)$$

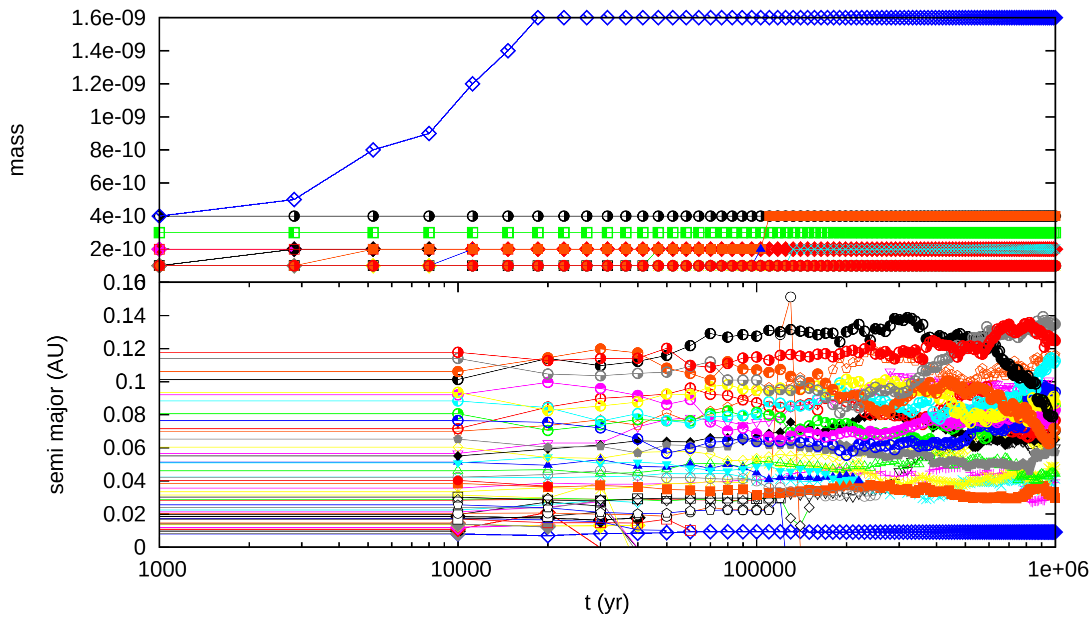

Figure 2. The mass and semi-major axis evolution of planetesimals in A1 series

Both k_{2p} and α_p are predicted to increase with time as a planet contracts and $\bar{R}_p$ decreases (Leconte et al. 2011). Using the models by Leconte et al. (2011) k_{2p} increase by 30% while α_p increases by 15% for a change in radius of about 20% (look at his κ for α_p). The factors of α_p and k_{2p} cancel out leaving the dependence on radius. This implies that we can consider J_2 primarily depending only on radius. As a planet contracts, radius decreases and J_2 would increase. This implies that at early times we can consider a smaller J_2 and as a consequence a slightly smaller Laplace radius. As the Laplace radius goes as $J_2^{1/5}$ this is a very small change (at most we might think about planets changing in radius by a factor of 2 and this corresponds to 15% change in J_2.

In summary, ignoring changes in J_2 is justified, and at most r_L could have changed by 15% due to changes in J_2. As $r_L \propto a_p^{3/5}$, with respect to the Hill radius $r_L/r_H \propto a_p^{-2/5}$ so if a planet migrates outward, the Laplace radius increases but decreases with respect to the Hill radius. If a moon is made at the Laplace radius when the planet is closer to the star, it would have been born at smaller radius than its current Laplace radius. Migration by the planet could induce a much more significant possible change (as a function of time) in the Laplace radius.

4 ESCAPE VELOCITY

The circular velocity

$$v_c = 0.7\text{km s}^{-1} \left(\frac{M_p}{M_{Neptune}}\right)^{\frac{1}{2}} \left(\frac{a_p}{0.01\text{AU}}\right)^{-\frac{1}{2}} \qquad (13)$$

where again I have used Neptune's current Laplace radius.

From Triton

$$v_{escape} = 0.46\text{km s}^{-1} \left(\frac{M_s}{M_{Triton}}\right)^{\frac{1}{2}} \left(\frac{R_s}{R_{Triton}}\right)^{-\frac{1}{2}} \qquad (14)$$

The escape velocity is $\propto R_s$ (assuming constant density). A comparison of these two numbers shows that collision velocities could be below the escape velocity exterior to the Laplace radius and for the largest moons. For smaller moons and at higher eccentricity we would be in the dispersion dominated regime (above escape velocity) and gravitational focusing would not be important.

5 SUMMARY

In this work, we study the moon formation in an extented circum-plamnetary disk which including both the regular region and the irrgular region. We have added the quadropole

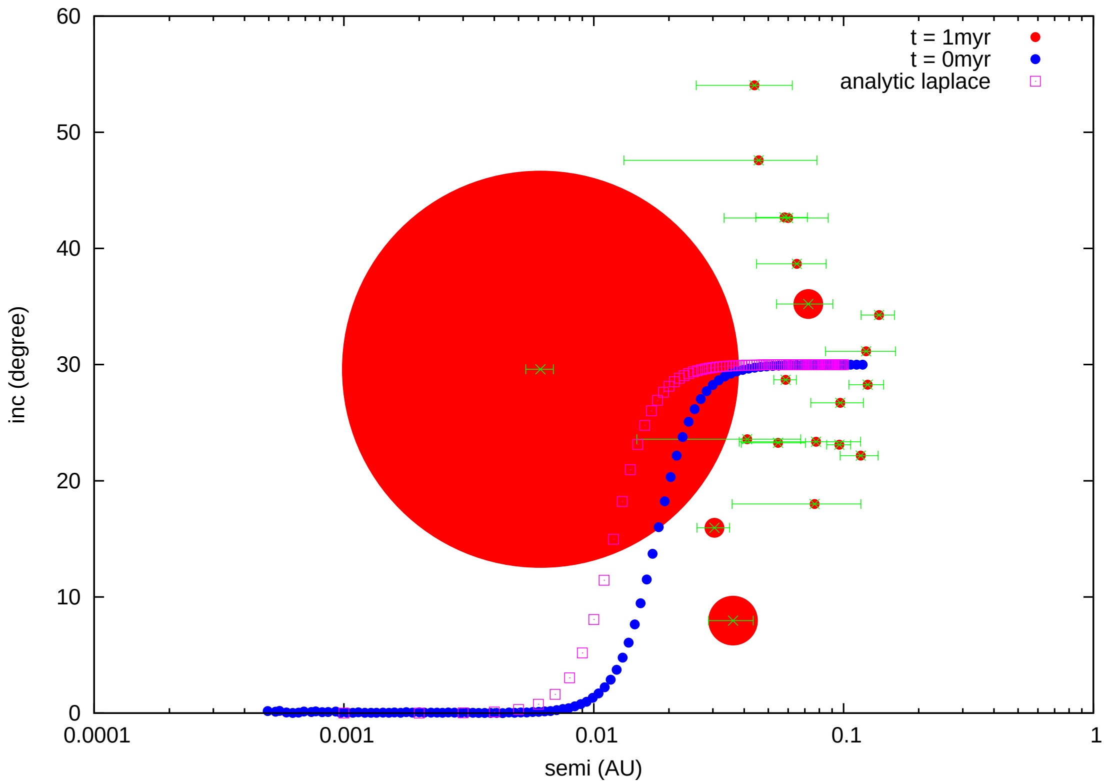

Figure 3. As same as Fig.1, but in the B1 series.

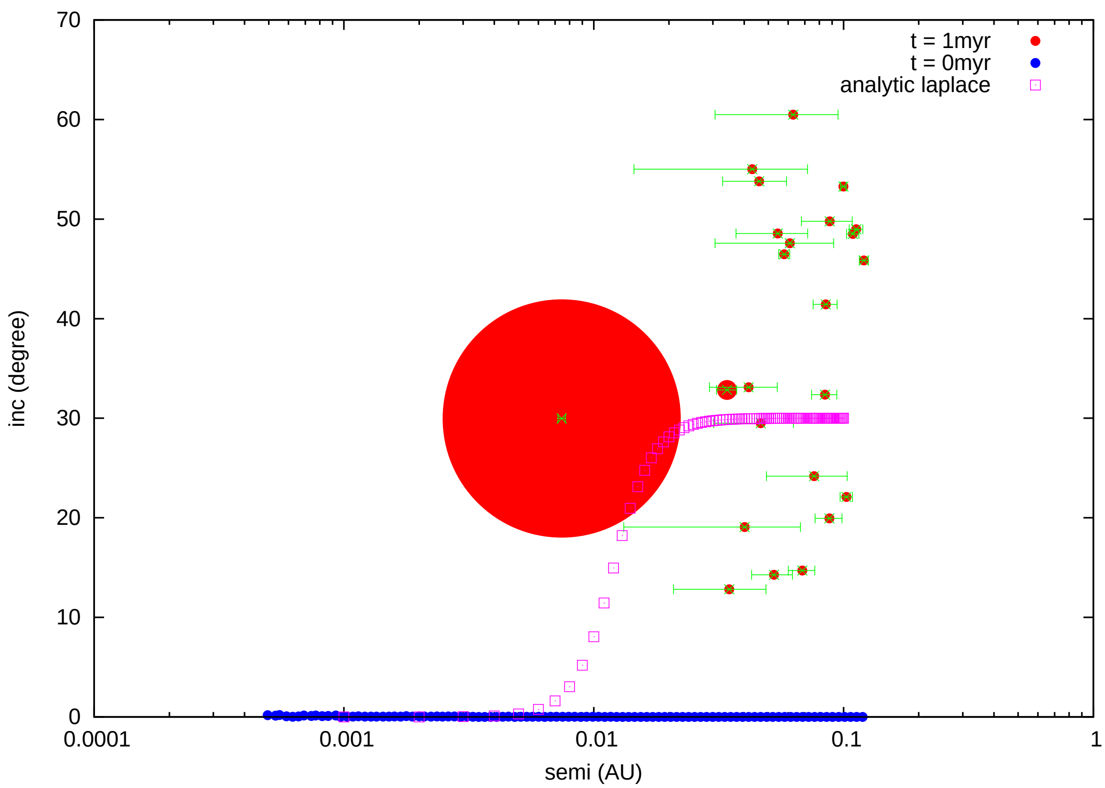

Figure 4. As same as Fig.1, but in the C1 series.

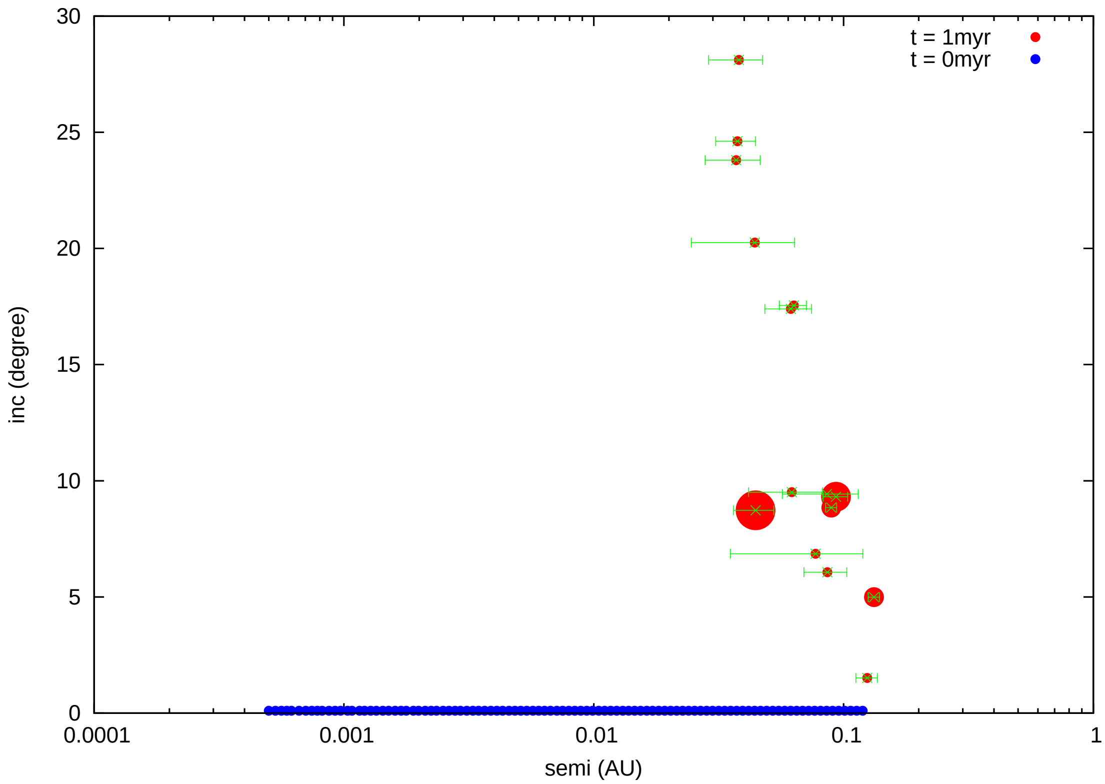

Figure 5. As same as Fig.1, but the planet is initially in zero obliquity.

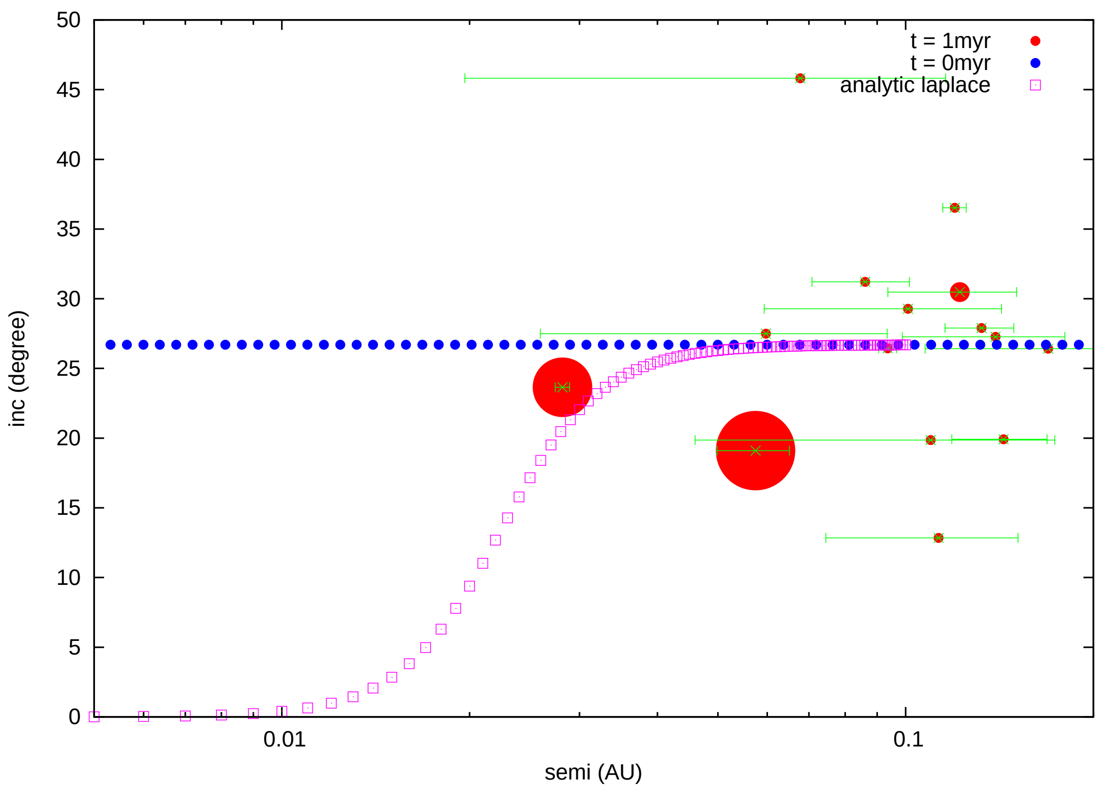

Figure 6. As same as Fig.1, but in the A3 series.

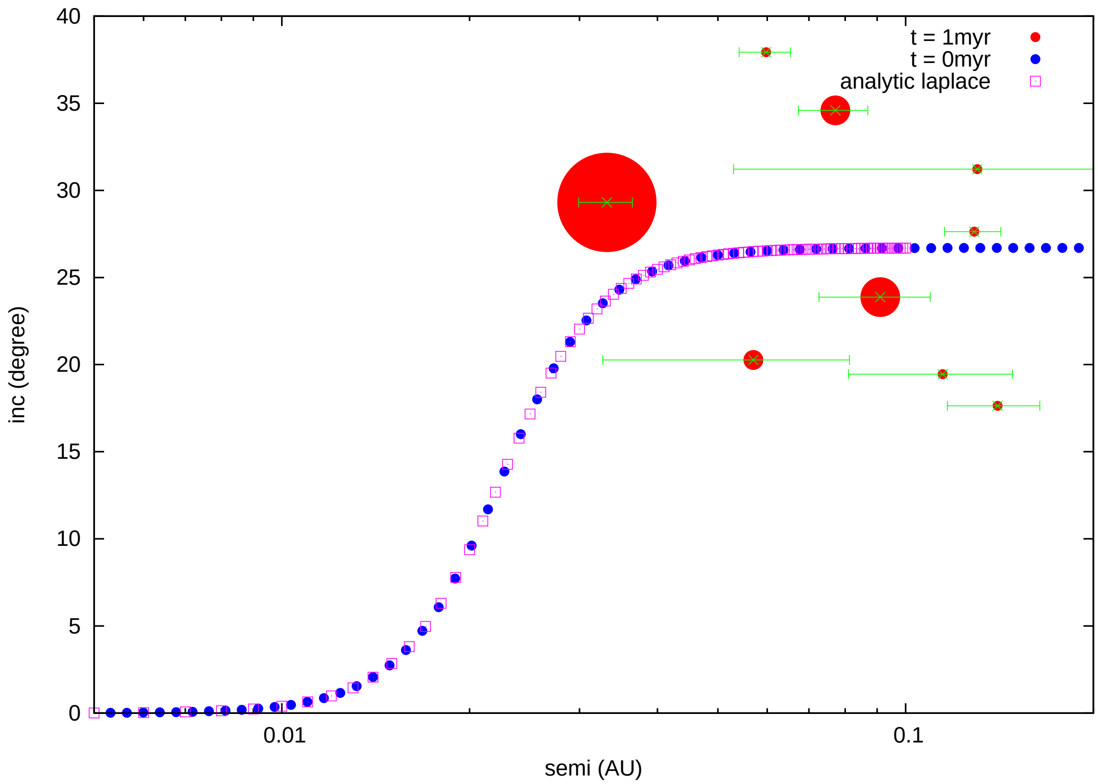

Figure 7. As same as Fig.1, but in the B3 series.

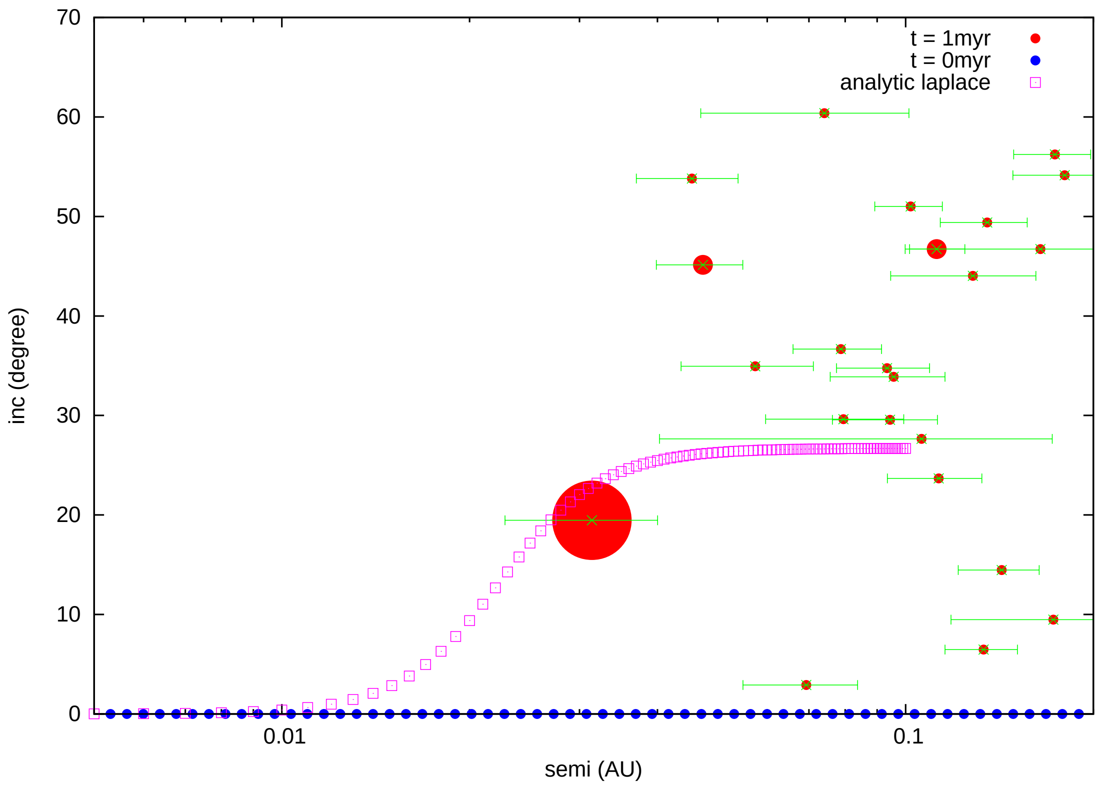

Figure 8. As same as Fig.1, but in the C3 series.

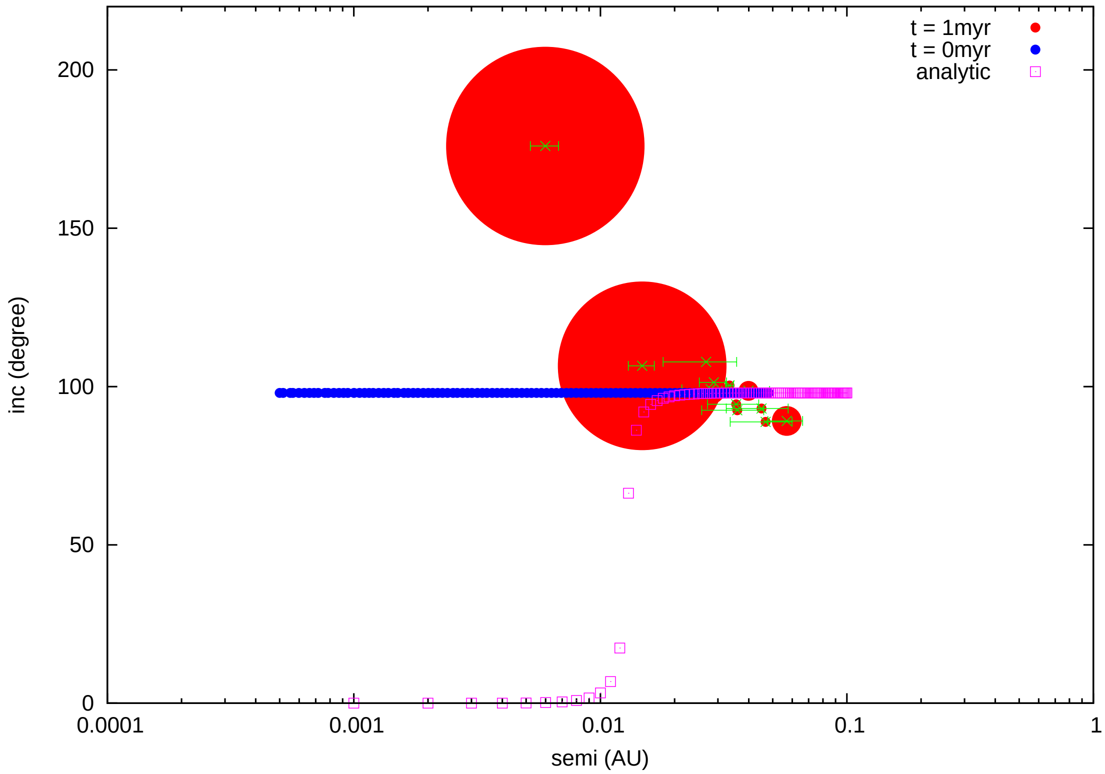

Figure 9. As same as Fig.1, but in the B2 series.

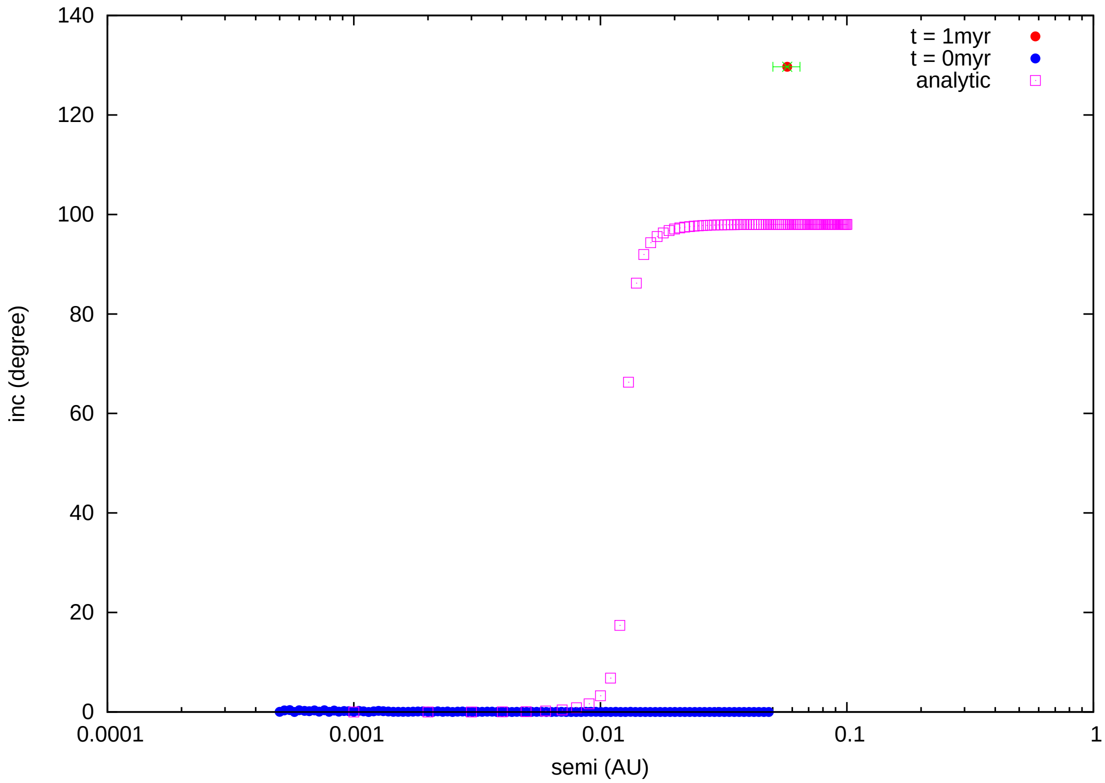

Figure 10. As same as Fig.1, but in the C2 series.

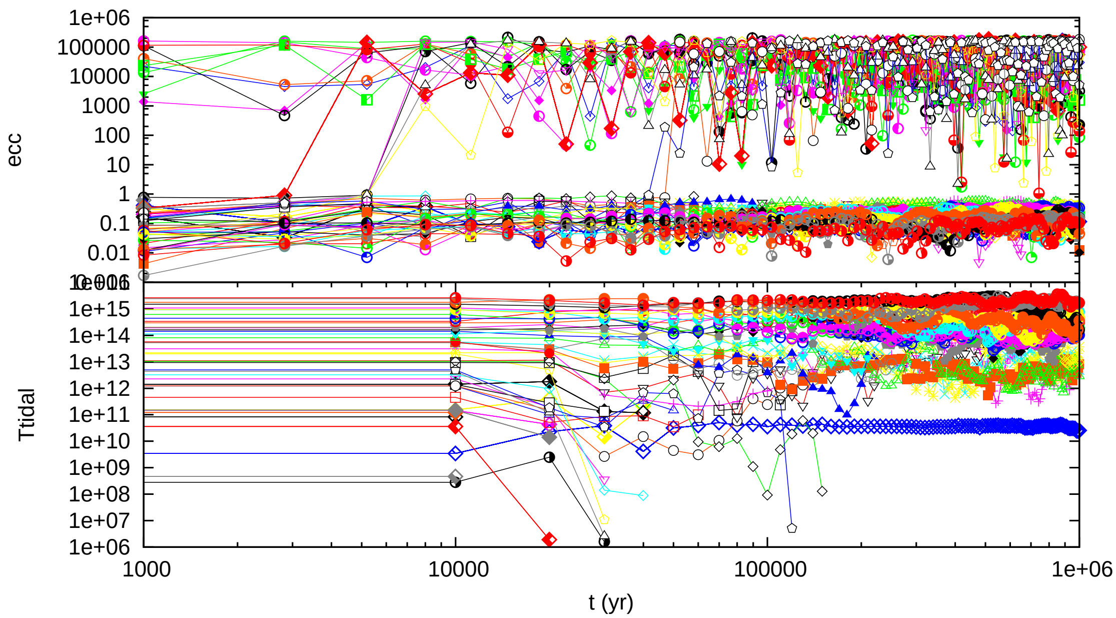

Figure 11. The eccentricity and tidal circularisation timescale evolution of planetesimals in A1 series, where the circularisation timescale is derived by equation8.

torque of host planet in previous Mercury code, and our simulation results certificant that planet obliquity affects moon formation.

Studies of terrestrial planet formation don't take into account a setting where eccentricities and inclinations can be increased by something like the Kozai-Lidov mechanism. This makes irregular satellite formation via collisions between planetesimals an interesting setting. A similar physical setting might occur with terrestrial planet formation in a triple star system. For example, a disk around a secondary star precesses due to the tidal force from a primary star, and if there is an additional star exterior to the binary system then there is an additional axis for precession (for example see EE-Cep models Galan et al. 2012).

Some of these simulations seem to show the formation of a massive object at r_L suggesting a formation model for a single giant moon, such as Titan. And we even obtain some retrograde moons in some cases indicate the possible scenario for Triton formation. However, there are still future work need to be done. Firstly, we need to understand the one big moon formation mechanism in detail, and why there is absence of moons inside laplace radius. In addtion, more N-body simulations are neccessary for the statistically study on the probability of forming moons on retrograde orbits like Triton.

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